

# Analysis on Multichannel Filter Banks-Based Tree-Structured Design for Communication System

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**Abstract:** This research aims to design and implement of tree-structured multichannel filter banks using MATLAB. The multichannel filter banks analysis are evaluated by the Digital Signal Processing (DSP) techniques. The multi rate analysis is suitable for sampling rate reduction and sampling rate increase on the digital filter design. When increasing sampling rate, filtering follows the up-sampling operation. The role of the filter is to attenuate unwanted periodic spectra which appear in the new baseband. The performance evaluation for tree-structured multichannel filter banks design is described in this research work. The experimental results for implemented design are implemented in this paper. The use of an appropriate filter enables one to convert a digital signal of a specified sampling rate into another signal with a target sampling rate without destroying the signal components of interest. The performance of multirate filtering for implemented design is evaluated by using MATLAB.

**Keywords:** DSP, Tree-Structured Multichannel Filter Banks, MATLAB, Digital Filter Design, Multirate Techniques

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## 1. Introduction

Multirate signal processing techniques are widely used in many areas of modern engineering such as communications, image processing, digital audio, and multimedia. The main advantage of a multirate system is the substantial decrease of computational complexity, and consequently, the cost reduction. The computational efficiency of multirate algorithms is based on the ability to use simultaneously different sampling rates in the different parts of the system. The sampling rate alterations generate the unwanted effects through the system: spectral aliasing in the sampling rate decrease, and spectral images in the sampling rate increase. As a consequence, the multirate processing might produce unacceptable derogations in the digital signal. The crucial role of multirate filtering is to enable the sampling rate conversion of the digital signal without significantly destroying the signal components of interest. The multirate filtering makes the general concept of multirate signal processing applicable in practice. [1-10]

For multirate filters, FIR (finite impulse response) or IIR

(infinite impulse response) transfer functions can be used for generating the overall system. The selection of the filter type depends on the criteria at hand. An FIR filter easily achieves a strictly linear-phase response, but requires a larger number of operations per output sample when compared with an equal magnitude response IIR filter. The linear-phase FIR filter is an adequate choice when the waveform of the signal has to be preserved. An advantage of the multirate design approach is the ability of improving significantly the efficiency of FIR filters thus making them very desirable in practice [11-15].

## 2. Two-Channel Filter Banks

The block diagram representing the analysis/synpaper two-channel filter bank with the processing unit between the analysis and synpaper parts is shown in Figure 2. In the analysis bank, the original signal  $x[n]$  is filtered using the lowpass/highpass filter pair  $[H_0(z), H_1(z)]$ , and the lowpass and highpass channel signals  $x_0[n]$  and  $x_1[n]$  are obtained. Therefore, their z-transforms  $X_0(z)$  and  $X_1(z)$  are given by

$$X_0(z)=H_0(z)X(z), \text{ and } X_1(z)=H_1(z)X(z) \quad (1)$$

Figure 3 illustrates typical magnitude frequency responses of  $H_0(z)$  and  $H_1(z)$ . The spectra of the filtered signals  $x_0[n]$  and  $x_1[n]$  occupy a half of the baseband of the original signal  $x[n]$ , and according to this,  $x_0[n]$  and  $x_1[n]$  can be further processed at the half of the input sampling rate. The filtered signals  $x_0[n]$  and  $x_1[n]$  are then down-sampled by-a-factor-of-two, and subband signals  $v_0[n]$  and  $v_1[n]$  are obtained. If the sampling rate at the input is  $F_0$ , the subband signal components  $v_0[n]$  and  $v_1[n]$  are sampled at the rate  $F_0/2$ .

Since  $v_0[n]$  and  $v_1[n]$  are the down-sampled versions of  $x_0[n]$  and  $x_1[n]$ , the z-transforms  $V_0(z)$  and  $V_1(z)$  are expressible in terms of the z-transforms  $X_0(z)$  and  $X_1(z)$ . It can be simply applied equation of  $T(z)$  to the z-transforms  $X_0(z)$  and  $X_1(z)$  with the down-sampling factor  $M = 2$ . Additionally, by introducing relations (3.1), it can be expressed  $V_1(z)$  and  $V_2(z)$  in terms of the z-transform of the input signal  $X(z)$ ,

$$V_0(z) = \frac{1}{2} [H_0(z^{1/2})X(z^{1/2}) + H_0(-z^{1/2})X(-z^{1/2})] \quad (2)$$

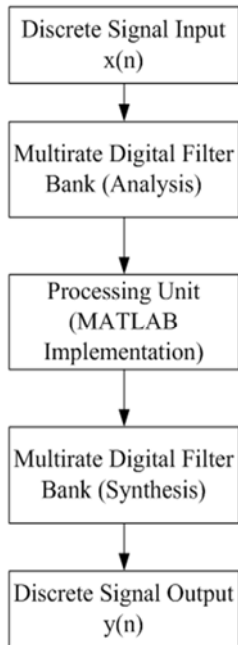


Figure 1. Proposed System Block Diagram.

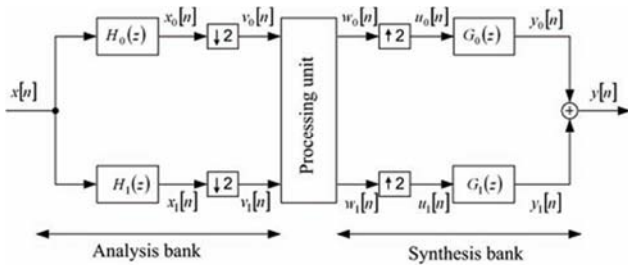


Figure 2. The two-channel analysis/synpaper filter bank.

The first terms in above equations represent the z-transforms of the desired decimated signal components, whereas the second terms represent the aliasing components that overlap in the basebands of the decimated signals. The signals  $v_0[n]$  and  $v_1[n]$  are processed in the processing unit,

usually coded and compressed. Before inputting to the synpaper bank, signals are usually decoded and decompressed. The coding and quantization errors in the processing unit may cause derogation of the signals. In that case the resulting signals  $w_0[n]$  and  $w_1[n]$  differ from the original signals  $v_0[n]$  and  $v_1[n]$  [16-20].

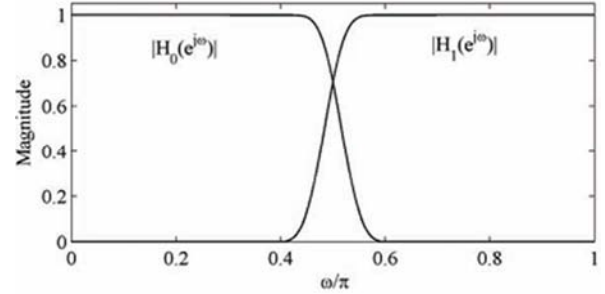


Figure 3. Typical magnitude responses of the lowpass filter  $H_0(z)$ , and the highpass filter  $H_1(z)$ .

$$V_1(z) = \frac{1}{2} [H_1(z^{1/2})X(z^{1/2}) + H_1(-z^{1/2})X(-z^{1/2})] \quad (3)$$

In order to examine the performances of the analysis/synpaper bank, the errors that may be produced in the processing unit are neglected. Hence, the future considerations of the analysis/synpaper filter bank in this section are evaluated under the assumption that

$$w_0[n] = v_0[n], \text{ and } w_1[n] = v_1[n] \quad (4)$$

In the synpaper bank, the two signal components are up-sampled-by-two first, then filtered by  $G_0(z)$  and  $G_1(z)$ , and finally added together to compose the output signal  $y[n]$ . The z-transforms of the up-sampled signals  $u_0[n]$  and  $u_1[n]$  follow directly from equation  $T(z)$  when applied for the up-sampling factor  $L = 2$ . With the assumption given in (12.3), i.e.,  $w_0[n] = v_0[n]$  and  $w_1[n] = v_1[n]$ , the z-transforms  $U_0(z)$  and  $U_1(z)$  become expressible in terms of  $X_0(z)$  and  $X_1(z)$ ,

$$U_0(z) = V_0(z^2) = \frac{1}{2} [X_0(z) + X_0(-z)] \quad (5)$$

$$U_1(z) = V_1(z^2) = \frac{1}{2} [X_1(z) + X_1(-z)] \quad (6)$$

With the up-sampling operation, the sampling rate increases from  $F_0/2$  to  $F_0$ . In above equations, the terms  $X_0(z)$  and  $X_1(z)$  represent the desired signal component without aliasing, and the terms  $X_0(-z)$  and  $X_1(-z)$  represent the unwanted aliased signal components. In the second step, signals  $u_0[n]$  and  $u_1[n]$  are processed by the lowpass/highpass synpaper filter pair  $G_0(z)$  and  $G_1(z)$ . Thereby, the z-transforms  $Y_0(z)$  and  $Y_1(z)$  of the output signals  $y_0[n]$  and  $y_1[n]$  are given by

$$Y_0(z) = G_0(z)U_0(z) = \frac{1}{2} [G_0(z)X_0(z) + G_0(z)X_0(-z)] \quad (7)$$

$$Y_1(z) = G_1(z)U_1(z) = \frac{1}{2} [G_1(z)X_1(z) + G_1(z)X_1(-z)] \quad (8)$$

Finally, in the third step the filtered signals  $y_0[n]$  and  $y_1[n]$  are added together to yield the output  $y[n]$ . Accordingly, the z-transform  $Y(z)$  is the sum of  $Y_0(z)$  and  $Y_1(z)$ ,

$$Y(z) = Y_0(z) + Y_1(z) \quad (9) \quad \text{component } X(-z)$$

Introducing the simplification, it can be arrived to the expression, which relates the  $z$ -transform of the output  $Y(z)$  with the  $z$ -transform of the input  $X(z)$  and with its aliased

$$[H_0(z)G_0(z) + H_1(z)G_1(z)]X(z) + \frac{1}{2}[H_0(-z)G_0(-z) + H_1(-z)G_1(-z)]X(-z) \quad (10)$$

The first term in above equation represents the input/output relation of the overall analysis synpaper filter bank without aliasing and imaging effects. The second term represents the effects of aliasing and imaging. This second term has to be eliminated by the proper combination of the transfer functions  $H_0(z)$ ,  $H_1(z)$ ,  $G_0(z)$  and  $G_1(z)$ . Conditions for the alias-free filter bank are considered in the next subsection.

### 3. Tree Structure Filter Banks

An approach for constructing a multichannel filter bank is based on a tree-structure, which uses the two-channel filter banks as building blocks. With the use of this approach, the multichannel filter banks with uniform and no uniform separation between the channels can be generated. If the two-channel filter banks satisfy the perfect-reconstruction (nearly perfect-reconstruction) property, the overall tree-structure filter bank also satisfies the perfect-reconstruction (nearly perfect-reconstruction) property. In this section, it could be shown examples of uniform and no uniform filter banks built on the basis of the two-channel filter banks. For the sake of simplicity, it could be used in this section the symbolic representations of the analysis and synpaper two-channel filter banks as shown in Figure 4. The single-input/two-output device  $A^{(k)}(z)$  symbolizes a two-channel analysis filter bank, and two-input/single-output device  $S^{(k)}(z)$  symbolizes a two-channel synpaper bank.

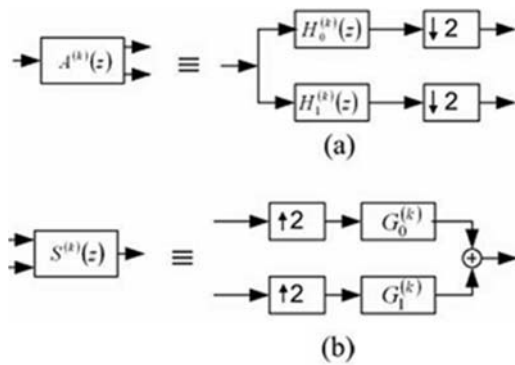


Figure 4. Symbolic representation of two-channel filter bank: (a) Analysis bank. (b) Synpaper bank.

### 4. Design of Tree-Structure Filter

A multichannel analysis/synpaper filter bank can be developed by iterating a two-channel QMF bank. Moreover, if the two-band QMF bank is of the perfect reconstruction type, the generated multiband structure also exhibits the perfect

reconstruction property.

By inserting a two-channel maximally decimated QMF bank in each channel of another two-channel maximally decimated QMF bank between the down-sampler and the up-sampler, it can be generated a four-channel maximally decimated QMF bank, as shown in Figure 5. Since the analysis and the synpaper filter banks are formed like a tree, the overall system is usually called a tree-structured filter bank. It should be noted that in the four-channel tree-structured filter bank of Figure 5, the two-channel QMF banks in the second level do not have to be identical.

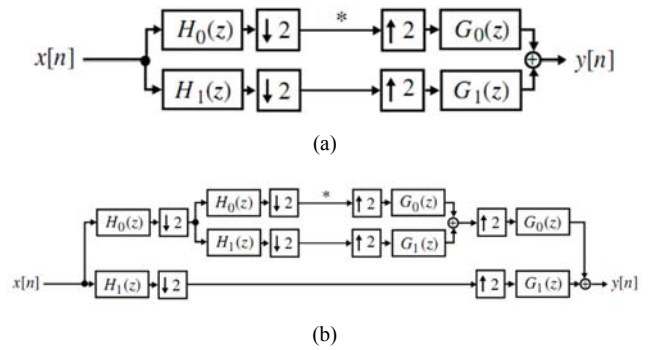


Figure 5. (a) A two-channel QMF bank, (b) a three-channel QMF bank derived from the two-channel QMF bank.

However, if they are different QMF banks with different analysis and synpaper filters, to compensate for the unequal gains and unequal delays of the two-channel systems, additional delays of appropriate values need to be inserted at the middle to ensure perfect reconstruction of the overall four-channel system. An equivalent representation of the four-channel QMF system of Figure 5(a) is shown in Figure 5(b).

The analysis and synpaper filters in the equivalent representation are related to those of the parent two-level tree-structured filter bank as follows:

$$H_{00}(z) = H_0(z)H_0(z^2), H_{01}(z) = H_0(z)H_1(z^2) \quad (11)$$

Because of the unequal passband widths of the analysis and synpaper filters, these structures belong to the class of nonuniform QMF banks. The tree-structured filter banks are also referred to as decade band QMF banks. Various other types of nonuniform filter banks can be generated by iterating branches of a parent uniform two-channel QMF in different forms. Nonuniform filter banks have been used in speech and image coding applications.

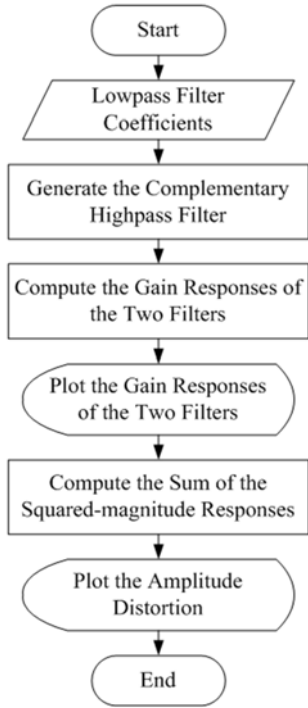


Figure 6. Flowchart of Tree-structure Filter Design Analysis.

## 5. Analysis of Multirate Filters

The analysis of single-stage multirate filters is performed at the rate the filter is operating. For decimators, the rate is equal to the input rate. For interpolators, the rate is equal to the output rate. For sample-rate converters, the rate of the filter is equal to the input rate multiplied by the interpolation factor. If a sampling frequency is given, it's assumed that it's the sampling frequency at which the filter is operating. The following plot overlays the magnitude response of a sample-rate converter, an interpolator, and a decimator. For the first filter, the input sampling frequency is 1000/5 and the output sampling frequency is 1000/3. For the interpolator, the input  $F_s$  is 1000/4 and the output  $F_s$  is 1000. Finally, for the decimator, the input  $F_s$  is 1000 and the output  $F_s$  is 1000/3.

Analysis of multistage filters is possible for multistage filters of the following form. Any of the blue, red, or green sections is optional. So it can be performed analysis on a multistage interpolator, a multistage decimator, or a multistage sample-rate converter. In performing the analysis, an equivalent overall filter is computed for the interpolation section and/or the decimation section as follows:

Where  $L = L_0 * L_1 * L_2 * \dots * L_m$ ;  $M = M_0 * M_1 * M_2 * \dots * M_n$ ;  
 $H(z) = H_1(z^{L_0 * L_1 * \dots * L_m}) * H_2(z^{L_0 * L_2 * \dots * L_m}) * \dots * H_m(z^{L_0})$ ; and  
 $G(z) = G_1(z^{M_0 * M_1 * \dots * M_n}) * G_2(z^{M_0 * M_2 * \dots * M_n}) * \dots * G_n(z^{M_0})$

Finally, the filters  $H(z)$ ,  $G(z)$ , and  $H_0(z)$  are all operating at the same rate and can be combined into a single filter on which the analysis is performed. If a sampling frequency is specified, it is assumed that this single overall filter is operating at that rate.

The analysis of a multistage interpolator is presented. It would be cascaded four interpolators to form a four-stage filter. The last interpolator will be a CIC filter. In this case, the sampling frequency specified corresponds to the output of the four-stage interpolator because this is the rate at which the equivalent filter is operating. It should be added some decimation stages to form a multistage sample-rate converter. The sampling frequency specified once again corresponds to the rate of the equivalent filter. This is the fastest rate in the entire system in this case. In addition to the multistage filter shown, analysis of a multistage filter where decimation occurs prior to interpolation is possible provided the overall interpolation and decimation factors are the same. Notice that this does not necessarily mean that there is an equal number of decimation and interpolation stages. Because the overall interpolation factor  $L$  is equal to the overall decimation factor  $M$ , both equivalent filters are operating at the same rate. If a sampling frequency is specified, it is assumed to be the rate at which both filters are operating. This would also be equal to the input and output rate for this case.

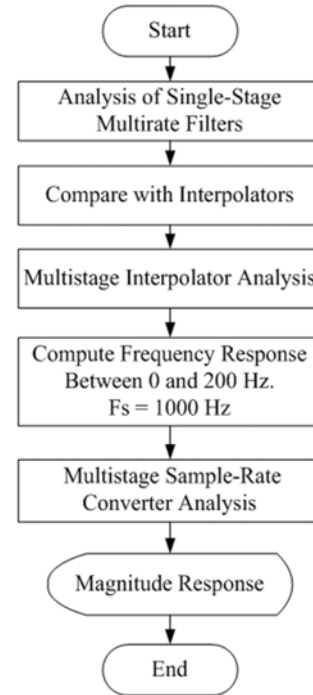


Figure 7. Flowchart of Multirate Analysis.

## 6. Design of Decimators/Interpolators

Typically lowpass filters are used for decimation and for interpolation. When decimating, lowpass filters are used to reduce the bandwidth of a signal prior to reducing the sampling rate. This is done to minimize aliasing due to the reduction in the sampling rate. When interpolating, lowpass filters are used to remove spectral images from the low-rate signal.

### 6.1. Design of Decimators

The specifications for the filter determine that a transition band of 2 Hz is acceptable between 23 and 25 Hz and that the

minimum attenuation for out of band components is 80 dB. Also that the maximum distortion for the components of interest is 0.05 dB (half the peak-to-peak passband ripple). An equiripple filter that meets these specs can be easily obtained as follows:

```
M = 4; % Decimation factor
Fp = 23; % Passband-edge frequency
Fst = 25; % Stopband-edge frequency
Ap = 0.1; % Passband peak-to-peak ripple
Ast = 80; % Minimum stopband attenuation
Fs = 200; % Sampling frequency
Hf = fdesign. Decimator (M,'lowpass', Fp, Fst, Ap, Ast, Fs)
Hm = design (Hf,'equiripple');
hfv = fvtool (Hm,'DesignMask','on','Color','White');
measure (Hm)
```

It is clear from the measurements that the design meets the specs. It may be preferable to use a multistage approach to obtain a more efficient design that meets the specs.

Nyquist filters are attractive for decimation and interpolation due to the fact that a  $1/M$  fraction of the number of coefficients is zero. The band of the Nyquist filter is typically set to be equal to the decimation factor, this centers the cutoff frequency at  $1/M \cdot F_s/2$ . The transition band is centered around  $1/4 \cdot 100 = 25$  Hz. A Kaiser Window design can be obtained in a straightforward manner. A more efficient design can be obtained through multistage techniques which results in two halfband filters cascaded.

It can be supposed the signal to be filtered has a flat spectrum. Once filtered, it acquires the spectral shape of the filter. After reducing the sampling rate, this spectrum is repeated with replicas centered on multiples of the new lower sampling frequency. An illustration of the spectrum of the decimated signal can be found from:

```
NFFT = 4096;
[H, f] = freqz (Hm (1), NFFT, 'whole', Fs);
```

Note that the replicas overlap somewhat, so aliasing is introduced. However, the aliasing only occurs in the transition band. That is, significant energy (above the prescribed 80 dB) from the first replica only aliases into the baseband between 24 and 25 Hz. Since the filter was transitioning in this region anyway, the signal has been distorted in that band and aliasing there is not important. On the other hand, notice that although it has been used the same transition width as with the lowpass design from above, it is actually retained a wider usable band (24 Hz rather than 23) when comparing this Nyquist design with the original lowpass design. To illustrate this, let's follow the same procedure to plot the spectrum of the decimated signal when the lowpass design from above is used.

```
[H, f] = freqz (Hm (1), NFFT, 'whole', Fs);
```

In this case, there is no significant overlap (above 80 dB) between replicas, however because the transition region started at 23 Hz, the resulting decimated signal has a smaller usable bandwidth.

## 6.2. Design of Interpolator

When interpolating a signal, the baseband response of the signal should be left as unaltered as possible. Interpolation is obtained by removing spectral replicas when the sampling rate

is increased.

It can be supposed a signal sampled at 48 Hz. If it is critically sampled, there is significant energy in the signal up to 24 Hz. If it could be wanted to interpolate by a factor of 4, it would be ideally designed a lowpass filter running at 192 Hz with a cutoff at 24 Hz. As with decimation, in practice an acceptable transition width needs to be incorporated into the design of the lowpass filter used for interpolation along with passband ripple and finite stopband attenuation.

```
L = 4; % Interpolation factor
Fp = 22; % Passband-edge frequency
Fst = 24; % Stopband-edge frequency
Ap = 0.1; % Passband peak-to-peak ripple
Ast = 80; % Minimum stopband attenuation
Fs = 192; % Sampling frequency
Hf = fdesign. Interpolator (L,'lowpass', Fp, Fst, Ap, Ast, Fs)
An equiripple design that meets the specs can be found in
the same manner as with decimators
Hm = design (Hf,'equiripple');
```

Notice that the filter has a gain of 12 dB which corresponds to a gain of 4 in linear units. In general interpolators will have a gain equal to the interpolation factor. This is needed for the signal being interpolated to maintain the same range after interpolation. The flowchart of decimator/interpolator is illustrated in Figure 8.

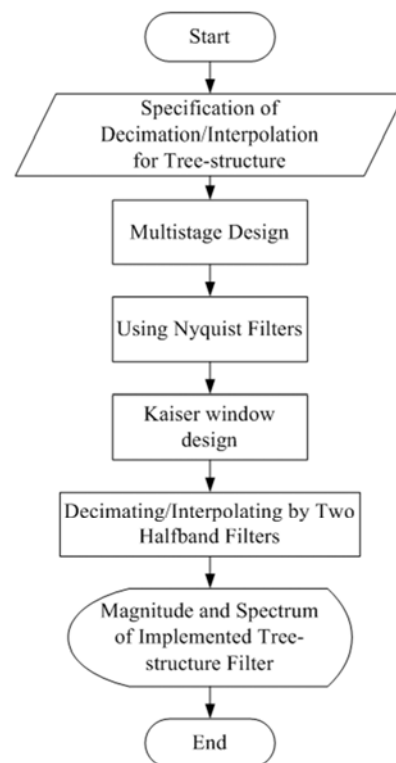


Figure 8. Flowchart of Decimation/Interpolation.

## 7. Simulation Results of Tree-Structure Filter Bank

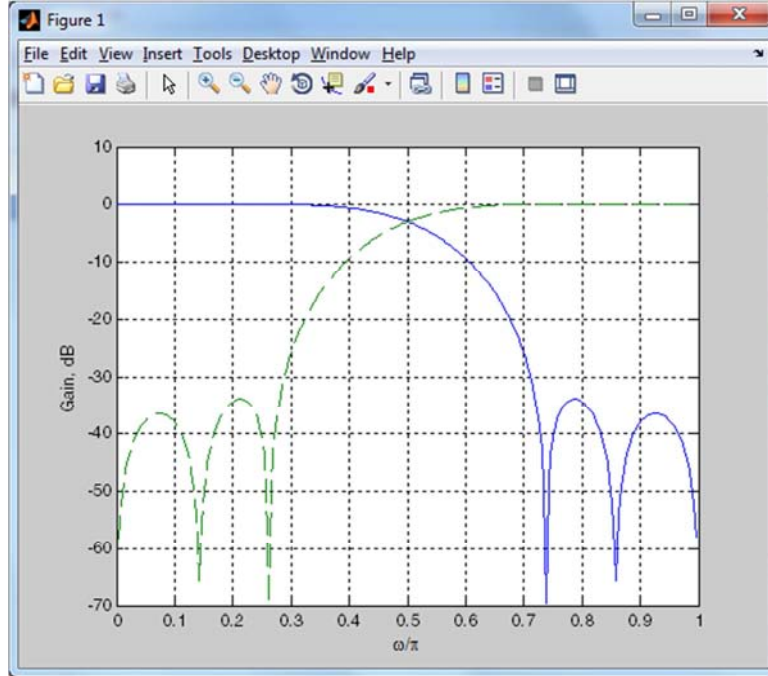
The passband of the cascade is the frequency range where



the passbands of the two filters overlap. On the other hand, the stopband of the cascade is formed from three different frequency ranges.

In two of the frequency ranges, the passband of one coincides with the stopband of the other, while in the third range, the two stopbands overlap. As a result, the gain

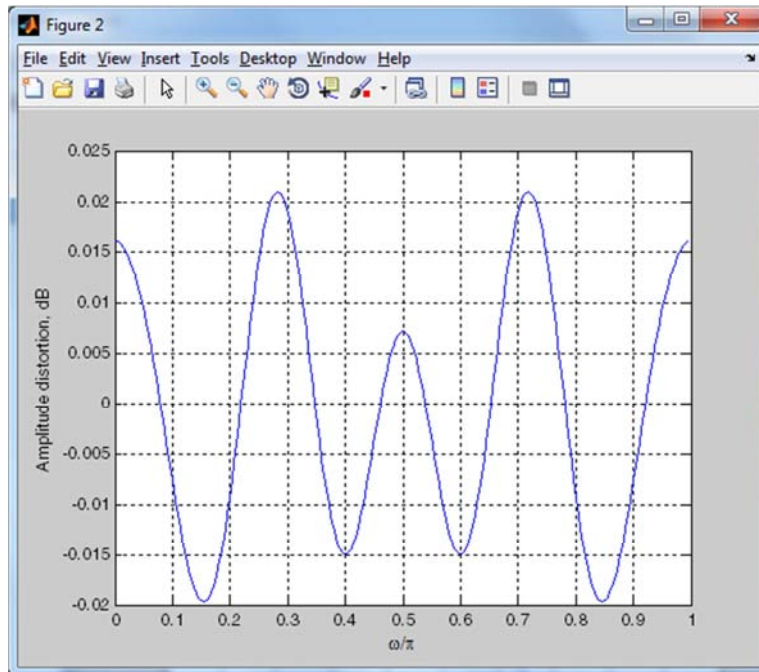
responses of the cascade in the three regions of the stopband are not equal, resulting in an uneven stopband attenuation characteristic. This type of behavior of the gain response can also be seen in Figure 9 and should be taken into account in the design of the tree-structured filter bank.



**Figure 9.** Gain Responses of the Two Analysis Filters.

By continuing the process of inserting a two-channel maximally decimated QMF bank, QMF banks with more than four channels can be easily constructed. It should be noted that the number of channels resulting from this approach is restricted to a power of 2; that is,  $L = 2^V$ . In addition, as

illustrated by Figure 10, the filters in the analysis (synpaper) branch have passbands of equal width, given by  $\pi/L$ . However, by a simple modification to the approach it can be designed QMF banks with analysis (synpaper) filters having passbands of unequal width.



**Figure 10.** Reconstruction Error in dB.

## 8. Simulation Results of Multirate Filter Analysis

All interpolators and decimators exhibit a lowpass response. The simplest interpolators, like the CIC interpolator and the hold interpolator have a poor lowpass response. However, they are easy to implement and do not require any multiplications to be performed in real-time. The following plot compares the lowpass response of four different interpolators. All of them have an interpolation factor of 4.

One can easily see the difference in the quality of the lowpass filter, depending on which type of interpolator is used. The CIC interpolator has more gain than the other interpolators. The comparison response for tree-structure filter with conventional filter is shown in Figure 11. The blue color response is for conventional FIR filter design and the other color responses are given the implemented filter design. The magnitude response for two channel tree-structure filter bank with respect to various operating frequency is illustrated in Figure 12.

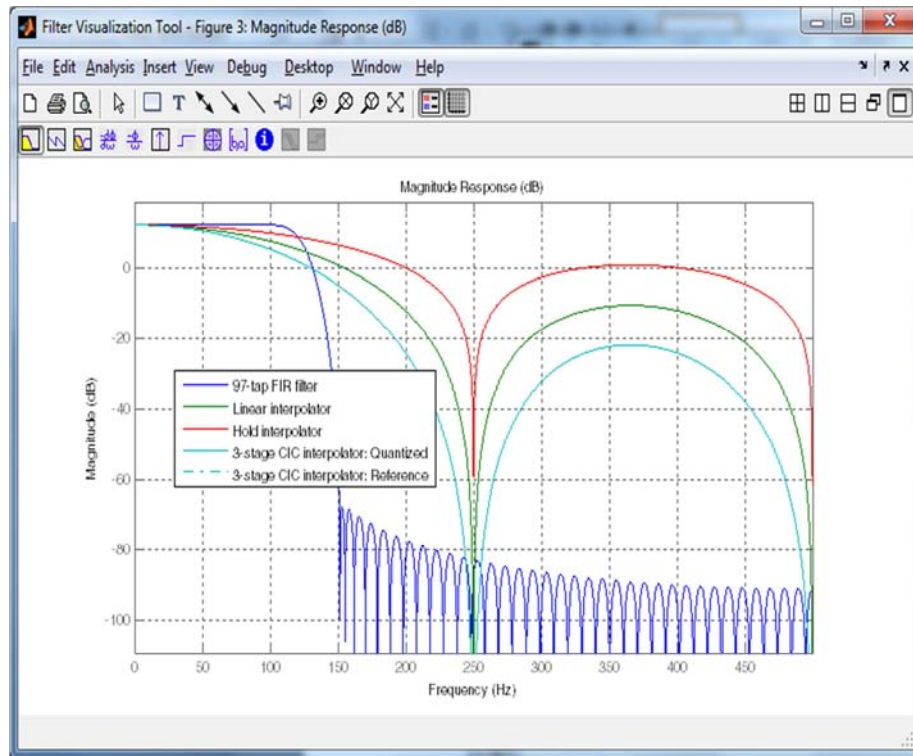


Figure 11. Comparison Response for Tree-structure Filter with Conventional Filter.

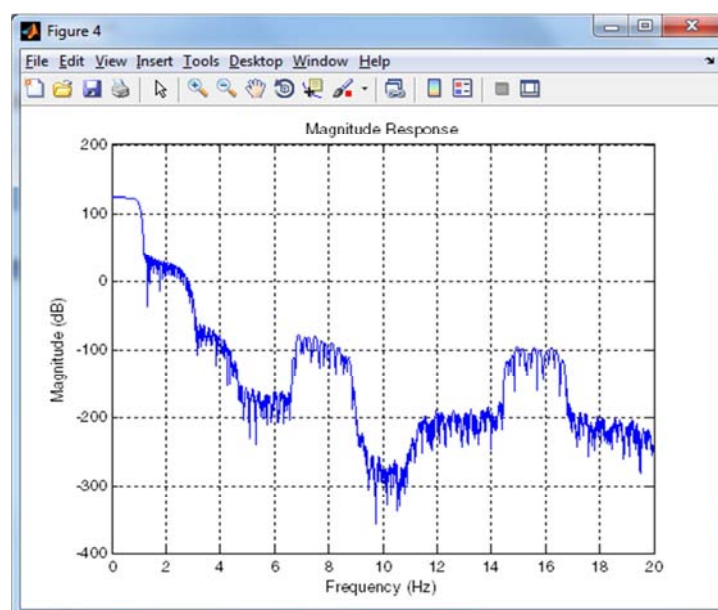


Figure 12. Magnitude Response for Two Channel Tree-structure Filter Bank.

## 9. Simulation Results of Decimator/Interpolator for Tree-Structure Filter Bank

When decimating, the bandwidth of a signal is reduced to an appropriate value so that minimal aliasing occurs when reducing the sampling rate. Suppose a signal that occupies the full Nyquist interval has a sampling rate of 200 Hz. The signal's energy extends up to 100 Hz. If the sampling rate is reduced by a factor of 4 to 50 Hz, significant aliasing will occur unless the bandwidth of the signal is also reduced by a factor of 4. Ideally, a perfect lowpass filter with a cutoff at 25

Hz is used. The components of signal between 0 and 25 Hz is slightly distorted by the passband ripple of a non-ideal lowpass filter; there was some aliasing due to the finite stopband attenuation of the filter; the filter would have a transition band which would distort the signal in such band. The amount of distortion introduced by each of these effects can be controlled by designing an appropriate filter. In general, to obtain a better filter, a higher filter order will be required. The screenshot result for response of two channel filter bank using decimation/interpolation methods is shown in Figure 13.

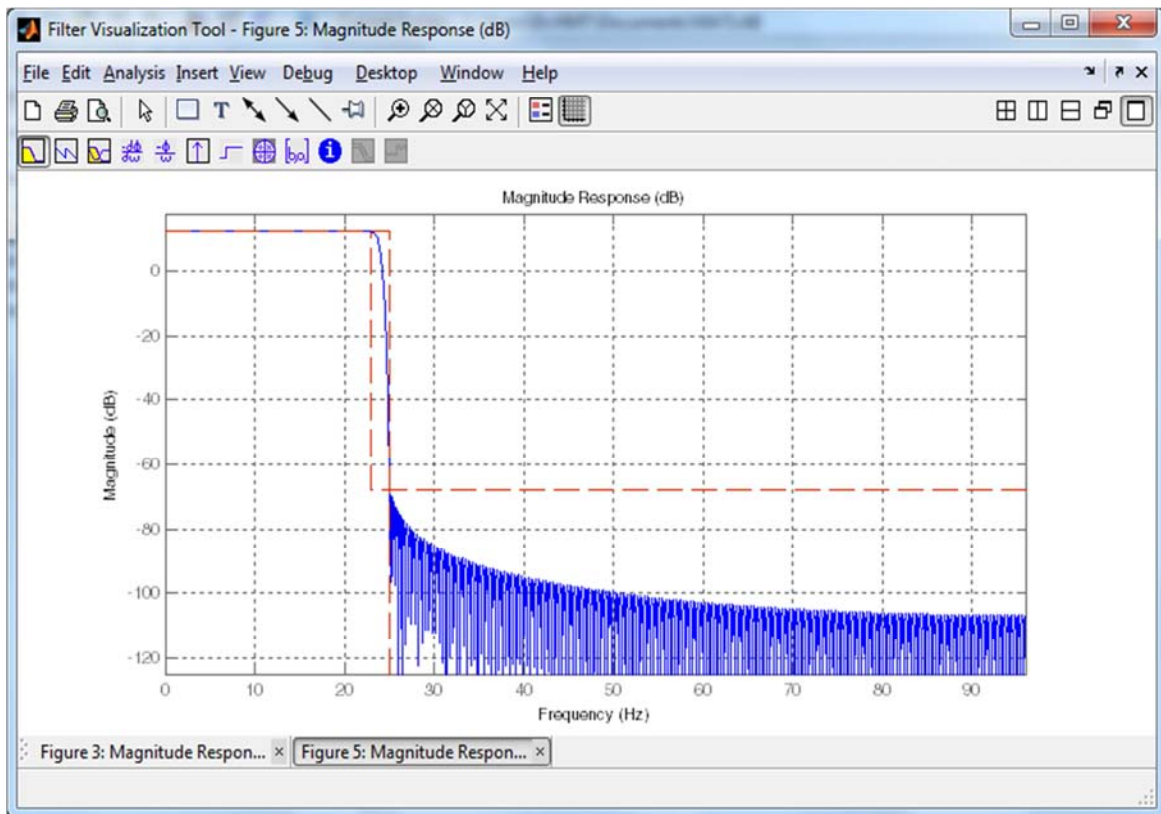


Figure 13. Screenshot Result for Response of Two Channel Filter Bank Using Decimation and Interpolation Methods.

When the decimation factor is 2, the Nyquist filter becomes a halfband filter. These filters are very attractive because just about half of their coefficients are equal to zero. Often, to design Nyquist filters when the band is an even number, it is desirable to perform a multistage design that uses halfband filters in some/all of the stages. As with other Nyquist filters, the halfbands are used for decimation, aliasing will occur only in the transition region. The frequency spectrum for constructed filter bank is illustrated in Figure 14.

Note that although the filter has a gain of 4, the interpolated signal has the same amplitude as the original signal. Similar to the decimation case, Nyquist filters are attractive for interpolation purposes. Moreover, given that there is a coefficient equal to zero every  $L$  samples, the use of Nyquist filters ensures that the samples from the input signal are

retained unaltered at the output. This is not the case for other lowpass filters when used for interpolation (on the other hand, distortion may be minimal in other filters, so this is not necessarily a huge deal).

In an analogous manner to decimation, when used for interpolation, Nyquist filters allow some degree of imaging. That is, some frequencies above the cutoff frequency are not attenuated by the value of  $A_{st}$ . However, this occurs only in the transition band of the filter. On the other hand, once again a wider portion of the baseband of the original signal is maintained intact when compared to a lowpass filter with stopband-edge at the ideal cutoff frequency when both filters have the same transition width. The discrete time signal response of implemented filter design is illustrated in Figure 15



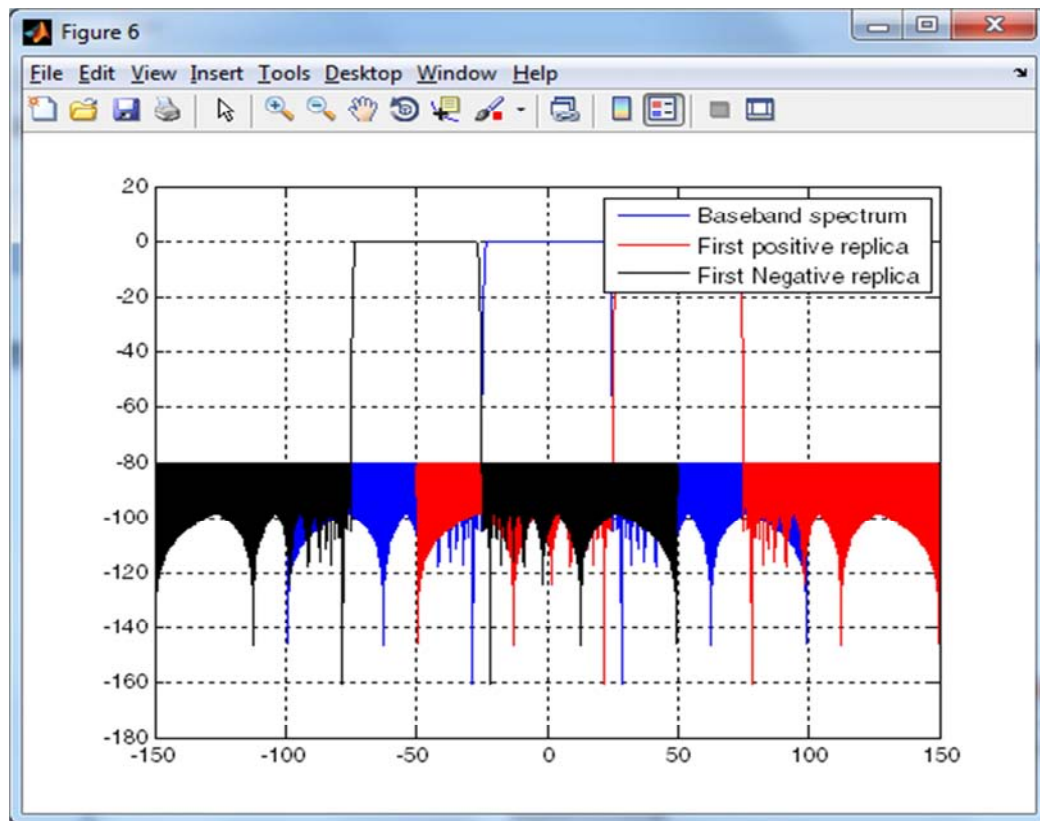


Figure 14. Frequency Spectrum for Constructed Filter Bank.

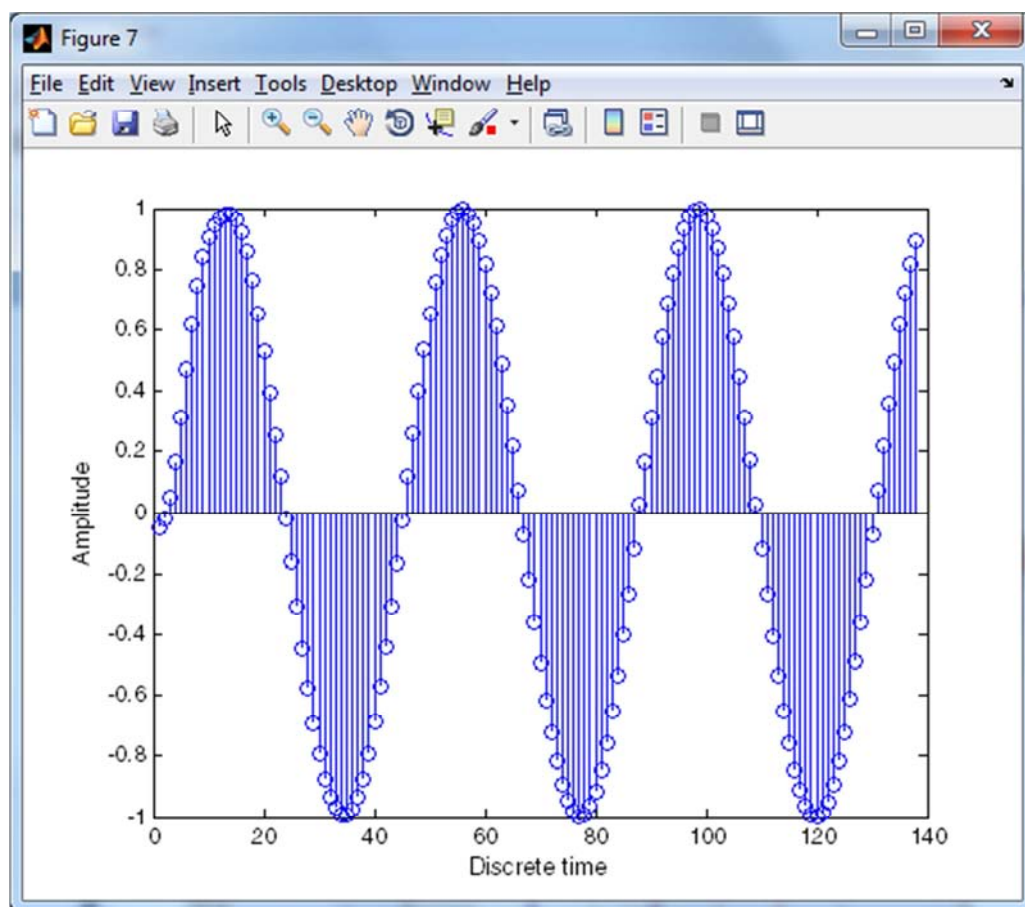


Figure 15. Discrete Time Signal Response of Implemented Filter Design.

## 10. Conclusion

Design and realization of the comb-based filters for decimators and interpolators has been developed. In this work, the structures of the CIC-based decimators and interpolators, discuss the corresponding frequency responses, and demonstrate the overall two-stage decimator constructed as the cascade of a CIC decimator and an FIR decimator. The condition coincides with the known one for the fractionally spaced equalizers when  $K = 1$ . The condition is not difficult to check when the ISI transfer function is known. In particular, it obtained a simplified version of the condition for an FIR non-maximally decimated multi-rate filter-bank pre-coder with  $N$  channels and the largest decimation, i.e.,  $K = N - 1$  which corresponds to the case of the smallest bandwidth expansion in the pre-coding. The condition can be stated as follows: All rotations of the zero set of the FIR transfer function  $H(z)$  at samples  $l \cdot N$  for  $l = 0, 1, 2, 3, \dots, N-1$  are disjoint from each other. These conditions are basically easy to satisfy. Thus, the approach in this paper suggests that the sampling rate that is times faster than the baud rate for the receiver may be good enough.

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