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On an Almost $C(\alpha)$ −Manifold Satisfying Certain Conditions on the Concircular Curvature Tensor

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Abstract: We classify almost $C(\alpha)-$manifolds, which satisfy the curvature conditions $\widetilde{Z}(\xi,X)R=0$, $\widetilde{Z}(\xi,X)\widetilde{Z}=0$, $\widetilde{Z}(\xi,X)S=0$ and $\widetilde{Z}(\xi,X)P=0$, where $\widetilde{Z}$ is the concircular curvature tensor, P is the Weyl projective curvature tensor, S is the Ricci tensor and R is Riemannian curvature tensor of manifold.

Keywords: Almost $C(\alpha)$ −Manifold, Concircular Curvature Tensor, Projective Curvature Tensor

1. Introduction

An odd-dimensional Riemannian manifold (M,g) is said to be an almost co-Hermitian or almost contact metric manifold if there exist on M a (1,1) tensor field ϕ, a vector field ξ (called the structure vector field) and 1-form η such that

$$\eta(\xi)=1, \qquad \phi^2(X)=-X+\eta(X)\xi \tag{1.1}$$

$$g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y) \tag{1.2}$$

$$\phi(\xi)=0, \quad \eta o\phi=0 \tag{1.3}$$

for any vector fields X,Y on M.

The Sasaki form (or fundamental 2-form) Φ of an almost co-Hermitian manifold (M,g,ϕ,ξ,η) is defined by

$$\Phi(X,Y)=g(X,\phi Y)$$

for all X,Y on M and this form satisfies $\eta\Lambda\Phi^n\neq 0$. This means that every almost co-Hermitian manifold is orientable and (η,Φ) defines an almost cosymplectic structure on M. If this associated structure is cosymplectic $d\Phi=d\eta=0$, then M is called an almost co-Kähler manifold. On the other hand, when $\Phi=d\eta$, the associated almost cosymplectic structure is a contact structure and is an almost Sasakian manifold. It is well known every contact manifold has an almost Sasakian structure.

The Nijenhuis tensor of type(1,1) −tensor field ϕ is type (1,2) $[\phi,\phi]$ defined by

$$[\phi,\phi](X,Y)=\phi^2[X,Y]+[\phi X,\phi Y]-\phi[\phi X,Y]$$
$$-\phi\,[X,\phi Y] \tag{1.4}$$

where $[X,Y]$ is the Lie bracket of $X,Y\in\chi(M)$.

On the other hand, an almost co-complex structure is called integrable if $[\phi,\phi]=0$ and normal $[\phi,\phi]+2d\eta\otimes\xi=0$. An integrable almost cocomplex structure is a cocomplex structure. A co-Kähler manifold (or normal cosymplectic manifold) is an integrable (or equivalently, a normal) almost co-Kähler manifold, while a Sasakian manifold is a normal almost contact metric manifold [3].

2. Preliminaries

In [2], N(k) − contact metric manifolds satisfying $\widetilde{Z}(\xi,X)\widetilde{Z}=\widetilde{Z}(\xi,X)R=R(\xi,X)\widetilde{Z}=0$ were classified.

In [1], $\widetilde{Z}(\xi,X)R=R(\xi,X)R=\widetilde{Z}(\xi,X)S=\widetilde{Z}(\xi,X)C=0$ on P −Sasakian manifolds and obtained the some results.

M. M. Tripathi and J. S. Kim gave a classification of (μ,κ) −manifolds satisfying the conditions $\widetilde{Z}(\xi,X)S=0$ [7].

Definition 2.1. An almost $C(\alpha)$ −manifold M is an almost co-Hermitian manifold such that the Riemann curvature tensor satisfies the following property, $\exists\alpha\in R$ such that

$$R(X,Y,W,Z)=R(X,Y,\phi Z,\phi W)$$
$$+\alpha\{-g(X,Z)g(Y,W)+g(X,W)g(Y,Z)$$
$$+g(X,\phi Z)g(Y,\phi W)$$
$$-g(X,\phi W)g(Y,\phi Z)\}. \tag{2.1}$$

Moreover, if such a manifold has constant ϕ −sectional curvature equal to c, then its curvature tensor is given by

$$R(X,Y)Z = \left(\frac{c+3\alpha}{4}\right)\{g(Y,Z)X - g(X,Z)Y\} + \left(\frac{c-\alpha}{4}\right)\{g(X,\phi Y)\phi Z - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z - \eta(X\eta(Z)Y + \eta(Y)\eta(Z)X - g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}, \quad (2.2)$$

for any $\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z} \in \chi(\boldsymbol{M})$.

A normal almost $C(\alpha)-$ manifold is said to be a $C(\alpha)-$manifold. For example, Co-Kählerian, Sasakian and Kenmotsu manifolds are $C(0), C(1)$ and $C(-1)-$manifolds, respectively [3].

Theorem 2.1.

(i) An almost co-Hermitian manifold $\boldsymbol{M}$ is $\boldsymbol{\alpha}-$Sasakian if and only if

$$(\nabla_X\phi)Y = \alpha\{g(X,Y)\xi - \eta(X)Y\}, \quad (2.3)$$

for all $X,Y \in \chi(M)$.

(ii) If M is $\alpha-$Sasakian, then ξ is Killing vector field and

$$\nabla_X\xi = -\alpha\phi X \quad (2.4)$$

(iii) An $\boldsymbol{\alpha}-$Sasakian manifold is a $\boldsymbol{C}(\boldsymbol{\alpha}^2)-$manifold [3].

Theorem 2.2.

(i) An almost co-Hermitian manifold $\boldsymbol{M}$ is an $\boldsymbol{\alpha}-$Kenmotsu manifold if and only if

$$(\nabla_X\phi)Y = \alpha\{g(\phi X,Y)\xi - \eta(Y)\phi X\}, \quad (2.5)$$

for all $X,Y \in \chi(M)$.

(iii) *An* $\boldsymbol{\alpha}-$Kenmotsu manifold is a $\boldsymbol{C}(-\boldsymbol{\alpha}^2)-$manifold.

3. An Almost $C(\alpha)-$Manifold Satisfying Certain Conditions on the Concircular Curvature Tensor

In this section, we will give the main results for this paper.

Let $\boldsymbol{M}$ be a $(\boldsymbol{2n+1})-$ dimensional almost $\boldsymbol{C}(\boldsymbol{\alpha})-$manifold and denote Riemannian curvature tensor of $\boldsymbol{R}$, then we have from (2.2), for $\mathbf{X} = \boldsymbol{\xi}$

$$R(\xi,Y)Z = \alpha\{g(Y,Z)\xi - \eta(Z)Y\}. \quad (3.1)$$

In the same way, choosing $\mathbf{Z} = \boldsymbol{\xi}$ in (2.2), we have

$$R(X,Y)\xi = \alpha\{\eta(Y)X - \eta(X)Y\}. \quad (3.2)$$

In (3.2), choosing $\mathbf{Y} = \boldsymbol{\xi}$, we obtain

$$R(X,\xi)\xi = \alpha\{X - \eta(X)\xi\}. \quad (3.3)$$

Also from (3.2), we obtain

$$\eta(R(X,Y)Z) = \alpha\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}. \quad (3.4)$$

From (2.2), we can state

$$R(X,e_i)e_i + R(X,\phi e_i)e_i + R(X,\xi)\xi = \left(\frac{c+3\alpha}{4}\right)\{nX - g(X,e_i)e_i + nX - g(X,\phi e_i)\phi e_i + X - g(X,\xi)\xi\} + \left(\frac{c-\alpha}{4}\right)\{3g(X,\phi e_i)\phi e_i - 2n\eta(X)\xi + 3g(X,\phi^2 e_i)\phi^2 e_i\eta(X)\xi - X\} \quad (3.5)$$

for $\{\boldsymbol{e_1},\boldsymbol{e_2},\dots,\boldsymbol{\phi e_1},\boldsymbol{\phi e_2},\dots,\boldsymbol{\phi e_n},\boldsymbol{\xi}\}$ orthonormal basis of $\boldsymbol{M}$. From (3.5), for $\in \chi(\boldsymbol{M})$, we obtain

$$S(X,Y) = \left(\frac{\alpha(3n-1)+c(n+1)}{2}\right) + \left(\frac{(\alpha-c)(n+1)}{2}\right)\eta(X)\eta(Y), \quad (3.6)$$

which is equivalent to

$$QX = \left(\frac{\alpha(3n-1)+c(n+1)}{2}\right)X + \left(\frac{(\alpha-c)(n+1)}{2}\right)\eta(X)\xi. \quad (3.7)$$

From (3.7) we can give the following corollary.

Corollary 3.1. An almost $\boldsymbol{C}(\boldsymbol{\alpha})-$manifold is always an $\boldsymbol{\eta}-$Einstein manifold.

Also, from (3.6), we can easily see

$$\tau = n[\alpha(3n+1) + c(n+1)], \quad (3.8)$$

$$S(X,\xi) = 2n\alpha\eta(X) \quad (3.9)$$

and

$$Q\xi = 2n\alpha\xi. \quad (3.10)$$

Definition 3.1. Let $(\boldsymbol{M},\boldsymbol{g})$ be an $(\boldsymbol{2n+1})-$dimensional Riemannian manifold. Then the Weyl concircular curvature tensor $\widetilde{\boldsymbol{Z}}$ is defined by

$$\widetilde{Z}(X,Y)W = R(X,Y)\mathrm{W} - \frac{\tau}{2n(2n+1)}\{\mathrm{g}(\mathrm{Y},\mathrm{W})\mathrm{X} - \mathrm{g}(\mathrm{X},\mathrm{W})\mathrm{Y}\} \quad (3.11)$$

for all $X,Y,W \in \chi(M)$, where τ is the scalar curvature of M[5].

In (3.11), choosing $X = \xi$, we obtain

$$\tilde{Z}(\xi,Y)W=\left(\alpha-\frac{\tau}{2n(2n+1)}\right)\{g(Y,W)\xi-\eta(W)Y\} \quad (3.12)$$

Theorem 3.1. Let M be a $(2n+1)$ −dimensional an almost $C(\alpha)$ −manifold. Then $\tilde{Z}(\xi,X)R=0$ if and only if M either has $c-$ sectional curvature or the scalar curvature $\tau=2n\alpha(2n+1)$.

Proof: Suppose that $\tilde{Z}(\xi,X)R=0$. Then from (3.11), we have

$$\begin{aligned}(\widetilde{Z(\xi,X)R})(U,W)Z &= \tilde{Z}(\xi,X)R(U,W)Z-R(\tilde{Z}(\xi,X)U,W)Z\\ &-R\big(U,\tilde{Z}(\xi,X)W\big)R-R(U,W)\tilde{Z}(\xi,X)Z\\ &=0. \end{aligned} \quad (3.13)$$

Using (3.12) in (3.13), we obtain

$$\begin{aligned}&=\left(\alpha-\frac{\tau}{2n(2n+1)}\right)\{g(X,R(U,W)Z)\xi-\eta(R(U,W)Z)X\\ &-g(X,U)R(\xi,W)Z+\eta(U)R(X,W)Z\\ &-g(X,W)R(U,\xi)Z+\eta(W)R(U,X)Z\\ &-g(X,Z)R(U,W)\xi+\eta(Z)R(U,W)X\}\\ &=0. \end{aligned} \quad (3.14)$$

Using (3.2), (3.3) and putting $U=\xi$ in (3.14), we get

$\left(\alpha-\frac{\tau}{2n(2n+1)}\right)\{R(X,U)W-\alpha\{g(U,W)X-g(X,W)U\}\}$=0.

Therefore, manifold has either $\alpha-$ sectional curvature or $\tau=2n\alpha(2n+1)$. This implies that $\alpha=c$.

Theorem 3.2. Let M be a $(2n+1)$ −dimensional an almost $C(\alpha)$ −manifold**.** $\tilde{Z}(\xi,X)\tilde{Z}=0$ if and only if M either has $\alpha-$ sectional curvature or the scalar curvature $\tau=2n\alpha(2n+1)$.

Proof: Suppose that $\tilde{Z}(\xi,X)\tilde{Z}=0$, we have

$$\begin{aligned}(\widetilde{Z(\xi,X)\tilde{Z}})(Y,U)W &= \tilde{Z}(\xi,X)\tilde{Z}(Y,U)W-\tilde{Z}(\tilde{Z}(\xi,X)Y,U)W\\ &-\tilde{Z}\big(Y,\tilde{Z}(\xi,X)U\big)W-\tilde{Z}(Y,U)\tilde{Z}(\xi,X)W\\ &=0. \end{aligned} \quad (3.15)$$

Using the equations (3.12) and (3.2), (3.3) in (3.15), we have

$$\begin{aligned}&=\tilde{Z}(\xi,X)R(Y,U)W-\tilde{Z}(R(\xi,X)Y,U)W\\ &-\tilde{Z}(Y,R(\xi,X)U)W-\tilde{Z}(Y,U)R(\xi,X)W\\ &+\left(\alpha-\frac{\tau}{2n(2n+1)}\right)\{g(Y,W)\tilde{Z}(\xi,X)U\\ &-g(U,W)\tilde{Z}(\xi,X)Y+g(X,Y)\tilde{Z}(\xi,U)W\\ &-\eta(Y)\tilde{Z}(X,U)W+g(X,U)\tilde{Z}(Y,\xi)W\\ &-\eta(U)\tilde{Z}(Y,X)W+g(X,W)\tilde{Z}(Y,U)\xi\\ &-\eta(W)\tilde{Z}(Y,U)X\}\\ &=0. \end{aligned} \quad (3.16)$$

Putting $Y=\xi$ in (3.16), we get

$\left(\alpha-\frac{\tau}{2n(2n+1)}\right)\{R(X,U)W-\alpha\{g(U,W)X-g(X,W)U\}\}$=0.

This tell us that $\boldsymbol{M}$ has either $\boldsymbol{\alpha}$ −sectional curvature or the scalar curvature $\boldsymbol{\tau}=\mathbf{2n\alpha(2n+1)}$.

The converse is obvious.

Theorem 3.3. Let $\mathbf{M}$ be a $(\mathbf{2n+1})$ −dimensional an almost $\mathbf{C(\alpha)}$ −manifold. Then $\widetilde{\mathbf{Z}}(\boldsymbol{\xi},\mathbf{X})\mathbf{S}=\mathbf{0}$ if and only if $\boldsymbol{M}$ reduce an Einstein manifold.

Proof: We suppose that $\widetilde{\mathbf{Z}}(\boldsymbol{\xi},\mathbf{X})\mathbf{S}=\mathbf{0}$, which implies that

$$\boldsymbol{S}\big(\widetilde{\mathbf{Z}}(\boldsymbol{\xi},\mathbf{X})\mathbf{U},\mathbf{W}\big)=+\boldsymbol{S}\big(\boldsymbol{U},\widetilde{\mathbf{Z}}(\boldsymbol{\xi},\boldsymbol{X})\boldsymbol{W}\big)=\mathbf{0}. \quad (\mathbf{3.17})$$

Using (3.12) in (3.17), we get

$$\begin{aligned}&\boldsymbol{S}\left(\boldsymbol{R}(\boldsymbol{\xi},\boldsymbol{X})\boldsymbol{U}-\frac{\boldsymbol{\tau}}{\mathbf{2n(2n+1)}}\{\boldsymbol{g}(\boldsymbol{X},\boldsymbol{U})\boldsymbol{\xi}-\boldsymbol{\eta}(\boldsymbol{U})\boldsymbol{X}\},\boldsymbol{W}\right)\\ &+\boldsymbol{S}\left(\boldsymbol{U},\boldsymbol{R}(\boldsymbol{\xi},\boldsymbol{X})\boldsymbol{W}-\frac{\boldsymbol{\tau}}{\mathbf{2n(2n+1)}}\{\boldsymbol{g}(\boldsymbol{X},\boldsymbol{W})\boldsymbol{\xi}-\boldsymbol{\eta}(\boldsymbol{W})\boldsymbol{X}\}\right)\\ &=\mathbf{0} \end{aligned} \quad (3.18)$$

Using (3.1), (3.9) in (3.18), we obtain

$$\begin{aligned}&=\mathbf{2n\boldsymbol{\alpha}^2}\boldsymbol{g}(\boldsymbol{X},\boldsymbol{U})\boldsymbol{\eta}(\boldsymbol{W})-\boldsymbol{\alpha\eta}(\boldsymbol{U})\boldsymbol{S}(\boldsymbol{X},\boldsymbol{W})\\ &-\frac{\boldsymbol{\tau}}{\mathbf{2n(2n+1)}}\big(\mathbf{2n}\boldsymbol{\alpha g}(\boldsymbol{X},\boldsymbol{U})\boldsymbol{\eta}(\boldsymbol{W})-\boldsymbol{\eta}(\boldsymbol{U})\boldsymbol{S}(\boldsymbol{X},\boldsymbol{W})\big)\\ &+\mathbf{2n\boldsymbol{\alpha}^2}\boldsymbol{g}(\boldsymbol{X},\boldsymbol{W})\boldsymbol{\eta}(\boldsymbol{U})-\boldsymbol{\alpha\eta}(\boldsymbol{W})\boldsymbol{S}(\boldsymbol{X},\boldsymbol{U})\\ &-\frac{\boldsymbol{\tau}}{\mathbf{2n(2n+1)}}\big(\mathbf{2n}\boldsymbol{\alpha g}(\boldsymbol{X},\boldsymbol{W})\boldsymbol{\eta}(\boldsymbol{U})-\boldsymbol{\eta}(\boldsymbol{W})\boldsymbol{S}(\boldsymbol{X},\boldsymbol{U})\big)\\ &=\mathbf{0}. \end{aligned} \quad (3.19)$$

Putting $\boldsymbol{U}=\boldsymbol{\xi}$ in (3.19), we get

$$\boldsymbol{S}(\boldsymbol{X},\boldsymbol{W})\left\{\frac{\boldsymbol{\tau}}{\mathbf{2n(2n+1)}}-\boldsymbol{\alpha}\right\}+\boldsymbol{g}(\boldsymbol{X},\boldsymbol{W})\left\{\mathbf{2n\boldsymbol{\alpha}^2}-\boldsymbol{\alpha}\frac{\boldsymbol{\tau}}{\mathbf{2n+1}}\right\}=\mathbf{0},$$

under the condition $\boldsymbol{\alpha}\neq\frac{\tau}{\mathbf{2n(2n+1)}}$,

$$\boldsymbol{S}(\boldsymbol{X},\boldsymbol{W})=\mathbf{2n}\boldsymbol{\alpha g}(\boldsymbol{X},\boldsymbol{W}).$$

Therefore, the manifold is Einstein manifold.

The converse is obvious.

If $\boldsymbol{M}$ is an Einstein manifold, the scalar curvature $\boldsymbol{\tau}$ of $\boldsymbol{M}$ is

$$\boldsymbol{\tau}=\mathbf{2n}\boldsymbol{\alpha}(\mathbf{2n+1}). \quad (\mathbf{3.20})$$

By corresponding (3.8) and (3.20) we obtain $\boldsymbol{\alpha}=\boldsymbol{c}$ which implies that $\boldsymbol{M}$ is of constant sectional curvature $\boldsymbol{c}$.

Definition 3.2. Let $(\mathbf{M},\mathbf{g})$ be a $(\mathbf{2n+1})-$ dimensional Riemannian manifold. Then Weyl projective curvature tensor $\boldsymbol{P}$ is defined by

$$\boldsymbol{P}(\boldsymbol{X},\boldsymbol{Y})\boldsymbol{Z}=\boldsymbol{R}(\boldsymbol{X},\boldsymbol{Y})\boldsymbol{Z}-\frac{\mathbf{1}}{\mathbf{2n}}\{\boldsymbol{S}(\boldsymbol{Y},\boldsymbol{Z})\boldsymbol{X}-\boldsymbol{S}(\boldsymbol{X},\boldsymbol{Z})\boldsymbol{Y}\}, \quad (\mathbf{3.21})$$

where $\boldsymbol{R}$ is Riemannian curvature tensor and $\boldsymbol{S}$ is Ricci tensor [5].

Theorem 3.2. Let $\boldsymbol{M}$ be a $(\mathbf{2n+1})$ −dimensional an almost $\boldsymbol{C}(\boldsymbol{\alpha})-$ manifold. Then, $\widetilde{\mathbf{Z}}(\boldsymbol{\xi},\mathbf{X})\mathbf{P}=\mathbf{0}$ if and only if $\boldsymbol{M}$ reduce an Einstein manifold.

Proof: Suppose that $\widetilde{\mathbf{Z}}(\boldsymbol{\xi},\mathbf{X})\mathbf{P}=\mathbf{0}$. Then we have,

$$\begin{aligned}(\widetilde{Z(\xi,X)\mathrm{P}})(Y,U)W &= \tilde{Z}(\xi,X)\mathrm{P}(Y,U)W-\mathrm{P}(\tilde{Z}(\xi,X)Y,U)W\\ &-\mathrm{P}\big(Y,\tilde{Z}(\xi,X)U\big)W-\mathrm{P}(Y,U)\tilde{Z}(\xi,X)W\\ &=0, \end{aligned} \quad (3.22)$$

for $X,Y,U,W\in\chi(M)$**.** Using (3.12) in (3.22), we get

$$\left(\alpha-\frac{\tau}{2n(2n+1)}\right)\{\alpha g(X,U)\eta(W)\xi-\alpha g(U,W)X+$$

$$\frac{1}{2n}\{S(U,W)X + S(X,W)\eta(U)\xi - S(U,X)\eta(W)\xi\}$$
$$-\alpha g(X,W)\eta(U)\xi + P(U,X)W\}$$
$$= 0. \qquad (3.23)$$

Taking inner product both sides of (3.23) by $\xi \in \chi(M)$, we obtain

$$\left(\alpha - \frac{\tau}{2n(2n+1)}\right)\{\alpha g(X,U)\eta(W) - \alpha g(U,W)\eta(X) +$$
$$\frac{1}{2n}\{S(U,W)\eta(X) + S(X,W)\eta(U) - S(U,X)\eta(W)\}$$
$$-\alpha g(X,W)\eta(U) + \eta(P(U,X)W)\}$$
$$= 0. \qquad (3.24)$$

Also making use of (3.21), we obtain

$$\eta(P(U,X)W) = \alpha\{g(X,W)\eta(U) - g(U,W)\eta(X)\}$$
$$-\frac{1}{2n}\{S(X,W)\eta(U) - S(U,W)\eta(X)\}. \qquad (3.25)$$

Using (3.25) in (3.24) and choosing $W = \xi$, we have provided that $\alpha \neq \frac{\tau}{2n(2n+1)}$,

$$S(U,X) = 2n\alpha g(U,X).$$

So, the manifold is an Einstein manifold. The converse is obvious.

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