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Existence of Coupled Solutions of BVP for ϕ-Laplacian Impulsive Differential Equations

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Abstract: In this paper, we study the existence of coupled solutions of anti-periodic boundary value problems for impulsive differential equations with ϕ-Laplacian operator. Based on a pair of coupled lower and upper solutions and appropriate Nagumo condition, we prove the existence of coupled solutions for anti-periodic impulsive differential equations boundary value problems with ϕ-Laplacian operator.

Keywords: Boundary Value Problems, Coupled Solutions, Impulsive Differential Equations, ϕ-Laplacian Operator

1. Introduction

In recent years, the study boundary value problems (BVPs for short) with p-Laplacian operator has been emerging as an important area and obtained a considerable attention. Since p-Laplacian operator appears in the study of flow through porous media ($p=3/2$), nonlinear elasticity ($p\geq 2$), glaciology ($1\leq p\leq 4/3$) and so on, there are many works about existence of solutions for differential equations with p-Laplacian operator [24, 25]. Usually, p-Laplacian operator is replaced by abstract and more general version ϕ-Laplacian operator, which lead to clearer expositions and a better understanding of the methods which ware employed to derive the existence results [12, 22, 23].

Moreover, impulsive differential equations have become an important aspect in some mathematical models of real processes and phenomena in science. There has a significant development in impulsive differential equations and impulse theory(see [2, 3, 14]). Moreover, p-Laplacian operator arises in turbulent filtration in porous media, non-Newtonian fluid flows and in many other application areas [10, 12].

Furthermore, the study of anti-periodic problem for nonlinear evolution equations is closely related to the study of periodic problem which was initiated by Okochi [17]. Anti-periodic problem which is a very important area of research has been extensively studied during the past decades, such as anti-periodic trigonometric polynomials [11] and anti-periodic wavelets [4]. Moreover, anti-periodic boundary conditions also appear in physics in a variety of situations (see [1, 13]) and difference and differential equations (see [6, 8, 19, 20]). The anti-periodic problem is a very important area of research.

In addition, we known that every T-anti-periodic solution gives rise to a $2T$-periodic solution if the nonlinearity f satisfy some symmetry condition. Indeed, the periodic and anti-periodic boundary value problems have attracted many researchers great interest (see [6, 8, 9, 15, 16, 19, 20, 21] and references therein). Recently, Guo and Gu [22] study a class of nonlinear impulsive differential equation with anti-periodic boundary condition:

$$(\phi(u'(t)))'=f(t,u(t),u'(t)) \quad a.e.\ t\in[0,T],P, \tag{1}$$

$$\begin{cases} I_k(u(t_k),u(t_k^+))=0, \\ M_k(u(t_k),u(t_k^+),u'(t_k),u'(t_k^+),u)=0, \end{cases} k=1,2,\cdots,p, \tag{2}$$

$$u(0)=-u(T),\ u'(0)=-u'(T), \tag{3}$$

where ϕ is an increasing homeomorphism from R to R, $f:[0,T]\times R^2\to R$ is a Carathéodory function. $P=\{t_1,\cdots,t_p:0=t_0<t_1<\cdots<t_p<t_{p+1}=T\}$, $I_k\in C^0(R^2)$, $M_k\in C^0(R^4\times C_P^1), k=1,\cdots,p$ are impulsive functions. C_P^1 will be given later. In [22], the authors obtained the existence of solution for anti-periodic boundary value problems (1)-(3)

for impulsive differential equations with ϕ -Laplacian operator. In this paper, we will continuous to consider the existence of coupled solutions for boundary value problems (1)-(3).

This paper is organized as follows: In section 2, we will state some preliminaries that will be used throughout the paper. In section 3, we will obtain the existence of coupled solutions for anti-periodic ϕ -Laplacian impulsive differential equations boundary value problems (1)-(3).

2. Preliminaries

In this section, we will introduce some definitions and preliminaries which are used throughout this paper.

For a given Banachspace E, let $C^0(E)$ be the set of all continuous functions $f: E \to R$. Let $C^m(I)$ be the set of functions u which are m times continuously differentiable on I with finite norm

$$\| u \|_{C^m(I)} = \max_{k=0,\cdots,m} \| u^{(k)} \|_\infty .$$

For $1 \le q \le \infty$, Let $L^q(I)$ be the set of Lebesgue measurable functions u on I such that $\| u \|_q$ is finite. $AC(I,q)$ denotes the set of absolutely continuous functions u on I satisfy $u' \in L^q(I)$. $W^{m,q}(I)$ denotes the set of functions $u \in C^{m-1}(I)$ and $u^{(m-1)} \in AC(I,q)$ with finite norm

$$\| u \|_{W^{m,q}(I)} = \max_{k=0,\cdots,m} \| u^{(k)} \|_q .$$

It is easy to see that $C^m(I)$ and $W^{m,q}(I)$ are Banach spaces and $W^{m,q}(I)$ is a usual Sobolev space.

Let $p \in N$. A finite subset P of the interval $[0,T]$ defined by

$$P = \{t_1, \cdots, t_p : 0 = t_0 < t_1 < \cdots < t_p < t_{p+1} = T\}.$$

Let $J_0 = [0, t_1]$ and $J_k = (t_k, t_{k+1}]$ for all $k = 1, \cdots, p$. For $m \in N \cup \{0\}$ and $1 \le q \le \infty$, we denote

$$C_P^m = \{u : [0,T] \to R : \ forall\ k = 0, \cdots, p, u \in C^m(J_k),\ there\ exist\ u^{(l)}(t_k^+),$$
$$k = 1, \cdots, p \ \ and\ u^{(l)}(t_k^-) = u^{(l)}(t_k), k = 1, \cdots, p+1; l = 0, \cdots, m\},$$
$$W_P^{m,q} = \{u : [0,T] \to R : u_{|J_k} \in W^{m,q}(J_k), k = 0, \cdots, p\}.$$

It is easy to verify that the spaces C_P^m and $W_P^{m,q}$ are Banach spaces with the norms

$$\| u \|_{C_P^m} = \max_{k=0,\cdots,p} \| u_{|J_k} \|_{C^m(J_k)} \quad and \quad \| u \|_{W_P^{m,q}} = \max_{k=0,\cdots,p} \| u_{|J_k} \|_{W^{m,q}(J_k)} .$$

We say that $f : [0,T] \times S \to R$ ($S \subset R^2$) satisfies the restricted Carathéodory conditions on $[0,T] \times S$ if

i. for each $x \in S$ the function $f(\cdot, x)$ is measurable on $[0,T]$;
ii. the function $f(t, \cdot)$ is continuous on S a.e. $t \in [0,T]$;
iii. for every compact set $K \subset S$, there exists a nonnegative function $\mu_K \in L^1(0,T)$ such that

$$| f(t,x) | \le \mu_K(t) \quad for\ \ a.e.\ t \in [0,T] \ \ and\ \ all\ \ x \in K.$$

In this paper, we use Car($[0,T] \times S$) to denote the set of functions satisfying the *restricted Carathéodory conditions*on $[0,T] \times S$. In what follows, $D^{\pm}$ and $D_{\pm}$ denote the Dini derivatives.

Definition 1. The functions $\alpha, \beta \in W_P^{1,\infty}$ such that $\alpha \le \beta$ are said to be a pair of coupled lower and upper solutions of problem (1)-(3) if α, β satisfy the following conditions:

(i) $D\_\alpha(t) \le D^+\alpha(t)$ for all $t \in [0,T], P$. Moreover, if $\tau \in [0,T], P$ such that $D\_\alpha(\tau) = D^+\alpha(\tau)$, then there exists $\varepsilon > 0$ such that

$$\alpha \in C^1([\tau - \varepsilon, \tau + \varepsilon]),$$

$$\varphi \circ \alpha' \in AC([\tau, \tau + \varepsilon])$$

and

$$(\varphi(\alpha'(t)))' \ge f(t, \alpha(t), \alpha'(t)) \quad a.e.\, t \in [\tau, \tau + \varepsilon].$$

(ii) $D\_\beta(t) \ge D^+\beta(t)$ for all $t \in [0,T], P$. Moreover, if $\tau \in [0,T], P$ such that $D\_\beta(\tau) = D^+\beta(\tau)$, then there exists $\varepsilon > 0$ such that

$$\beta \in C^1([\tau - \varepsilon, \tau + \varepsilon]),$$

$$\varphi \circ \beta' \in AC([\tau, \tau + \varepsilon])$$

and

$$(\varphi(\beta'(t)))' \le f(t, \beta(t), \beta'(t)) \quad a.e.\, t \in [\tau, \tau + \varepsilon].$$

(iii) For all $k = 1, \cdots, p$, $I_k(\alpha(t_k), \cdot)$ are injective and there exist $D^+\alpha(t_k)$, $D\_\alpha(t_k)$, $D_+\beta(t_k)$, $D^-\beta(t_k) \in R$ such that

$$I_k(\alpha(t_k), \alpha(t_k^+)) = 0 \le M_k(\alpha(t_k), \alpha(t_k^+), D\_\alpha(t_k), D^+\alpha(t_k), \alpha),$$
$$I_k(\beta(t_k), \beta(t_k^+)) = 0 \ge M_k(\beta(t_k), \beta(t_k^+), D^-\beta(t_k), D_+\beta(t_k), \beta).$$

and there exist $D^+\alpha(0)$, $D\_\alpha(T)$, $D_+\beta(0)$, $D^-\beta(T) \in R$ such that

$$\alpha(0) + \beta(T) = 0 \le D^+\alpha(0) + D^-\beta(T),$$
$$\alpha(T) + \beta(0) = 0 \ge D\_\alpha(T) + D_+\beta(0).$$

Definition 2. Given a function $u \in C_P^1$ is called a solution of the problem (1)-(3) if $\varphi \circ u' \in W_P^{1,1}$ and u satisfies (1) and fulfills conditions (2) and (3).

Definition 3. Assume that $f \in$ Car($[0,T] \times R^2$) and

$\alpha,\beta\in W_P^{1,\infty}$ satisfying $\alpha(t)\le\beta(t)$ for $\forall t\in[0,T]$. We say that f satisfies a Nagumo condition with respect to α and β if, for $k=1,\cdots,p$, there exist $\phi_k\in C[0,\infty)$ and $w\in L^q(0,T), 1\le q\le\infty$, such that $\phi_k>0$ on $[0,\infty)$,

$$|f(t,u,v)|\le w(t)\phi_k(|v|) \quad on \quad J_k\times[\alpha(t),\beta(t)]\times R.$$

Moreover, there exists a constant $K=K(\alpha,\beta)$ with $K>\max\{r_k,\|\alpha'\|_\infty,\|\beta'\|_\infty\}$, such that

$$\int_{\varphi(r_k)}^{\varphi(K)}\frac{(\varphi^{-1}(x))^{(q-1)/q}}{\phi_k(\varphi^{-1}(x))}dx>\|w\|_{J_k,q}\eta_k^{(q-1)/q}, \quad or$$
$$-\int_{\varphi(-K)}^{\varphi(-r_k)}\frac{(\varphi^{-1}(x))^{(q-1)/q}}{\phi_k(-\varphi^{-1}(x))}dx>\|w\|_{J_k,q}\eta_k^{(q-1)/q}, \tag{4}$$

where $\eta_k=\sup_{t\in J_k}\beta(t)-\inf_{t\in J_k}\alpha(t)$ and $r_k=\frac{1}{t_{k+1}-t_k}\max\{\beta(t_{k+1}^-)-\alpha(t_k^+),\ \beta(t_k^+)-\alpha(t_{k+1}^-)\}$. Any constant such $K>\max\{r_k:k=0,\cdots,p\}>0$ will be called a Nagumo constant.

Throughout this paper, we impose the following hypotheses:

(H_1)The function $\varphi:R\to R$ is a continuous and strictly increasing.

(H_2)The BVP (1)-(3) has a pair of coupled lower and upper solutions α and β.

(H_3) $f\in \mathrm{Car}([0,T]\times R^2)$ and satisfies a Nagumo condition with respect to α and β.

(H_4)The functions $I_k\in C^0(R^2)$ are non-decreasing in the first variable for $k=1,\cdots,p$, and the functions $M_k\in C^0(R^4\times C_P^1)$ are non-increasing in the third variable and non-decreasing in the fourth and fifth variables.

3. Existence Results of Coupled Solutions

This section is devoted to proving the existence of coupled solutions for anti-periodic impulsive differential equations boundary value problems with ϕ-Laplacian operator. Firstly, we state the following existence and uniqueness result.

Lemma 1.(*Lemma 7 of [23]*)Assume that $\tilde f\in L^1[0,T]$ and $A_k,B_k\in R$ for each $k=0,\cdots,p$. Suppose that $\bar\varphi:R\to R$ is a strictly increasing function satisfies $\bar\varphi(R)=R$. Then the non homogeneous impulsive Dirichlet problem

$$\begin{cases}(\bar\varphi(u'(t)))'=\tilde f(t) \quad a.e.\, t\in[0,T],\\ u(t_k)=B_{k-1},\, u(t_k^+)=A_k,\ k=1,2,\cdots,p,\\ u(0)=A_0,\ u(T)=B_p,\end{cases}$$

has a unique solution u, which can be written in the form

$$u(t)=A_k+\int_{t_k}^t\bar\varphi^{-1}\Big(\int_{t_k}^z\tilde f(s)ds+\tau_k\Big)dz,\ t\in J_k,\ k=0,\cdots,p,$$

where τ_k is the unique solution of the equation

$$B_k-A_k=\int_{t_k}^{t_{k+1}}\bar\varphi^{-1}\Big(\int_{t_k}^z\tilde f(s)ds+\tau_k\Big)dz.$$

Next, let us consider the following functions

$$\delta_K(y)=\min\{K,\max\{y,-K\}\} \quad for\ all\ \ y\in R,$$

where K is the constant introduced in definition 2.3,

$$\rho(t,u)=\min\{\beta(t),\max\{u,\alpha(t)\}\} \quad for\ \ (t,u)\in[0,T]\times R,$$

coupled with functionals $A_k,B_k:C_P^1\to R$ given by

$$A_0(u)=\rho(0,-u(T)),$$
$$B_p(u)=\rho(T,u(T)-u'(0)-u'(T)),$$
$$A_k(u)=\rho(t_k^+,u(t_k^+)+I_k(u(t_k),u(t_k^+))),\ \ k=1,\cdots,p,$$
$$B_{k-1}(u)=\rho(t_k,u(t_k)+M_k(u(t_k),u(t_k^+),u'(t_k),u'(t_k^+),u)),\ \ k=1,\cdots,p.$$

Moreover, for each $u\in C_P^1$ we consider a function $\tilde f_u:[0,T]\to R$ defined by

$$\tilde f_u(t)=f(t,\rho(t,u(t)),\delta_K(\frac{d}{dt}\rho(t,u(t)))).$$

The function $\tilde f_u$ is well defined according to the following result (by redefining function $\frac{d}{dt}\rho(t,u(t))$ as zero when it does not exist). It can be proved in a similar way to Lemma 2 in [24].

Lemma 2. For given $u,u_n\in C_P^1$ such that $u_n\to u$ in C_P^1, then

(i) $\frac{d}{dt}\rho(t,u(t))$ exists for a.e. $t\in[0,T],P$;

(ii) $\frac{d}{dt}\rho(t,u_n(t))\to\frac{d}{dt}\rho(t,u(t))$ for a.e. $t\in[0,T],P$.

Now, we can define a strictly increasing homeomorphism $\bar\varphi:R\to R$ by:

$$x\in R\to\bar\varphi(x)=\begin{cases}\varphi(x), & |x|\le K,\\ \frac{\varphi(K)-\varphi(-K)}{2K}x-\frac{1}{2}(\varphi(K)+\varphi(-K)), & |x|>K.\end{cases}$$

In the following, we are in a position to prove the existence theorem for our considering problems.

Lemma 3. (*Theorem 3.3 of [22]*)Assume that (H_1)-(H_4) hold. Then there exists at least one solution u of the problem (1)-(3) such that

$$\alpha(t)\le u(t)\le\beta(t)$$

and

$$|u'(t)|\le K,\ t\in[0,T],$$

where $K = K(\alpha,\beta)$ is the constant introduced in Definition 2.3.

Next, we are devoted to the existence of coupled solutions. We first introduce the following definition.

Definition 4. The functions x, y are called coupled solutions of problems (1)-(3) if $x, y \in C_P^1$ and satisfy (1)-(2) and

$$x(0) = -y(T), \tag{5}$$

$$x'(0) = -y'(T), \tag{6}$$

$$y(0) = -x(T), \tag{7}$$

$$y'(0) = -x'(T). \tag{8}$$

Remark If the coupled solutions x and y of problem (1)-(3) satisfy $x = y$, the $x = y$ is a solution of problem (1)-(3).

Next, we give the existence of coupled solutions for problems (1)-(3).

Theorem 5. Assume hypotheses (H_1)-(H_4) hold. Then there exists at least a pair of coupled solutions $x, y \in C_P^1$ of the impulsive differential equations boundary value problem (1)-(3) such that

$$x, y \in [\alpha,\beta] = \{u : \alpha(t) \le u(t) \le \beta(t), t \in [0,T]\}, \tag{9}$$

and

$$|x'(t)| \le K \quad for \quad t \in [0,T],$$

$$|y'(t)| \le K \quad for \quad t \in [0,T],$$

where $K = K(\alpha,\beta)$ is the constant introduced in Definition 2.3.

Proof. Let us define ρ, A_k, B_{k-1} for each $k = 1,\cdots,p$ in the same way as above, and construct a modified problem (P^*) similar to the proof of Lemma 3, that is

$$\begin{cases} (\bar{\varphi}(x'(t)))' = \tilde{f}_x(t), & a.e.\ t \in [0,T], P, \\ (\bar{\varphi}(y'(t)))' = \tilde{f}_y(t), & a.e.\ t \in [0,T], P, \\ x(t_k) = B_{k-1}(x), \quad y(t_k) = B_{k-1}(y), & k = 1,2,\cdots,p, \\ x(t_k^+) = A_k(x), \quad y(t_k^+) = A_k(y), & k = 1,2,\cdots,p, \\ x(0) = A_0(x), \quad y(0) = A_0(y), & \\ x(T) = B_p(x), \quad y(T) = B_p(y), & \end{cases}$$

where

$$A_0(x) = \rho(0, -y(T)),$$

$$B_p(x) = \rho(T, x(T) - y'(0) - x'(T)),$$

$$A_0(y) = \rho(0, -x(T)),$$

$$B_p(y) = \rho(T, y(T) - x'(0) - y'(T)).$$

From the proof of the Lemma 3, there exists a couple of solutions $x, y \in C_P^1$ such that

$$\alpha \le x \le \beta,$$

$$\alpha \le y \le \beta,$$

and

$$|x'(t)| \le K, \ |y'(t)| \le K \quad for \quad t \in [0,T].$$

Furthermore, x, y satisfy the condition (2). Now, to prove that (5)-(8) is verified, it suffices to prove that

$$\alpha(0) \le -y(T) \le \beta(0), \tag{10}$$

$$\alpha(0) \le -x(T) \le \beta(0), \tag{11}$$

$$\alpha(T) \le x(T) - y'(0) - x'(T) \le \beta(T), \tag{12}$$

$$\alpha(T) \le y(T) - x'(0) - y'(T) \le \beta(T). \tag{13}$$

Firstly, we will prove (10), by contradiction, if $\alpha(0) > -y(T)$, then by $\alpha \le y \le \beta$, we have

$$\alpha(0) > -y(T) \ge -\beta(T),$$

which contradict to $\alpha(0) + \beta(T) = 0$. Moreover, $-y(T) \le \beta(0)$ can be proved similarly.

As the same way, we can obtain that the inequality (10) is holds. Thus we have

$$x(0) = -y(T), \ y(0) = -x(T). \tag{14}$$

Assume that the first inequality if (11) isn't holds, as a consequence, we have

$$x(T) = \alpha(T)$$

and

$$y'(0) + x'(T) > 0.$$

From (14) and $\alpha(T) + \beta(0) = 0$, we have

$$y(0) = -x(T) = -\alpha(T) = \beta(0).$$

From these facts and the relation $\alpha \le x, y \le \beta$, we have

$$x'(T) \le D_{-}\alpha(T), \ y'(0) \le D_{+}\beta(0),$$

thus

$$0 < y'(0) + x'(T) \le D_{+}\beta(0) + D_{-}\alpha(T) \le 0.$$

It is a contradiction. Moreover, the inequality in (13) be obtain in a similar way. Hence inequalities (11)-(12) are hold, that is to say x, y satisfy (5)-(8).

Therefore, the functions x, y is a coupled solutions of the problem (1)-(3), which completes the proof.

4. Conclusion

In this paper, we mainly discuss the existence of coupled solutions of anti-periodic boundary value problems for impulsive differential equations with ϕ-Laplacian operator. To give the existence results of coupled solutions for the problem (1)-(3), we first introduce a pair of coupled lower and upper solutions (see Definition 1), Then, we provide and prove the existence results of coupled solutions for anti-periodic ϕ-Laplacian impulsive differential equations boundary value problems based on a pair of coupled lower and upper solutions and appropriate Nagumo condition (Theorem 5).

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