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New types of chaotic maps

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Abstract: In this paper, we will study a new class of chaotic maps on locally compact Hausdorff spaces called Lambda -type chaotic maps and θ-type chaotic maps. The Lambda -type chaotic map implies chaotic map which implies θ-type chaotic map. Further, the definition of topological Lambda -type chaos implies John Tylar definition which implies topological θ-type chaos definition. Relationships with some other types of chaotic maps defined on topological spaces are given.

Keywords: Lambda -Type Chaos, Chaotic Maps, Irresolute Maps, Transitivity

1. Introduction

In this research paper, we have studied new types of topological transitivity. This is intended as a survey article on topological transitivity of a discrete system given by a λ-irresolute self-map of a compact topological space. On one hand it introduces postgraduate students to the study of new types of topological transitivity and gives an overview of results on the topic, but, on the other hand, it covers some of the recent developments. We studied a new type of transitive map called λ-type transitive and investigate some of its properties. Relationships with some other type of transitive maps are given. We list some relevant properties of the λ-type transitive map. and θ-type transitive and studied notions of λ- minimal mapping. We have proved that every λ-type transitive map is topologically θ-type transitive map but the converse not necessarily true. In 1986, Maki [1] continued the work of Levine and Dunham on generalized closed sets and closure operators by introducing the notion of Λ -sets in topological spaces. A subset A of a space X is called a Λ -set if it coincides with its kernel (saturated set), i.e. to the intersection of all open supersets of A. A subset A of a space X is called λ-closed [2] if A=L∩C, where L is a Λ-set and C is a closed set. The complement of a λ-closed set is called λ-open set. We denote the collection of all λ-open (resp. λ-closed) sets by λO(X) (resp. λC(X)).A point x ∈ X is called λ-cluster point of a subset A ⊂ X [3, 4] if for every λ-open set U of X containing x A∩U ≠ϕ . The set of all λ-cluster points is called the λ-closure of A and is denoted by $Cl_{\lambda}(A)$. A point x ∈ X is said to be a λ-interior point of a subset A ⊂ X if there exists a λ-open set U containing x such that U ⊂ A. The set of all λ-interior points of A is said to be the λ-interior of A and is denoted by $Int_{\lambda}(A)$.

Maps and of course irresolute maps stand among the most important notions in the whole of pure and applied mathematical science. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. In 1972, Crossley and Hildebrand [5] introduced the notion of irresoluteness. . Many different forms of irresolute maps have been introduced over the years. Andrijevic [6] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. A subset A of a topological space X is called regular open if A=Int(Cl(A)), and regular closed if X\A is regular open, or equivalently if A=Cl(Int(A)). It is well known that A subset of X is regular open if and only if A=Int(G), where G is closed and A is said to be regular closed if and only if A=Cl(U), where U is open set.

In this research paper, we studied a new class of topological λ-type transitive maps and θ-type transitive maps[7-8]. We also studied some of their properties. Relationships with some other type of transitive maps are given. We will list some relevant properties of λ-type transitive map and θ-type transitive map.

2. Preliminaries and Definitions

Definition2.1. [7] By a topological system we mean a pair (X,f), where X is a locally compact Hausdorff topological space (the phase space), and $f:X\to X$ is a continuous function. The dynamics of the system is given by $x_{n+1}=f(x_n),\ x_0\in X,\ n\in \mathrm{N}$ and the solution passing through x is the sequence $\{f(x_n)\}$ *where* $n\in \mathrm{N}$.

Definition 2.2 Let $x\in X$, then the set $\{x,f(x),f^2(x),\ldots\}$ is called an orbit of x under f and is denoted by $O_f(x)$, and the sequence $x,f(x),f^2(x),\ldots$ is called the trajectory of x.

Suppose for some $x\in X$, sequence $x,f(x),f^2(x),\ldots$ converges to some point say $x_0\in X$, then we must have $f(x_0)=x_0$, because f is continuous. Such points we call as fixed points. We say that the point x is attracted by the fixed point x_0. The set of all points in X attracted by x_0 is called the stable set or the basin of attraction of the fixed point x0 and is denoted by $W_f(x_0)$. A fixed point x_0 is said to be attracting if its stable set is a neighborhood of it.

A point $x\in X$ is said to be periodic if there exists a positive integer $n\in \mathbf{N}$ such that $f^n(x)=x$. The set of all periodic points of f is denoted by Per(f).

Let (X,f) be a topological system. If X is compact Hausdorff space, then the following hold:

1 The set of all fixed points is a closed subset of X.
2 Orbits of any two periodic points are either identical or disjoint.
3 If a trajectory converges. it converges to a fixed point.
4 Every orbit is an invariant set. The orbits of periodic points are minimal invariant sets.
5 A subset of X is invariant if and only if it is a union of orbits.
6 The closure of an invariant set is also invariant.
7 The set of all periodic points is an invariant set.
8 For each subset A of X, the set $\bigcup_{n=0}^{\infty} f^n(A)$ is the smallest invariant set containing A.

Recall that a subset S is a Λ-set (resp. a V-set) if and only if it is an intersection (resp. a union) of open (resp. closed) sets and that a subset A of a topological space X is called a λ-open set (or λ-closed) if A = L ∩F, where L is a Λ -set and F is closed. Complements of λ-closed sets will be called λ-open.

Definition 2.3. [4]A function $f:X\to X$ is called λ-irresolute if the inverse image of each λ-open set is a λ-open set in X.

Example 2.4[7] Let (X, τ) be a topological space such that X={a, b, c,} and τ ={ϕ, {a} ,{a, b},X}. We have λO(X, τ)={ϕ, X, {a}, {c}, {a, b}, {a, c}, {b, c}. Define the map f : X→X as follows f(c)= a, f(b)= b, f(a)= c. Then f is λ-irresolute.

Definition 2.5. A topological space (X,τ) is irreducible if every pair of nonempty open subsets of the space X has a nonempty intersection In the study of dynamics on a topological space, it is natural and convenient to break the topological space into its irreducible parts and investigate the dynamics on each part. The topological property that precludes such decomposition is called topological transitivity. In [7], Mohammed Nokhas Murad introduced the definitions of topological λ-type transitive and topologically λ-mixing maps as follows

Definition 2.6. [7] Let (X, τ) be a topological space, $f: X\to X$ be λ-irresolute map, then the map f is called λ-type transitive if for every pair of non-empty λ-open sets U and V in X there is a positive integer n such that $f^n(U)\cap V\neq\phi$..

Definition 2.7.[7] Let (X, τ) be a topological space, $f: X\to X$ be λ-irresolute map, then the map f is called topologically λ-mixing if, given any nonempty λ-open subsets $U,V\subseteq X$ $\exists\, N\geq 1$ such that $f^n(U)\cap V\neq\phi$ for all n>N. Clearly if f is topologically λ-mixing then it is also λ-transitive but not conversely

Definition 2.8[7] Two topological systems (X, f) and (Y, g) are said to be conjugate if there is a homeomorphism $h:X\to Y$ such that $h\circ f=g\circ h$

First of all, any property of topological systems must face the obvious question: Is it preserved under topological conjugation? That is to say, if f has property P and if we have a commutative diagram

$$\begin{array}{ccc} X & \xrightarrow{f} & X \\ {\scriptstyle h}\downarrow & & \downarrow{\scriptstyle h} \\ Y & \xrightarrow[g]{} & Y \end{array}$$

Where (X,f) and (Y,g) are topological systems and h is a homeomorphism, then, is g necessarily has property P? Certainly transitivity and the existence of dense periodic points are preserved as they are purely topological conditions.

Definition 2.9[9] Let (X, τ) be a topological space , $f:X\to X$ be θ-irresolute map then f is said to be topologically θ-type transitive if every pair of non-empty θ-open sets U and V in X there is a positive integer n such that $f^n(U)\cap V\neq\phi$..

In [8] We introduced a new definition of chaos for θ-irresolute map $f:X\to X$ of a locally compact Hausdorff topological space X, so called θ-type chaos. The definition of John Tylar implies θ-type chaos, furthermore, John Tylar definition coincides with Devaney's definition for chaos when the topological space happens to be a metric space.

Definition 2.10 [8] Let (X, f) be a topological system, where X is a locally compact Hausdorff space. Then, f is said to be θ-type chaotic map on X provided that for any nonempty θ-open sets U and V in X, ∃ a periodic point $p\in X$ such that $U\cap O_f(p)\neq\phi$ and $V\cap O_f(p)\neq\phi$.

3. Topologically λ-Type Transitive Maps and λr-Conjugacy

We will investigate some properties of λ-type transitive and θ-type transitive and prove some results associated with these new definitions of topological transitivity. We studied some properties and characterizations of such maps.

Definition 3.1 Recall that a subset A of a space X is called λ-dense in X if $Cl_{\lambda}(A)=X$, we can define equivalent definition that a subset A is said to be λ-dense if for any x in X either x in A or it is a λ-limit point for A.

Remark 3.2 [7] Any λ-dense subset in X intersects any λ-open set in X.

Definition 3.3 [7] A subset A of a topological space (X, τ) is said to be nowhere λ-dense, if its λ-closure has an empty λ-interior, that is, $\mathrm{int}_{\lambda}(Cl_{\lambda}(A))=\phi$.

Definition 3.4 if for $x\in X$ the set $\{f^{n}(x):\ n\in \mathrm{N}\}$ is dense in X then x is said to have a dense orbit. If there exists such an $x\in X$, then f is said to have a dense orbit.

Definition3.5.[7] A function $f:X\to X$ is called λr-homeomorphism if f is λ-irresolute bijective and $f^{-1}:X\to X$ is λ-irresolute.

Definition 3.6 [7] Two topological systems $f:X\to X$, $x_{n+1}=f(x_n)$ and $g:Y\to Y$, $y_{n+1}=g(y_n)$ are said to be topologically λr -conjugate if there is λr -homeomorphism $h:X\to Y$ such that $h\circ f=g\circ h$ *(i.e. $h(f(x))=g(h(x))$).* We will call h a topological λr -conjugacy. Then

1 f and g have the same kind of dynamics.
2 $h^{-1}:Y\to X$ is a topological conjugacy.
3 $h\circ f^{n}=g^{n}\circ h$ for each n in N.
4 $x\in X$ is a periodic point of f if and only if h(x) is a periodic point of g.
5 If x is a periodic point of the map f with stable set $W_f(x)$, then the stable set of h(x) is $h(W_f(x))$.
6 The periodic points of f are dense in X if and only if the periodic points of g are dense in Y.
7 f is chaotic on X if and only if g is chaotic in Y.

Remark 3.7

If $\{x_0, x_1, x_2, \ldots\}$ denotes an orbit of $x_{n+1}=f(x_n)$ then $\{y_0=h(x_0),\ \ y_1=h(x_1),\ \ y_2=h(x_2),\ldots\}$ yields an. In particular, h maps periodic orbits of f onto periodic orbits of g, since $y_{n+1}=h(x_{n+1})=h(f(x_n))=g(h(x_n)=g(y_n)$, i.e. f and g have the same kind of dynamics.

Theorem 3.8.[7] For a λ-irresolute map f : X → X, where X is a topological space, the following are equivalent:.

1 f is topologically λ-type transitive;
2 Any Proper λ-closed subset $A\subset X \ni f(A)\subseteq A$ is nowhere λ-dense;
3 $\forall\ A\subseteq X\ni f(A)\subseteq A$, A is either λ-dense or nowhere λ-dense;
4 *Any subset* $A\subseteq X\ni f^{-1}(A)\subseteq A$ with non-empty λ-interior is λ-dense.

Proposition 3.9[7] if $f:X\to X$ *and* $g:Y\to Y$ are λr -conjugated by the λr-homeomorphism $h:Y\to X$ .Then for all y ∈ Y the orbit $O_g(y)$ is λ-dense in Y if and only if the orbit $O_f(h(y))$ of h(y) is λ-dense in X.

Proposition 3.10[7] Let X be a λ-compact space without isolated point, if there exists a λ-dense orbit, that is there exists x0 ∈X such that the set $O_f(x_0)$ is λ- dense then the function f is λ-type transitive .

Proposition3.11 [7] if $f:X\to X$ *and* $g:Y\to Y$ are λr -conjugate. Then

(1) f is λ-type transitive if and only if g is λ-type transitive;
(2) f is λ-minimal if and only if g is λ-minimal;
(3) f is topologically λ-mixing if and only if g is topologically λ-mixing.

Definition 3.12 (i) Recall that a space X is said to be 2nd countable if it has a countable basis.(ii) A space X is said to be of First Category if it is a countable union of nowhere dense subsets of X. It is of second Category if it is not of First Category.

Theorem 3.13[7] Let X be a non-empty locally λ-compact Hausdorff space. Then the intersection of a countable collection of λ-open λ-dense subsets of X is λ-dense in X. Moreover, X is of second Category.

Definition 3.14 A space (X, τλ) , where τ^{λ} is the set of all λ-open subsets of X, is said to be second λ-type countable if and only if and only if the λ-topology of X has a countable basis.

Definition 3.15 A space X is said to be λ-type separable if X contains a countable λ-dense subset.

Corollary 3.16 Recall that a subset A of a space (X,τ) is λ-dense if and only if $A\cap U\neq\phi$ for all $U\in\tau^{\lambda}$ other than $U=\phi$.

Proposition 3.17: if $p\in X$ is λ-transitive point of the topological system (X,f), then every point belongs to the orbit of the point $p\in X$ is also λ -transitive.

4. New Types of Chaos of Topological Spaces

In [8] We introduced a new definition of chaos for λ-irresolute self-map $f:X\to X$ of a locally compact Hausdorff topological space X, so called λ-type chaos. This new definition implies John Tylar definition which coincides with Devaney's definition for chaos when the topological space happens to be a metric space.

Definition 4.1[7] Let (X,f) be a topological system, the dynamics is obtained by iterating the map. Then, f is said to be topologically λ-chaos type or λ-type chaotic map on X provided that for any nonempty λ-open sets U and V in X, there is a periodic point $p\in X$ such that $U\cap O_f(p)\neq\phi$ and $V\cap O_f(p)\neq\phi$.

5. Conclusion

There are the following results:

5.1 The definition of topologically λ-type chaos implies John Tylar definition which coincides with Devaney's definition for chaos when the topological space happens to be a metric space.

5.2 Topologically λ-type chaotic map implies chaotic map which implies θ-type chaotic maps

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