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Generalized Nörlund summability of fuzzy real numbers

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Abstract: Fuzzy set, mathematical modelling in order to some uncertainty in 1965 was described by L. A. Zadeh [7]. In studies on fuzzy sets, fuzzy numbers [5], fuzzy relations [5], fuzzy function [5], fuzzy sequence [4] is defined as concepts. After Nörlund fuzzy and blurry Riez summability have been identified [6]. In this study, fuzzy Generalized Nörlund summability have been defined and Generalized Nörlund summability necessary and sufficient conditions to ensure the regular was investigated.

Keywords: Generalized Nörlund Summability, Nörlund Mean Fuzzy, Fuzzy Mean Riesz, Cesaro Mean Fuzzy

1. Introduction

This section will be the basic concepts of fuzzy sets.

Definition 1.1. A fuzzy set A on the universe X is a set defined by a membership function μ_A representing a mapping

$$\mu_A : X \to [0,1].$$

Here the value $\mu_A(x)$ for the fuzzy set A is called the membership value or the grade of membership of $x \in X$. The membership value represents the degree of x belonging to the fuzzy set *A*. [5]

Definition 1.2. Let D denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on the real line *R*. For $X, Y \in D$, we define

$$d(X,Y) = \max\left(|a_1 - b_1|, |a_2 - b_2|\right)$$

where $X = [a_2, a_2]$, $Y = [b_2, b_2]$. It is known that (D, d) is a complete metric space [6].

Definition 1.3. A fuzzy real number *X* is called convex if $X(t) \geq X(s) \wedge X(r) = \min(X(s), X(r))$, where $s < t < r$. If there exists $t_0 \in IR$, such that $X(t_0) = 1$, then the fuzzy real number *X* is called normal. A fuzzy real number *X* is a fuzzy set on *R* and is a mapping

$$X : IR \to I(=[0,1])$$

associating each real number t with its grade of membership $X(t)$ [6].

A fuzzy real number *X* is said to be upper-semicontinuous if for each $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$, for all $a \in I$ is open in the usual topology of *R* [6].

The set of all upper-semicontinuous, normal, convex fuzzy number is denoted by $R(I)$ [6].

Definition 1.4. The α-level set of a fuzzy real number *X*, for $0 < \alpha \leq 1$ denoted by X^α is defined as $X^\alpha = \{t \in R : X(t) \geq \alpha\}$; for $\alpha = 0$, it is the closure of the strong 0-cut (that is, the closure of the set $\{t \in IR : X(t) > 0\}$). Throughout the article α means $\alpha \in (0,1]$ unless otherwise stated [6].

Theorem 1.1. Let *A* is a fuzzy number if and only if there exists a closed interval $[a,b] \neq \varnothing$ such that

$$\mu_A(x) = \begin{cases} 1 & , x \in [a,b] \\ l(x) & , x \in (-\infty, a) \\ r(x) & , x \in (b, \infty) \end{cases}$$

where *l* is a function from $(-\infty, a)$ to $[0,1]$ that is monotonic increasing, continuous from the right, and such that $l(x) = 0$ for $x \in (-\infty, w_1)$; r is a function from (b, ∞) to $[0,1]$ that is monotonic decreasing, continuous from the left, and such that $r(x) = 0$ for $x \in (w_2, \infty)$. [3]

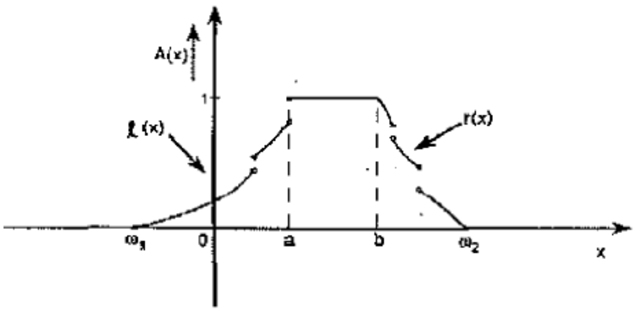

Figure 1.1. Fuzzy number [3]

Definition 1.5. The set R of all real numbers can be embedded in $R(I)$. For each $r \in R$, $\overline{r} \in R$ is defined by

$$\overline{r}(t) = \begin{cases} 1, t = r \\ 0, t \neq r \end{cases} \text{ [6].}$$

Definition 1.6. Let $\overline{d} : R(I)XR(I) \to IR$ be defined by

$$\overline{d}(X,Y) = \sup_{0 \le \alpha \le 1} d(X^{\alpha}, Y^{\alpha}).$$

Then $\overline{d}$ defines a metric on $R(I)$. It is well known that $(R(I), \overline{d})$ is a complete metric space. The additive identity and multiplicative identity in $R(I)$ are denoted by $\overline{0}$ and $\overline{1}$ respectively [6].

2. Preliminaries

Fuzzy sequence defined on fuzzy sets in this section will be Reisz and Nörlund averages. A fuzzy set of samples to be explained.

Definition 2.1. A sequence $\overline{A} = (\overline{A}_k)$ of fuzzy numbers is said to be convergent to the fuzzy number $\overline{A}_0$, written as $\lim_{k \to \infty} \overline{A}_k = \overline{A}_0$, if for every $\varepsilon > 0$ there exists a positive integer N such that $d(\overline{A}_k, \overline{A}_0) < \varepsilon$ for every $k > N$. [1]

Example 2.1.

$$\overline{A}_k(x) = \begin{cases} \dfrac{k}{k+2}x + \dfrac{2-3k}{k+2} & , \quad x \in \left[\dfrac{3k-2}{k}, 4\right] \\ \dfrac{-k}{k+2}x + \dfrac{5k+2}{k+2} & , \quad x \in \left[4, \dfrac{5k+2}{k}\right] \\ 0 & , \quad x \notin \left[\dfrac{3k-2}{k}, \dfrac{5k+2}{k}\right] \end{cases}$$

from of $\overline{A} = (\overline{A}_k)$ Consider the fuzzy number sequence. Limit of this sequence,

$$\overline{A}_0(x) = \begin{cases} x - 3 & , \quad x \in [3,4] \\ -x + 5 & , \quad x \in [4,5] \\ 0 & , \quad x \notin [3,5] \end{cases}.$$

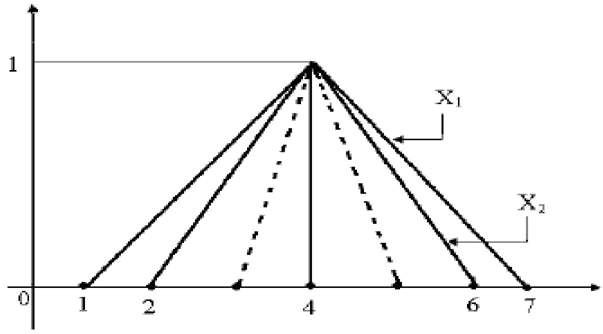

Figure 1.2. *$(\overline{A}_k)$ Fuzzy number sequence $\overline{A}_0$ the convergence of the fuzzy number [2]*

Definition 2.2. Let (p_n) be a sequence of non-negative real numbers which are not all zero and

$$P_n = p_1 + p_2 + ... + p_n$$

for all $n \in N$. A sequence $(\overline{s}_n)$ of fuzzy real numbers is said to be summable by Nörlund mean $(\overline{N}, p)$ to $\overline{L}$, if

$$\overline{d}\left(\frac{1}{P_n}\sum_{v=1}^{n} p_{n-v+1}\overline{s}_v, \overline{L}\right) \to 0$$

as $n \to \infty$.

Definition 2.2. Let (p_n) be a sequence of non-negative real numbers which are not all zero and

$$P_n = p_1 + p_2 + ... + p_n$$

for all $n \in N$. A sequence $(\overline{s}_n)$ of fuzzy real numbers is said to be summable by Riesz mean $(\overline{R}, p)$ to $\overline{L}$, if

$$\overline{d}\left(\frac{1}{P_n}\sum_{v=1}^{n} p_v\overline{s}_v, \overline{L}\right) \to 0$$

as $n \to \infty$.

3. Generalized Nörlund Summability of Fuzzy Real Numbers

This section will be defined the fuzzy generalized Nörlund summability and explored regularity conditions on the summability.

Definition 3.1. Let (p_n) , (q_n) be a sequence of non-negative real numbers which are not all zero and

$$r_n = \sum_{v=0}^{n} p_{n-v}q_v$$

for all $n \in N$. A sequence $(\overline{s}_n)$ of fuzzy real numbers is said to be summable by Nörlund summability $(\overline{N}, p, q)$ to $\overline{L}$, if

$$\overline{d}\left(\frac{1}{r_n}\sum_{v=1}^{n} p_{n-v+1}q_v\overline{s}_v, \overline{L}\right) \to 0$$

as $n \to \infty$.

Theorem 3.1. Let (p_n), (q_n), (r_n) be sequences defined in the defenition 2.1. Then, $(\overline{N}, p, q)$ regular if and only if $\lim\limits_{n\to\infty}\frac{p_n}{r_n} = 0$.

Proof: Sufficiency: Let $(\overline{s}_n)$ be any any convergent sequence of fuzzy real numbers, and $\lim\limits_{n\to\infty}\overline{s}_n = \overline{L}$. Without loss of generality, we may assume that $\overline{L} = \overline{0}$. For a fixed $\varepsilon > 0$ there exists no such that, $\overline{d}(s_n, \overline{0}) < H$ for $n \ge n_0$. If $\lim\limits_{n\to\infty}\frac{p_n}{r_n} = 0$ then for all $\varepsilon > 0$ there exists $n_1 \in IN$ such that,

$$\frac{p_n}{r_n} < \frac{\varepsilon}{2H\max(n_0, n_1)}$$

for $n > n_1$. Let $n_2 = \max(n_0, n_1)$ and assume $\overline{d}(\overline{s}_n, \overline{0}) < \frac{\varepsilon}{2}$ for all $n > n_2$. Then,

$$\frac{p_n}{r_n} < \frac{\varepsilon}{2H\max(n_0, n_1)}.$$

So for all $n > n_2$ we obtain.

$$\overline{d}\left(\frac{1}{r_n}\sum_{v=1}^{n} p_{n-v+1}q_v\overline{s}_v, \overline{0}\right)$$

$$\le \overline{d}\left(\frac{1}{r_n}\sum_{v=1}^{n_2} p_{n-v+1}q_v\overline{s}_v, \overline{0}\right) + \overline{d}\left(\frac{1}{r_n}\sum_{v=n_2+1}^{n} p_{n-v+1}q_v\overline{s}_v, \overline{0}\right)$$

$$= \overline{d}\left(\frac{1}{r_n}(p_n q_1\overline{s}_1 + \ldots + p_{n-n_2+1}q_{n_2}\overline{s}_{n_2}), \overline{0}\right) + $$
$$+ \overline{d}\left(\frac{1}{r_n}(p_{n-n_2}q_{n_2+1}\overline{s}_{n_2+1} + \ldots + p_1 q_n\overline{s}_n), \overline{0}\right)$$

$$= \frac{p_n q_1}{r_n}\overline{d}(\overline{s}_1, \overline{0}) + \ldots + \frac{p_{n-n_2+1}q_{n_2}}{r_n}\overline{d}(\overline{s}_{n_2}, \overline{0}) + $$
$$+ \frac{p_{n-n_2}q_{n_2+1}}{r_n}\overline{d}(\overline{s}_{n_2+1}, \overline{0}) + \ldots + \frac{p_1 q_n}{r_n}\overline{d}(\overline{s}_n, \overline{0})$$

$$\le \frac{p_n q_1}{r_n}H + \ldots + \frac{p_{n-n_2+1}q_{n_2}}{r_n}H + \frac{p_{n-n_2}q_{n_2+1}}{r_n}.\frac{\varepsilon}{2} + \ldots + \frac{p_1 q_n}{r_n}\frac{\varepsilon}{2}$$

$$\le \frac{\varepsilon}{2Hn_2}H + \ldots + \frac{\varepsilon}{2Hn_2}H + \left(\frac{p_{n-n_2}}{r_n} + \ldots + \frac{p_1}{r_n}\right)\frac{\varepsilon}{2}$$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Necessity: Let $(\overline{N}, p, q)$ to get regular and $\overline{e}_1 = (\overline{1}, \overline{0}, \overline{0}, \ldots) = (\overline{s}_v)$ consider the sequence. Then,

$$\lim_{v\to\infty}\overline{s}_v = \overline{0}$$

and so

$$\lim_{n\to\infty}\frac{p_n}{r_n} = 0$$

since

$$\lim_{n\to\infty}\overline{d}\left(\sum_{v=1}^{n}\frac{p_{n-v+1}q_v}{r_n}\overline{e}_1, \overline{0}\right).$$

4. Results

In this section, fuzzy generalized summability Nörlund been identified and will be investigated regularity conditions on the summability.

$(\overline{N}, p, q)$ regular if and only if $\lim\limits_{n\to\infty}\frac{p_n}{r_n} = 0$ we obtained.

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