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A decision-making phase-space model for fairness assessment

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Abstract: Toward the goal of delineating the underlying decision-making process in relation to fairness, a mathematical model describing the decision criteria is derived. In this fairness-decision model, the decision-making criteria are limited to choose between fairness, equity/disparity and monetary gain. In this model, the decision threshold criteria are represented by the graphical location of the decision space in the fairness-equity quadrant. The fairness decision criterion is determined by the relativistic fairness stimulus-response function representing the relationship between fairness and disparity. The disparity/equity decision criterion is determined by the disparity of the monetary offer. The decision threshold is represented by the graphical intersection between the fairness stimulus-response function and the disparity function. The analysis shows that monetary gain or loss is a consequence of the decision, rather than a decision criterion, unless the decision is already predetermined. The analysis also shows that the paradoxical decisions that seem to be irrational (such as rejecting hyper-equitable offers) are, in fact, logically consistent without being paradoxical or irrational. It is resulted from a bias in fairness perception that shifts the fairness stimulus-response function up/down or left/right around the four fairness-equity quadrants. If either fairness or equity/disparity were used as the sole criterion for decision, no paradox would exist. It is only when both fairness and equity/disparity were used as the decision criteria simultaneously that would have resulted in a paradoxical decision under certain circumstances. But such paradox is merely a shift/bias in the fairness perception without being irrational, as predicted by the present relativistic fairness-equity model.

Keywords: Decision-Making, Fairness Bias, Equity, Rational Decision, Monetary Gain, Ultimatum Game

1. Introduction

This paper will present a computational model of the rational decision-making process when confronted with a choice to choose between fairness and monetary gain as the decision criteria using a relativistic fairness-equity model. In order to identify the criteria in which a decision is made using a cognitive model, we will study the decision-making process under the special circumstance in which the decision is limited to choose between fairness and monetary gain. Then the criteria for making a decision can be delineated between the two choices, such that the threshold to accept or reject a fair or unfair offer can be determined in relation to these two decision variables — fairness and monetary gain.

1.1. Theoretical Derivation for Decision-Making as an Optimization Process to maximize the Desired Outcome

Theoretically, the decision-making process is an optimization process to maximize the desired outcome. If the desired outcome is dependent on a single decision variable, such as monetary gain, then the decision is simply a maximization process to increase the monetary gain as much as possible. Similarly, if the desired outcome is dependent on fairness, then the decision is simply maximizing the level of fairness as much as possible.

If the desired outcome is dependent on two decision variables, such as fairness and monetary gain, then the decision-making process requires maximization of both fairness and monetary gain. If the maximization of fairness does not conflict with the monetary gain, then the decision can be made to maximize both. But if maximizing one of the variables conflicts with the maximization of the other variable, then a choice has to be made in the decision to optimize one or the other, but not both.

1.2. Decision Dilemma and Conflict Resolution

The selection of the decision choice is often governed by the desired outcome in which the decision-maker has to decide which of the two variables is more important to choose to optimize. A conflict in decision occurs when maximization of one variable will minimize the other, making it impossible to maximize both. Thus, the decision requires choosing between one of the two variables to maximize, when no other alternatives are available.

For the decision to choose between fairness and monetary gain, it is often assumed that monetary gain will override fairness for the decision in economic transactions, while fairness will override monetary gain for the decision in social transactions. But sometimes, there is the paradoxical decision that people can choose to forgo maximization of either fairness or monetary gain to obtain the desired outcome that seems counter-intuitive.

This paper will explore the theoretical relationship between these two decision criteria, and determine that a logically consistent decision can be made by choosing the fairness criterion, without necessarily choosing the monetary gain criterion to resolve the conflict. Experimental confirmation of the decision model is provided in the companion paper [1] to confirm that the decision can be made using fairness as the decision criterion without necessarily relying on monetary gain as a criterion

In examining the decision-making process, many studies use fairness as a factor to determine how decisions are made in economic transactions [2-5] and distributive justice [6-8]. Fairness is also used as a factor to determine how decisions are affected in social interactions [9-21]. Because what is considered as fair (or unfair) is often biased by an individual's subjective perception, and this bias can alter the decision made by an individual. Thus, it is important to delineate the underlying decision-making criteria so that we can quantify which factor is more important in influencing a decision. Humans are not the only species that use fairness as a criterion for making decisions, primates also use fairness as a factor to make their decisions [22]. Thus, the decision-making process is conserved across species in evolution from primates to humans, which suggests that there is a generalizable universal principle underlying the decision-making process.

1.3. Ultimatum Game as a Tool to Determine the Decision-Making Process in Relation to Fairness

Decisions based on fairness have been studied extensively using the classical Ultimatum Game (UG) experimental paradigm in behavioral economics [2, 23-27]. UG is a split-the-money game where the human subject's decision-making process is deduced from the decision to accept or reject the monetary offer, depending on whether the offer is perceived as fair or not [27]. The rule of the UG is that a proposer offers an amount of money to share with the responder. The responder is asked to make a decision to accept or reject the proposed offer. If the responder decides to accept the money, both keep the money; otherwise, both lose the money.

Thus, the decision to accept or reject the offer in UG depends on whether it is better to maximize the monetary gain or maximize fairness in the decision criterion. This provides a useful tool to determine which decision variable — fairness or monetary gain — is more important to use as the decision criterion.

Since the rule of UG requires losing the money if the responder rejects the offer, it creates a conflict for inequitable offers, in which the responder cannot maximize fairness and monetary gain at the same time. If the responder chooses money, it would not be fair. If the responder chooses fairness, it cannot gain the money. Thus, it creates a dilemma for the responder to decide which of the two decision criteria is more important to maximize. This provides the condition in which the underlying decision-making process can be examined theoretically, using a logically consistent model, without violating any logical reasoning, or contradicting any decision criteria.

Numerous computational models for hypothesizing the decision-making process based on fairness have been developed to describe how fairness is evolved in UG [25, 26, 28-34] using economic game theories [4-7, 35, 36]. We will introduce a different theoretical model to account for the decision-making process that can use a single criterion — fairness — without requiring choosing both fairness and monetary gain as the criteria to resolve the dilemma. Previous decision-making model has incorporated the relativity of fairness considerations to describe how fairness and monetary gain/loss considerations without compromising the decision for fairness over monetary gain [1, 37-40]. This paper will derive a novel decision-making criterion using the geometric quadrant of the decision-space in the fairness-equity stimulus-response function for determining how a decision is made (see Fig. 1 below).

1.4. Relativity in Fairness Assessment in the Decision-Making Process

In assessing fairness in the decision-making process, there is an implicit comparison between two entities — self-regarding and other-regarding concerns [13, 14, 41]. Without such comparison, equality and fairness would not exist. When a comparison is made, it is usually based on one frame of reference relative to another (i.e., comparing between *self* and *others*). For example, when someone asks us how fair it is, it usually involves an implicit computation to compare others relative to ourselves.

In computing *subjective* fairness, it compares *self* in relative to *others*, using a *self*-centered frame of reference in the comparison. When the frame of reference is switched from a *self*-centered one to an *other*-centered one, fairness is also changed from fair into unfair relatively — without changing the amount of disparity between them.

On the other hand, *objective* fairness is computed by comparing the disparity relative to both parties (*self* and *others*) using a neutral party's (a third person's) standpoint. Thus, objective fairness is computed by including *other*-

regarding concerns using an *other*-centered frame of reference, while subjective fairness is computed by including only *self*-regarding concerns using a *self*-centered frame of reference. Thus, the decision using fairness as the criterion can change depending on whether a *self*-centered or an *other*-centered frame of reference is used as the decision criterion.

2. The Relativistic Fairness-Equity Model

Expressing the above relativistic relationships mathematically, let us define **f** as a quantifiable measure of fairness as a vector, and **d** as the disparity vector between self and others. Then the level of fairness perception in relation to disparity is given by:

$$\mathbf{f} = k \cdot f(\mathbf{d}) + b \quad (1)$$

where k is the fairness sensitivity coefficient, b is a constant representing the baseline fairness level, and $f(\mathbf{d})$ is a function of the disparity vector, which can be either a linear or a nonlinear function. The disparity measure is a relativistic measure that is opposite to the equity measure.

Without loss of generality, the disparity vector (**d**) is a vector difference between *oneself* and *others* when comparing a quantity — in the case of UG, the monetary difference — between two persons in the proposed monetary offer. The disparity measure can take on a positive or a negative value, depending on whether the disparity is in favor of *oneself* in the comparison. For instance, if an offer is a bigger amount to *oneself* than the amount to the *other person*, then the disparity is a positive value. If the offer is a lesser amount to *oneself* than the amount to the *other person*, then the disparity is a negative value. If the offer is the same for both the *self* and the *other person*, then the disparity is zero.

Since the vector **d** is a signed quantity, Eq. 1 automatically accounts for the relativity of fairness — what is fair (**f**) for the *self* is unfair (–**f**) for the *other person*. This relativity in fairness is automatically computed by the change in the sign of disparity from a positive (**d**) vector to a negative (–**d**) vector, when the frame of reference is switched from a *self-centered* frame of reference to an *other-centered* frame of reference.

2.1. Decision Threshold Using Fairness as a Decision Criterion

Note that Eq. 1 also corresponds to the classical stimulus-response (SR) function for fairness in physiological or psychological systems. This fairness stimulus-response function also corresponds to the input/output (I/O) function in computer science. The stimulus is disparity, and the response is fairness. For the UG paradigm, the stimulus is the amount of monetary disparity between the two persons in the offer (or the offer-ratio), which will result in either monetary gain or loss if the responder accepts or rejects the offer, respectively.

The stimulus-response function is usually a non-linear sigmoidal function in psychological or physiological systems, rather than a linear function. Since the operating range of most living systems lies in the linear physiological region (in the middle of the sigmoidal stimulus-response function), for simplicity, we will use this linear operating range as a first approximation in our model. That is, given the disparity stimulus **d**, a person will respond with a fairness perception computed according to Eq. 1.

If the decision is based on fairness as a criterion, then the fairness stimulus-response function can be used to determine the fairness threshold in which a person decides to switch from a rejection decision to an acceptance decision. Thus, using this relativistic fairness-equity model, it will allow us to quantify the threshold in which a decision is made, and determine whether monetary gain can be captured in the fairness decision, without using monetary gain as a decision criterion.

2.2. Relativity in Fairness Assessment by Including both Self-Regarding and Other-Regarding Concerns

If the decision incorporates self-regarding concerns, it uses the self-centered frame of reference to evaluate fairness for the decision criterion. If the decision incorporates other-regarding concerns, then it uses the other-centered frame of reference to evaluate fairness for the decision criterion.

This relativistic model of fairness can account for both *self*-centered fairness (i.e., how fair it is to "*me*") and *other*-centered fairness (i.e., how fair it is to "*you*") by Eq. 1. That is, the equation implicitly incorporates not only a *self*-centered perspective of fairness (using a local frame of reference), but also an *other*-centered (*non-self*) view of fairness (using a global frame of reference).

2.3. Switching Frame of Reference in the Evaluation of Fairness Perception

By default, this vectorial model has already encapsulated the inclusion of reference frame implicitly by the signed vector, **d**, in which relative fairness is computed — i.e., "fairness to me" is computed by $\mathbf{f} = k \bullet f(\mathbf{d})$, while "fairness to you" is computed by the opposite vector, $\mathbf{f} = k \bullet f(-\mathbf{d})$.

To explicitly express the relativity of fairness, let us denote "fairness to me" as **f** (using a self-centered frame of reference), and "fairness to you" as **f′** (using the other-centered frame of reference), with the primed notation. Then "fairness to others" is given by:

$$\mathbf{f}' = k' \cdot f(\mathbf{d}') + b' \quad (2)$$

Thus, the decision threshold can be determined by either Eq. 1 or Eq. 2, depending on whether only the self-regarding concerns is incorporated into the decision or the other-regarding concerns are also incorporated into the decision.

2.4. Derivation of Decision Criterion Based on Fairness

If the decision is based on fairness, then the criterion to accept or reject an offer is determined by the level of fairness.

Let's say that the decision threshold, θ, is located at neutral fairness level ($\theta = 0$), then the decision is to accept the offer if it is fair, and reject the offer if it is unfair. The decision, δ, would be quantified by:

$$\delta = \begin{cases} +1, & \text{if } \mathbf{f} \geq 0 \\ -1, & \text{if } \mathbf{f} \geq 0 \end{cases} \tag{3}$$

where $\delta = +1$ represents an acceptance decision while $\delta = -1$ represents a rejection decision. If the decision threshold is located at a positive fairness level ($\theta > 0$) for a fair perception or a negative fairness level ($\theta < 0$) for an unfair perception, then the decision is determined by:

$$\delta = \begin{cases} +1, & \text{if } \mathbf{f} \geq \theta \\ -1, & \text{if } \mathbf{f} \geq \theta \end{cases} \tag{4}$$

2.5. Fairness Bias by Shifting the Baseline Level of Fairness Perception

The baseline level of fairness perception is given by the y-intercept of the stimulus-response function $\mathbf{f} = k\bullet f(\mathbf{d}) + b$, i.e., the constant b in Eq. 1. Thus, any bias in the fairness baseline level is represented by a change in the constant, b. If the baseline bias is toward a more fair level, then the constant, b, will increase. If the baseline bias is toward the unfair level, then the constant, b, will decrease. This quantification of this fairness bias will allow us to determine how a decision can be affected by a change in the fairness baseline level.

2.6. Decisions Bias Resulted from Changing the Baseline Level of Fairness Perception

Let us assume, without loss of generality, that the decision criterion is fairness, then the decision would be determined by the level of fairness perceived by the person. Furthermore, if the decision threshold were set according to the fairness level as defined by Eq. 4, then any change in fairness baseline level would alter the decision threshold accordingly.

That is, any bias in the fairness perception will also bias the decision. In other words, if the decision to accept is determined by fairness, and if the decision to reject is determined by unfairness, then when the fairness perception is shifted/switched from fair to unfair. The decision, δ, will also change/switch from acceptance ($\delta = +1$) to rejection ($\delta = -1$) according to Eq. 3, if the decision threshold is set at $\theta = 0$. For any other non-zero decision threshold, the decision, d, is given by Eq. 4.

The above logic is generally assumed in the decision-making process when fairness is used as the criterion by most of the UG studies [42-47]. But there are exceptions to the above assumption that seem paradoxical. Sometimes, humans accept unfair offers, while other times they may reject fair offers. When this occurs, it is often assumed that the decision is either irrational or the decision is made using some other criteria other than fairness [42-47]. But this assertion may not be necessary. We will show below, by using the relativistic fairness-equity model, that the decision can still be made with fairness as the criterion without being irrational, and without incorporating some other factors other than fairness as the criterion.

2.7. Fairness Bias by Changing the Fairness Sensitivity

Fairness perception can also be biased by a change in fairness sensitivity rather than a change in fairness baseline. Fairness sensitivity is quantified by the slope, k, of the stimulus-response function in Eq. 1. If the slope, k, increases, the sensitivity to fairness is heightened with a much more exaggerated sense of fairness. If the slope, k, decreases, the sensitivity to fairness is diminished with an indifference perception to fairness.

Thus, there are two types of fairness biases — baseline bias and sensitivity bias. Baseline bias affects the sense of what is fair or unfair, whereas sensitivity bias affects the heightened or diminished awareness of fairness or unfairness. Baseline bias is quantified by the y-intercept, b, and sensitivity bias is quantified by the slope, k, of the fairness stimulus-response function, $\mathbf{f} = k\bullet f(\mathbf{d}) + b$, in Eq. 1.

By the same token, if the decision criteria were based on fairness, then fairness baseline, fairness sensitivity, or both can bias the decision. Thus, a decision may be altered by changing the y-intercept, b, or the slope, k, of Eq. 1. This summarizes the dependence of decision on fairness biases mathematically.

3. Graphical Representation of the Decision Phase-Space Quadrants

Let us represent the objective disparity, $\mathbf{d}$, graphically by the x-axis (independent axis), and the subjective fairness, $\mathbf{f}$, by the y-axis (dependent axis) based on the fairness stimulus-response function, $\mathbf{f} = k\bullet f(\mathbf{d}) + b$, in Eq. 1 (see Fig. 1). The same graph is essentially divided into the left-half and the right-half by the y-axis representing inequity (hypo-equity) and hyper-equity, respectively. The graph is also divided into the upper-half and the lower-half by the x-axis, representing a fair and an unfair perception, respectively. When a decision is made, it is made based on the condition of fairness and equity according to the specific quadrant as described below (see Fig. 1).

3.1. Interpretation of the Decision-Space in the Relativistic Fairness-Equity Quadrants

Combining the above fairness and equity interpretations, the decision-space in which the decision is made can also be subdivided by four quadrants (see Fig. 1):

(a) Upper-left "fair and inequitable" quadrant;
(b) Upper-right "fair and hyper-equitable" quadrant;
(c) Lower-right "unfair and hyper-equitable" quadrant;
(d) Lower-left "unfair and inequitable" quadrant.

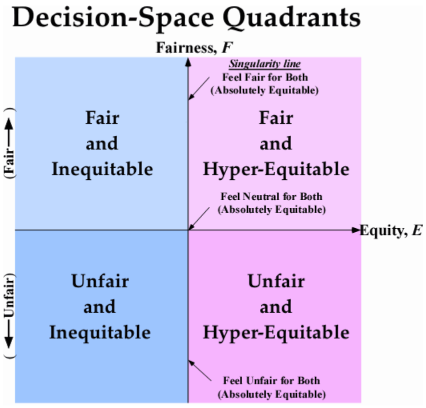

Figure 1. *Graphical representation of the four quadrants of decision-space. Each quadrant represents the subjective fairness perception of fairness and unfairness with respect to disparity (whether it is inequitable or hyper-equitable) in the decision-making process.*

The interpretations of the fairness perception in each of the quadrant are provided below:

(a) If the decision is made in the upper-left quadrant decision-space, then it is a lenient decision — it is based on the condition of feeling fair even though it is inequitable (see Fig. 2).

(b) If the decision is made in the upper-right quadrant decision-space, then it is a fair decision — it is based on the condition of feeling fair when it is hyper-equitable (see Fig. 2).

(c) If the decision is made in the lower-right quadrant decision-space, then it is a greedy decision — it is based on the condition of feeling unfair, even though it is hyper-equitable (see Fig. 2).

(d) If the decision is made in the lower-left quadrant decision-space, then it is an unfair decision — it is based on the condition of feeling unfair when it is inequitable (see Fig. 2).

3.2. Relativistic Interpretation of the Fairness-Equity Quadrants when the Frame of Reference is Switched

If the frame of reference for evaluating fairness is switched from *self* to *others*, then the fairness-equity quadrant graph would become a mirror image of the decision-space graph in Fig. 1. That is, what is hyper-equitable to *self* is inequitable to *others*, and vice versa. Thus, these graphs represent *subjective* fairness based on their own frame of reference. The only exception to this subjectivity is the center dividing line at the absolute equitable offer (disparity $\mathbf{d}$ = 0 at x-axis origin), where it is equitable to both *self* and *others*, objectively. At this vertical y-axis, the proposed offer is absolutely equitable for both *self* and *others.* Thus, the dividing vertical line represents *objective* fairness relative to any neutral third party (independent of the relative self-centered or other-centered frame of reference).

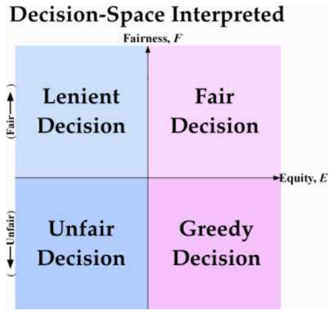

Figure 2. *Graphical interpretation of the decision-space in each quadrant.*

3.3. Decision Criterion Based on Offer-Ratio

If the decision criterion were based on the monetary offer-ratio in UG, then it also corresponds to the decision criterion based on the disparity variable in the relativistic fairness-equity model. For instance, if the acceptance decision criterion were set at a specific offer-ratio (at a specific disparity), then the decision space would be divided vertically into two halves instead of four quadrants. The vertically dividing-line is the decision threshold that is based on disparity. This dividing-line is a given by:

$$\mathbf{d} = \varepsilon \tag{5}$$

where ε is the specific offer-ratio (or disparity) used to determine an acceptance decision. The decision criterion based on disparity is given by:

$$\delta = \begin{cases} +1, & \text{if } \mathbf{d} \ge \varepsilon \\ -1, & \text{if } \mathbf{d} < \varepsilon \end{cases} \tag{6}$$

3.4. Determination of the Decision Threshold using Both Fairness and Disparity Criteria

Given that the fairness stimulus-response function in Eq. 4 is used as one of the decision criteria and the disparity in Eq. 6 as the other criterion, then the intersection of these two decision thresholds would determine the exact location (quadrant) within the fairness-equity space in which the decision were made. If both fairness and disparity were used as the criteria, then the decision space where the decision is made is given by:

$$\delta = \begin{cases} +1, & \text{if } \mathbf{f} \ge \theta \text{ and } \mathbf{d} \ge \varepsilon \\ -1, & \text{if } \mathbf{f} < \theta \text{ and } \mathbf{d} < \varepsilon \end{cases} \tag{7}$$

Any decisions made outside of the decision space in Eq. 7 would appear as irrational, because it is inconsistent with using both fairness and disparity as the criteria. But such paradoxical decisions are not necessarily irrational, but rather caused by using solely one decision variable as the criterion — such as using either fairness or disparity as the criterion. Examples of such paradoxical decision spaces are:

$$\delta = \begin{cases} +1, & \text{if } \mathbf{f} \ge \theta \text{ and } \mathbf{d} < \varepsilon \\ -1, & \text{if } \mathbf{f} < \theta \text{ and } \mathbf{d} \ge \varepsilon \end{cases} \tag{8}$$

and

$$\delta = \begin{cases} +1, & \text{if } \mathbf{f} < \theta \text{ and } \mathbf{d} \geq \varepsilon \\ -1, & \text{if } \mathbf{f} \geq \theta \text{ and } \mathbf{d} < \varepsilon \end{cases} \quad (9)$$

It is only paradoxical if both criteria were used, as in Eq. 8 and Eq. 9. But if one of the criteria were used, as in Eq. 4 or Eq. 6, no paradox or irrationality would exit. The paradoxical decision spaces in Eq. 8 and Eq. 9 would merely be a subspace captured by either Eq. 4 or Eq. 6, resolving the paradox or irrationality. That is, if a person decides based solely on the fairness criterion, irrespective of the disparity in the monetary offer, or if money is not an issue for the person, then it is perfectly rational to reject money, because money is not an issue. There can be many other reasons to reject an equitable or accept an inequitable, nonetheless monetary gain/loss is not one of the criteria.

3.5. Identification of Decision Criteria in the Decision Space with Respect to the Fairness-Equity Quadrant

Given that the specific perception of fairness and equity can be represented by the fairness-equity quadrants, we can identify the decision criteria by the graphic location of the quadrant in which the decision threshold is located. That is, if the acceptance decision is located in the hyper-fair and hyper-equitable (upper-right) quadrant, and the rejection decision is located in the unfair and inequitable (lower-left) quadrant, then the decision made is often considered as logical/rational. The exact location of the decision threshold in these quadrants is dependent on the fairness biases, as reviewed in the above sections.

3.6. Rational Decisions due to a Shifting of the Decision Space into a Paradoxical Fairness-Equity Quadrant

On the other hand, if the decision is located in the hyper-fair and inequitable (upper-left) quadrant, then the decision appears to be paradoxical, when a person considers inequitable offers as fair in the decision. Most often, this paradoxical decision is assumed to be irrational, but in fact, is logically consistent with the relativistic fairness-equity model. This is because the location of the decision criterion is merely being shifted to the upper-left quadrant by the fairness biases in the stimulus-response function. Thus, this results in a decision bias that seems paradoxical or illogical, but it is merely caused by a shift of the decision space into a different fairness-equity quadrant, without contradicting any logical principles for fairness assessment or decision-making. It is merely a result of the fairness bias, which subsequently affects the decision.

Similarly, if the decision is located in the unfair and hyper-equitable (lower-right) quadrant, then the decision appears to be paradoxical when a person considers hyper-equitable offers as unfair in the decision. This paradoxical decision is also appeared to be irrational, but in fact, is logically consistent with a shift in the decision space into the lower-right fairness-equity quadrant, without contradicting any logical principles for fairness assessment or decision-making. The paradoxical decision can be identified as a shift of the decision space in the fairness-equity quadrant caused by a shift in the fairness bias.

3.7. Decisions Based on Fairness Criterion Rather than the Monetary Gain Criterion

Because the amount of monetary gain or loss in UG is directly linked to the amount of disparity in the offer when a person accepts or rejects the offer, an acceptance decision would result in a monetary gain, and a rejection decision would result in monetary loss. Thus, if monetary gain or loss were the criterion for the acceptance or rejection decision, then monetary offer of any amount would always result in an acceptance decision, independent of fairness. Therefore, the decision space for acceptance decision would span all four fairness-equity quadrants.

Thus, the monetary gain or loss is a consequence of the decision rather than the criterion of decision in UG. That is, if a person accepts the money, it will always be a monetary gain. If a person rejects the money, it will always be a monetary loss. The monetary gain or loss is caused by the decision. If the decision were to use monetary gain or loss as the criterion, then the decision is already predetermined, without any regards to fairness or disparity. If the decision were not predetermined by the consequence of monetary gain or loss, then a person could use fairness, disparity/equity or both in the decision criterion (assuming fairness and disparity/equity were the two given choices in the decision, as in the UG paradigm).

If either one criterion — fairness or disparity/equity — were used as the decision criterion, then no paradoxical or seemingly irrational decision would exist. If both criteria — fairness and disparity/equity — were used as the decision criteria simultaneously, then there are some conditions in which the decision may appear to be paradoxical (as in Eq. 8 or Eq. 9). But such paradoxical decision is merely a shift of the decision criterion into the decision space, which is caused by a shift in the fairness perception (i.e., caused a fairness bias) rather than being irrational. The experimental evidence in the companion paper [1] also showed that human subjects behaved precisely as predicted by the relativistic fairness-equity model, which is logically consistent without being irrational when they rejected the monetary offer.

4. Summary

The mathematics of the decision-making process using fairness and disparity as the decision criteria is derived theoretically using a relativistic fairness-equity model. The results show that the logically consistent decisions can be made using either fairness or disparity, or both criteria, without being irrational or paradoxical. The monetary gain or loss is a consequence of the decision in UG rather than a decision criterion, unless the decisions were predetermined by the monetary gain or loss. These logically consistent decision criteria were deduced graphically by the location of the fairness-equity quadrant in which the decisions were made. The location of the decision space quantifies the rationale in which the decisions were made, i.e., the decision criteria used in making such a decision.

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