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Emerging Importance of EVs in the Green Grid Era

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Abstract: A very plausible picture of the near future–some decades from now–would be the Green Grid Era and the emerging importance of EVs in the spectrum, where by Green Grid we mean the advanced Smart Grid in which essential part of the whole electric power is supplied by distributed generation, i.e. it is generated by renewable energy including solar energy, biogas, geothermal energy, etc. The crucial ingredients in this grid system are enhanced batteries and the participation of EVs in the grid as a pool of electricity. Our primary concern in this paper is the control of EV participation in the Green Grid, and in particular the study of linear systems and elucidation of various state equations in [HHP] etc.

Keywords: Green Grid, Distributed generation, EV participation, renewable energy, linear systems

1. Introduction and Preliminaries

The biggest defect of electricity is that it cannot be stored in large quantity and therefore must be kept producing, which is one of two main excuses of big power companies for their bulldozing plans of building new power stations including Nukes. The only major storage unit in most power stations is the pumped storage systems, i.e. in the form of potential energy. Electricity can be stored in batteries (or super-capacitors) only in small quantity.

The main ingredient of the cost of electricity is the cost of the high-voltage power lines, which compose a grid (system). In general, the operation cost of a grid depends highly on the PAR (peak-to-average ratio) in aggregate load demand. For example, there is usually at least one major peak in a daily (residential as well as industrial) load demand profile and some 10 hours peak during the whole year.

Here arises the second of two main excuses of big power companies for building new power stations: To assure reliable service including the supply at peak hours, the companies must produce power superseding the peak value. This makes the value of PAR higher and can significantly increase the generation cost since the grid will be highly underutilized most of time.

2. State Space Representation and the Visualization Principle

Let $x = x(t) \in R^n, u = u(t) \in R^r$ and $y = y(t) \in R^m$ be the state function, input function and output function, respectively. We write $\dot{x}$ for $\frac{d}{dt}x$.

Then a state equation for a linear system is usually given as the system of DEs (differential equations)

$$\begin{cases} \dot{x} = Ax + Bu, \\ y = Cx + Du \end{cases} \tag{2.1}$$

where $A \in M_{n,n}(R), B, C, D$ are given constant matrices.

It is usually the case that if we work in the frequency domain, the things are much easier. The Laplace transform has the effect of shifting from the time domain to frequency domain and as a version of the Fourier transform, it is invertible, i.e. it restores the information in the frequency domain into the time domain. Taking the Laplace transform of (2.1) with $x(0) = o$, we obtain

$$\begin{cases} sX(s) = AX(s) + BU(s) \\ Y(s) = CX(s) + DU(s), \end{cases} \tag{2.2}$$

which we solve as

$$Y(s) = G(s)U(s), \tag{2.3}$$

where

$$G(s) = C(sI - A)^{-1}B + D, \tag{2.4}$$

and where I indicates the identity matrix, which is sometimes denoted I_n to show its size.

In general, supposing that the initial values of all the signals in a system are 0, we call the ratio of output/input of the signal, the transfer function, and denote it by $G(s), \Phi(s)$, etc. We may suppose so because if the system is in equilibrium, then we may take the values of parameters at that moment as standard and may suppose the initial values to be 0.

(2.4) is called the state space representation (form, realization, description, characterization) of the transfer function $G(s)$ of the system (2.1), and is written as

$$G(s) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{2.5}$$

2.1. Preliminaries

The three main ingredients in (electrical) circuits are coil (L), condenser (C) and resistance (R). The inverse electromotive force generated by these component is given respectively by

$$e_L = -L\frac{di}{dt}, \quad e_C = -\int_0^t i dt, \quad e_R = -Ri, \tag{2.6}$$

where $i = i\ (t)$ and $e = e\ (t)$ with subscript indicates the current and the voltage of the prescribed component resp. Hereafter we write u for the voltage e.

The governing law of the circuits is the Kirchhoff laws which have two versions. The first law is the one used for node analysis to the effect that the sum of currents flowing into a node is 0 while the second law is the one used for loop analysis which is to the effect that the sum of all electro-motive forces in a closed circuit is 0.

2.2. Control Plant I

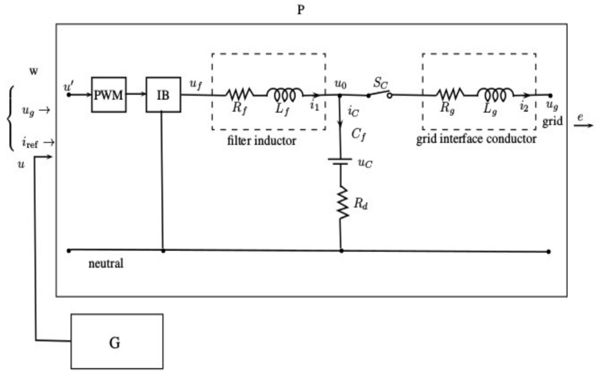

Figure 1. *Control plant* P .

Here PWM=Power Width Modulation and IB=Inverter Bridge.

First consider the LC filter consisting of L_f, C_f in Fig. 1. As a consequence of the Kirchhoff law we have

$$i_1 = i_c + i_2. \tag{2.7}$$

The potential difference $u_0 - u_f$ and $-u_0 = 0 - u_0$ are given respectively by

$$u_0 - u_f = -R_f i_1 - L_f\frac{di_1}{dt} \tag{2.8}$$

and

$$-u_0 = -u_C - R_d i_C, \tag{2.9}$$

where $u_C = \int_0^t i_C dt$.

Hence we conclude that

$$R_f i_1 + L_f\frac{di_1}{dt} = u_f - u_C - R_d i_C = u_f - u_C - R_d(i_1 - i_2) \tag{2.10}$$

whence that

$$\frac{di_1}{dt} = \frac{1}{L_f}(-(R_f + R_d)i_1 + R_d i_2 - u_C + u_f) \tag{2.11}$$

or

$$\frac{di_1}{dt} = \frac{1}{L_f}(-(R_f + R_d)i_1 + R_d i_2 - u_C + u_g + u) \tag{2.12}$$

on using the relation $u_f = u + u_g$.

Also for the grid filter consisting of L_g, C_g, the potential difference $u_g - u_0$ is given by

$$u_g - u_0 = -R_g i_2 - L_g\frac{di_2}{dt}, \tag{2.13}$$

so that from (2.9), we obtain

$$R_g i_2 + L_g\frac{di_2}{dt} = u_C + R_d(i_1 - i_2) - u_g, \tag{2.14}$$

whence

$$\frac{di_2}{dt} = \frac{1}{L_g}(R_d i_1 - (R_g + R_d)i_2 + u_C - u_g). \tag{2.15}$$

Finally

$$\frac{du_C}{dt} = i_C = i_1 - i_2. \tag{2.16}$$

We may express the control plant P in [ZH, pp. 83-84] in the form of (2.1). Let

$$x = \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix}, \quad w = \begin{pmatrix} u_g \\ i_{ref} \end{pmatrix}, \quad y = e = i_{ref} - i_2 \tag{2.17}$$

be the state variable, the external input and the output, respectively. Hence, writing

$$A = \begin{pmatrix} -\frac{R_f + R_g}{L_f} & \frac{R_d}{L_f} & -\frac{1}{L_f} \\ \frac{R_d}{L_g} & -\frac{R_g + R_d}{L_f} & -\frac{1}{L_g} \\ \frac{1}{C_f} & -\frac{1}{C_f} & 0 \end{pmatrix},$$

$$B = (B_1, B_2), \quad B_1 = \begin{pmatrix} \frac{1}{L_f} & 0 \\ -\frac{1}{L_f} & 0 \\ 0 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} \frac{1}{L_f} \\ 0 \\ 0 \end{pmatrix},$$

$$C_1 = (-R_d, R_d, -1), \quad D_1 = (0,1), \quad D_2 = 0, \tag{2.18}$$

Then by viewing B, D in (2.1) as block decompositions,

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u, \\ y = C_1 x + D_1 w + D_2 u \end{cases} \tag{2.19}$$

amounts to (2.1).

2.3. Control Plant II

It is a challenge to keep both the THD (Total Harmonic Distortion) low of the inverter local load voltage u_0 and the gird current (the current i_2 flowing through the grid interface inductor). The inverter LCL plant (control plant P) may be thought of as a cascaded control structure consisting of an inner loop voltage controller and an outer loop of current controller.

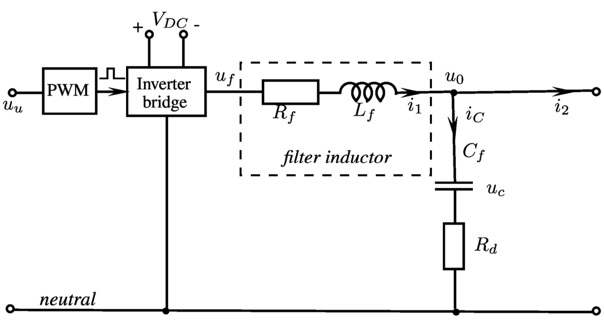

Figure 2. *The inner loop voltage controller.*

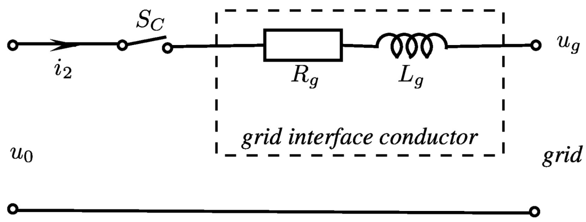

Figure 3. *The outer loop current controller.*

$$x_v = \begin{pmatrix} i_1 \\ u_c \end{pmatrix}, w_v = (w, u),$$

$$w_v = \begin{pmatrix} i_2 \\ u_{ref} \end{pmatrix}, \quad y_v = e_v = u_{ref} - u_0 \tag{2.20}$$

be the state variable, the external input and the output signal (which is the tracking error), respectively.

The reasoning is similar to the one given for (2.1). Hence we conclude that

$$R_f i_1 + L_f \frac{di_1}{dt} = u_f - u_c - R_d i_c = u_f - u_C - R_d (i_1 - i_2) \tag{2.21}$$

whence that

$$\frac{di_1}{dt} = \frac{1}{L_f}(-(R_f + R_d)i_1 + R_d i_2 - u_c + u_0 + u_v) \tag{2.22}$$

on using the relation $u_f = u_v$.

3. Unification

In this section we shall unify linear systems (2.1) given above. For this we view (2.6) as impedance operators $Z = Z(t)$ with current I flowing through it, i.e.

$$Z_L I = -L\frac{dI}{dt}, \quad Z_C I = -\int_0^t I dt, \quad Z_R = -RI, \tag{3.1}$$

where L, C , and R indicate the coil, condenser and resister, respectively. We consider the cascade connection Z of two impedances $Z_i = Z_i(t), i = 1,2$ with the potential u and u_0 , with the current I flowing from u to u_0 , thus the voltage difference is $u_0 - u$. Then we have

$$u_0 - u = (Z_1 + Z_2)I, \tag{3.2}$$

where the addition of impedances is in the sense of addition of operators, i.e. $Z_1 I + Z_2 I$.

Suppose Z_2 is a parallel connection of the coil with inductance L and a resister with resistance r and that the current flowing the coil is i and that Z_1 is a resistance with resistance R_1 .Then

$$Z_2 I = -L\frac{di}{dt} = -r(I-i)\ ,\ \ u_0 - u = -R_1 I - r(I-i)\ . \qquad (3.3)$$

Substituting the first equality in (3.3) into (3.2), we obtain

$$u_0 - u = -(R_1 + r)I + ri$$

or

$$I = \frac{r}{R_1 + r}i - \frac{1}{R_1 + r}(u_0 - u)\ . \qquad (3.4)$$

Substituting this in (3.3), we obtain

$$L\frac{di}{dt} = r\left(\frac{r}{R_1 + r} - 1\right)i - \frac{r}{R_1 + r}(u_0 - u)\ ,$$

whence

$$\frac{di}{dt} = -\frac{R_1 r}{(R_1 + r)L}i - \frac{r}{(R_1 + r)L}(u_0 - u)\ . \qquad (3.5)$$

More generally, we consider the combination of two such cascade connections at node u_0. Two impedances $Z_i = Z_i(t), i = 3, 4$ with the potential $u = 0$ and u_0 and with the current I_2 flowing from u to u_0. Then we have

$$u_0 = (Z_3 + Z_4)I_2\ . \qquad (3.6)$$

We choose $Z_3 = Z_C = u_C$ (condenser) and Z_4 a resister with resistance R_4. Then (3.6) becomes

$$u_0 = u_C + R_4(I_1 - I_2), \qquad (3.7)$$

where I_1 is the current flowing the impedances Z_1, Z_2 in (3.2). Hence substituting (3.7) in (3.5), we conclude

Theorem 3.1. If two cascade connections of two impedances $Z_i, i = 1, ..., 4$ are connected at the node u_0 with voltage difference $u_0 - u$ and i flowing through it, then (3.3) in the form

$$u_0 - u = -R_1 I - L\frac{di}{dt} \qquad (3.8)$$

describes the whole paradigm as either

$$\frac{di}{dt} = -\frac{1}{(R_1 + r)L}(rR_1 i - R_4 I_1)$$

$$-\frac{R_4}{(R_1 + r)L}I_2 - \frac{1}{(R_1 + r)L}(u_C - u). \qquad (3.9)$$

or

$$\frac{di}{dt} = -\frac{R_1 + R_4}{L}I_1 + \frac{R_4}{L}I_2 - \frac{1}{L}(u_C - u), \qquad (3.10)$$

where u_C is described in Corollary 3.1.

Corollary 3.1. The cascade connection (3.5) of two impedances Z_1 (resister) and Z_2 (the parallel connection of a coil and a resister) is a special case of (3.9) with $I_1 = I_2$ and (therefore) $u_C = u_0$. (2.11) is a special case with $I_1 = i_1, I_2 = i_2$.

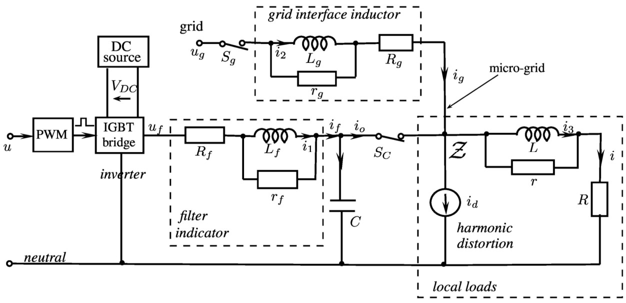

Figure 4. *The three-phase inverter system (represented as a single one).*

Example 1. We consider the three cascade connections $Z_i, i =$ 1, 2, 3 connected at a node Z with the flowing-in currents i_f, i_g and flowing-out currents i, i_d (d is for harmonic distortion). And the configurations of each Z_i are similar. Z_1 indicates the filter inductor with Z_2 a parallel connection of the coil with inductance L_f and a resistor with resistance r_f and that the current flowing the coil is i_1 and that Z_1 is a resister with resistance R_f .

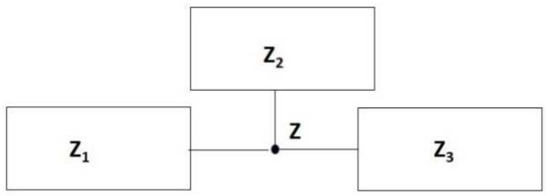

Figure 5. *Cascade connection.*

Other two are similar as given Table 3.1 below.
In this case, (3.4) reads

$$i_f = \frac{r_f}{R_f + r_f}i_1 - \frac{1}{R_f + r_f}(u_0 - u), \qquad (3.11)$$

$$i_g = \frac{r_g}{R_g + rg}i_2 - \frac{1}{R_g + r_g}(u_0 - u_g),$$

$$i = \frac{r}{R\ + r}i_3 - \frac{1}{R\ + r}u_0.$$

Hence from Theorem 3.1 it follows that

$$\frac{di_1}{dt} = -\frac{R_f r_f}{(R_f + r_f)L_f}i_1 - \frac{r_f}{(R_f + r_f)L_f}(u_0 - u)\ , \qquad (3.12)$$

$$\frac{di_2}{dt} = -\frac{R_g r_g}{(R_g + r_g)L_g} i_2 - \frac{r_g}{(R_g + r_g)L_g}(u_0 - u_g) ,$$

$$\frac{di_3}{dt} = -\frac{R\ r}{(R + r\ )L} i_3 - \frac{r}{(R + r)L} u_0 .$$

We want to add one more state variable u_0 which is the electro-motive force u_C generated by the condenser C. Since the current flowing the condenser is $i_f - i_0$, we have

$$\frac{d}{dt} u_0 = \frac{1}{C}(i_f - i_0) . \qquad (3.13)$$

At the node Z, we have by the Kirchhoff law,

$$i_0 + i_g = i + i_d ,$$

whence

$$i_0 = i + i_d - i_g \qquad (3.14)$$

Hence (3.13) amounts to

$$\frac{d}{dt} u_0 = \frac{1}{C}(i_f + i_g - i - i_d) . \qquad (3.15)$$

Substituting (3.11) in this, we deduce that

$$\frac{d}{dt} u_0 = \frac{1}{C}\Big(\frac{r_f}{R_f + r_f} i_1 + \frac{r_g}{R_g + r_g} i_2 - \frac{r}{R + r} i_3$$
$$-\Big(\frac{r_f}{R_f + r_f} + \frac{r_g}{R_g + r_g} + \frac{r}{R + r}\Big) u_0$$
$$- i_d + \frac{1}{R_g + r_g} u_g + \frac{1}{R_f + r_f} u\Big). \qquad (3.16)$$

We put

$$x = \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ u_0 \end{pmatrix},\ u = \begin{pmatrix} w \\ u \end{pmatrix},\ w = \begin{pmatrix} i_d \\ u_g \\ i_{ref} \\ u \end{pmatrix},\ y = \begin{pmatrix} e \\ i_0 \end{pmatrix}, \qquad (3.17)$$

where $e = u_{ref} - u_0$. Then, putting

$$A = \begin{pmatrix} -\frac{R_f r_f}{(R_f + r_f)L_f} & 0 & 0 & -\frac{r_f}{(R_f + r_f)L_f} \\ 0 & -\frac{R_g r_g}{(R_g + r_g)L_g} & 0 & -\frac{r_g}{(R_g + r_g)L_g} \\ 0 & 0 & -\frac{Rr}{(R + r)L} & -\frac{r}{(R + r)L} \\ \frac{r_f}{R_f + r_f} & \frac{r_g}{R_g + r_g} & -\frac{r}{R + r} & -\frac{1}{C}\Big(\frac{r_f}{R_f + r_f} + \frac{r_g}{R_g + r_g} + \frac{r}{R + r}\Big) \end{pmatrix} \qquad (3.18)$$

$$B = (B_1, B_2) ,$$

$$B_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{r_g}{(R_g + r_g)L_g} & 0 \\ 0 & 0 & 0 \\ \frac{1}{C} & \frac{1}{(R_g + r_g)C} & 0 \end{pmatrix},$$

$$B_3 = \begin{pmatrix} \frac{r_f}{(R_f + r_f)L_f} \\ 0 \\ 0 \\ \frac{1}{(R_f + r_f)C} \end{pmatrix},$$

$$C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \quad C_1 = (0, 0, 0, -1) ,$$

$$C_2 = \Big(0, -\frac{r_g}{R_g + r_g}, \frac{r}{R + r}, \frac{1}{R + r} + \frac{1}{R_g + r_g}\Big) ,$$

$$D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & -\frac{1}{R_g + r_g} & 0 & 0 \end{pmatrix},$$

(3.12) and (3.16) lead to

$$\frac{d}{dt} x = Ax + Bu = Ax + (B_1, B_2)\begin{pmatrix} w \\ u \end{pmatrix}, \qquad (3.19)$$

$$y = Cx + Du = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} x + \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}\begin{pmatrix} w \\ u \end{pmatrix}.$$

Table 3.1. *Components of* Z_i's .

Connection	Coil	C.resist	C.current	Resistance	Current
Z_1	L_f	r_f	i_1	R_f	i_f
Z_2	L_g	r_g	i_2	R_g	i_g
Z_3	L	r	i_3	R	i_0

4. H^∞ -Controllers

4.1. H^∞ *-Control Problem*

Following [Kim, p. 7, p. 67], we first give the definition of a chain scattering representation of a system. Suppose $z \in R^m, y \in R^q, w \in R^r$ and $u \in R^p$ denote errors to be corrected, observation output, exogenous input, and control input, respectively and that they are related by

$$\begin{pmatrix} z \\ y \end{pmatrix} = P \begin{pmatrix} w \\ u \end{pmatrix}, \tag{4.1}$$

where

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}. \tag{4.2}$$

According to the embedding principle, this is to be thought of as corresponding to the second equality in (2.1). (4.1) means that

$$z = P_{11}w + P_{12}u\ ,\ \ y = P_{21}w + P_{22}u\ . \tag{4.3}$$

Suppose that y is fed back to u by

$$u = Cy\ , \tag{4.4}$$

where C is a controller. Multiplying the second equality in (4.3) by S and incorporating (4.4), we find that

$$u = Cy = CP_{21}w + CP_{22}u,$$

whence

$$u = (I - P_{22}K)^{-1}CP_{21}w\ .$$

Substituting this in (4.3), we find that

$$z = \Phi w\ , \tag{4.5}$$

where Φ is given by

$$\Phi = P_{11} + P_{12}(E - P_{22}C)^{-1}CP_{21} \tag{4.6}$$

and is referred to as the closed-loop transfer function Φ . (4.6) is sometimes referred to as a linear fractional transformation and denoted by $LF(P;K)$.

H^∞ -controller problem

Find a controller K such that the closed-loop system is internally stable and the transfer function Φ satisfies

$$\|\Phi\|_\infty < \gamma \tag{4.7}$$

for a positive constant γ .

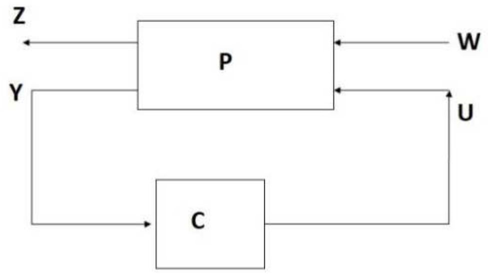

Figure 6. *Control plant*

4.2. Chain Scattering Representation

Assume that P_{21} is a (square) regular matrix (whence $q = r$). Then from the second equality of (4.3), we obtain

$$w = P_{21}^{-1}(y - P_{22}u) = -P_{21}^{-1}P_{22}u + P_{21}^{-1}y\ . \tag{4.8}$$

Substituting (4.8) in the first equality of (4.3), we deduce that

$$z = (P_{12} - P_{11}P_{21}^{-1}P_{22})u + P_{11}P_{21}^{-1}y\ . \tag{4.9}$$

Hence putting

$$\Theta = CHAIN(P) = \begin{pmatrix} P_{12} - P_{11}P_{21}^{-1}P_{22} & P_{11}P_{21}^{-1} \\ -P_{21}^{-1}P_{22} & P_{21}^{-1} \end{pmatrix} = \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix}, \tag{4.10}$$

which is usually referred to as a chain scattering representation of P , we obtain an equivalent form of (4.1)

$$\begin{pmatrix} z \\ w \end{pmatrix} = CHAIN(P)\begin{pmatrix} u \\ y \end{pmatrix} = \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix}\begin{pmatrix} y \\ u \end{pmatrix}. \tag{4.11}$$

Substituting (4.4), (4.11) becomes

$$\begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} \Theta_{11}S + \Theta_{12} \\ \Theta_{21}S + \Theta_{22} \end{pmatrix} u\ ,$$

whence we deduce that the closed-loop transfer function is expressed also as

$$\Phi = (\Theta_{11}S + \Theta_{12})(\Theta_{21}S + \Theta_{22})^{-1} = \Theta S\ , \tag{4.12}$$

the linear fractional transformation (which is referred to as a homographic transformation and denoted by $HM(\Phi;S)$, where in the last equality we mean the action of Θ on the variable S . We must impose the non-constant condition $|\Theta| \neq 0$. Then $\Theta \in GL_{m+r}(R)$. If S is obtained from S' under the action of $\Theta', S = \Theta' S'$, then its composition J with

(4.12) yields $JS' = \Phi\Phi' = \Theta\Theta'S'$, i.e.

$$J = \Theta\Theta',\ HM(\Theta; HM(\Theta'; S)) = HM(\Theta\Theta'; S), \quad (4.13)$$

which is referred to as the cascade connection or the cascade structure of Θ and Θ'.

Thus the chain-scattering representation of a system allows us to treat the feedback connection as a cascade connection.

4.3. Siegel Upper Space

Let * denote the conjugate transpose of a square matrix: $S^* = {}^t\overline{S}$ and let the imaginary part of S defined by $\mathrm{Im}\, S = \frac{1}{2j}(S - S^*)$. Let H_n be the Siegel upper half-space consisting of all the matrices S (recall Eq. (4.4)) whose imaginary parts are positive definite ($\mathrm{Im}\, S > 0$—imaginary parts of all eigen values are positive) and satisfies $S = {}^t S$:

$$H_n = \{S \in M_n(C) \mid \mathrm{Im}\, S > 0, S = {}^t S\} \quad (4.14)$$

and let Sp(n,R) denote the symplectic group of order n:

$$\mathrm{Sp}(n, R) = \left\{ \Theta = \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix} \middle| \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix}^{-1} = \begin{pmatrix} \Theta_{22} & -{}^t\Theta_{12} \\ -{}^t\Theta_{21} & \Theta_{11} \end{pmatrix} \right\}. \quad (4.15)$$

The action of Sp(n,R) on H_n is defined by (4.12) which we restate as

$$\Theta S = (\Theta_{11}S + \Theta_{12})(\Theta_{21}S + \Theta_{22})^{-1} (= \Phi), \quad (4.16)$$

Theorem 4.1. For a controller S living in the Siegel upper space, its rotation $Z = -jS$ lies in the right half-space RHS. i.e. stable having positive real parts. For the controller Z, the feedback connection

$$-ju = Z(-jy) \quad (4.17)$$

is accommodated in the cascade connection of the chain scattering representation Θ (4.13), which is then viewed as the action (4.13) of $\Theta \in$ Sp(n,R) on $S \in H_n$:

$$(\Theta\Theta')S = \Theta(\Theta'S); \quad (4.18)$$

or $HM(\Theta; HM(\Theta'; S)) = HM(\Theta\Theta'; S)$,where Θ is subject to the condition

$${}^t\overline{\Theta}U\Theta = U, \quad (4.19)$$

with $U = \begin{pmatrix} O & I_n \\ -I_n & O \end{pmatrix}$. An FOPID controller (see below), being a unity feedback connection, is also accommodated in this framework.

Remark 4.1. With action, we may introduce the orbit decomposition of H_n and whence the fundamental domain. We note that in the special case of $n = 1$, we have $H_1 = H$ and $\mathrm{Sp}(1;R) = SL_n(R)$ and the theory of modular forms of one variable is well-known. Siegel modular forms are a generalization of the one variable case into several variables. As in the case of the sushmna principle in [7], there is a need to rotate the upper half-space into the right half-space RHS, which is a counter part of the right-half plane RHP. In the case of Siegel modular forms, the matrices are constant, while in control theory, they are analytic functions (mostly rational functions analytic in RHP). A general theory would be useful for controlling theory.

4.4. FOPID

“FO" means “Fractional order and "“PID" refers to “Proportional, Integral, Differential", whence “Proportional" means just constant times the input function $e(t)$, “Integral" means the fractional order integration $I_t^{\lambda} D_t^{-\lambda}$ of $e(t)$ ($\lambda >$ 0), and “Differential" the fractional order differentiation D_t^{δ} of $e(t)$ ($\delta > 0$). The FO $PI^{\lambda}D^{\delta}$ controller (control signal in the time domain) is one of the most refined feed-forward compensator defined as the operator

$$u = (P + I + D)e, \quad (4.20)$$

where

$$Pu = K_p u,\quad Iu = K_i D_t^{-\lambda},\quad Du = K_d D_t^{\delta}, \quad (4.21)$$

where u is the input function, e is the deviation and K_p, K_i, K_d are constant parameters which are to be specified ($K_p -$ the position feedback gain, $K_d -$ the velocity feedback gain). DE (4.21) translates into the state equation

$$Y(s) = C(s)E(s), \quad (4.22)$$

where U, Y indicate the Laplace transforms of u, y, respectively and G is the compensator continuous transfer function

$$C(s) = K_p + K_i s^{-\lambda} + K_d s^{\delta}. \quad (4.23)$$

The derivation of (4.23) from (4.21) depends on the following. The general fractional calculus operator ${}_aD_t^{\alpha}$ is symbolically stated as

$$
{}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}}, & \mathrm{Re}\,\alpha > 0 \\ 1, & \mathrm{Re}\,\alpha = 0 \\ \displaystyle\int_a^t \frac{1}{dt^{\alpha}}, & \mathrm{Re}\,\alpha < 0, \end{cases} \qquad (4.24)
$$

where a and t are the lower and upper limits of integration and α is the order of calculus.

More precisely, the definition of the fractional differo-integral is given by the Riemann-Liouville expression

$$
{}_aD_t^{\alpha}f(t) = \frac{1}{\Gamma(1-\{\alpha\})}\left(\frac{d}{dt}\right)^{\alpha-\{\alpha\}+1}\int_a^t (t-\tau)^{-\{\alpha\}}f(\tau)d\tau \qquad (4.25)
$$

where $\{\alpha\} = \alpha - [\alpha]$ indicates the fractional part of α, with $[\alpha]$ the integral part of α. Thus we are also led to the Riemann-Liouville fractional integral transform:

$$
\mathrm{RL}[f] = \frac{1}{\Gamma(\mu)}\int_0^y (y-x)^{\mu-1}f(x)dx \qquad (4.26)
$$

For more details we refer to [10].

5. Cyber Attack Impact on Smart Grid

However, wherever there is light, there is shadow. Since the smart gird highly depends on the information technologies based on communications systems, it has the same vulnerabilities as the present Internet has. The complexity of integration (of information technology in traditional power grid), diversity of system vendors, urge for timely solutions etc. all lead to increased risk of cyber attack.

We apply the theory of graph-based dynamical systems [2], [9].

$$
\frac{d}{dt}x = f(x,u), \qquad (5.1)
$$

where x, u indicates the state and an input, respectively. This is quite suited for the purpose of assessing the cyber-attack impact on the smart grid. For there is a need of relating a cyber-attack to physical consequences in the electrical network. A dynamical system paradigm gives a flexible framework to model the cause-effect relationships between the cyber data and electric grid states signals. The work is in progress.

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References

[1] Committee of the driving system of EVs, Electrical Engineering Society, The latest technologies of EVs, Ohm-sha, Tokyo 1999.

[2] X. Feng, T. Ziurntos and K. L. Butler-Purry, Dynamic load management for NG IPS ships, Proc. IEEE Power Engineering Society General Meeting, Minneapolis 2010.

[3] F. S. Grodins, Control theory and biological systems, Columbia Univ. Press, New York and London 1963.

[4] Y. Hayakawa, Systems and their controlling, Ohmsha, Tokyo 2008 (in Japanese).

[5] J. W. Helton and O. Merino, Classical control using H1 methods, Theory, optimization and design, SIAM, Philadelphia 1998.

[6] H. Hossain, Z. Han and H. V. Poor, Smart Grid communications and networking, Cambridge UP, Cambridge 2014.

[7] S. Kanemitsu and H. Tsukada, Vistas of Special Functions, World Scientific, Singapore etc. 2007.

[8] H. Kimura, Chain scattering approach to H1-control, Birkhäuser, Boston- Basel-Berlin 1997.

[9] D. Kundurm S, Mashayehk, T. Ziurntos and K. L. Butler-Purry, Cyberattack impact analysis of smart grid, in H. Hossain, Z. Han and H. V. Poor, Smart Grid communications and networking, Cambridge UP, Cambridge 2014, 354-372.

[10] F.-H. Li, N.-L. Wang and S. Kanemitsu, Number Theory and its Applications, World Scientific, Singapore etc. 2013.

[11] M. Okamura, Electric double layer capacitor and energy capacitor system (ECS),1 Nikkan-kogyo-Shinbun-sha, Tokyo 1999.

[12] R. C. Qiu and P. Antonik, Smart Grid and Big Data: Theory and Practice, Wiley, New York 2014.

[13] K. Tsukamoto, EV wars, Asahi Publ. House, Tokyo 2010.

[14] M. Ye, Z.-F.Bai and B.-G.Cao, Robust H2/Hinfinity control for regenerative braking of electric vehicles, ICCA 2007. IEEE International Conference on Control and Automation, 2007, 1366 - 1370.

[15] R. C. Qiu and P. Antonik, Smart Grid and Big Data: Theory and Practice, Wiley, New York 2014.

[16] Carol L. Stimmel, Big Data Analytics Strategies for the Smart Grid, CRC Press, New York 2015.

[17] Q.-Z. Zong and T. Hornik, Control of power inverters in renewable energy and smart grid integration, Wiley, New York 2014.