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2-D Problem of Magneto-Thermoelastic Medium Under the Effect of Different Fields with Two-Temperature and 3PHL Model

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Abstract: Three-phase-lag theory of thermoelasticity is employed to study the deformation of thermo-elastic solid half-space under hydrostatic initial stress, rotation, magnetic field and gravity with two-temperature. The normal mode analysis is used to obtain the analytical expressions of the displacement components, force stress, thermodynamic temperature and conductive temperature. The numerical results are given and presented graphically when mechanical and thermal force is applied. Comparisons are made with the results predicted by the three-phase-lag model, Green-Naghdi III and Lord-Shulman theories.

Keywords: Initial Stress, Three-Phase-Lag, Gravity, Rotation, Magnetic Field, Two-Temperature

1. Introduction

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. Hetnarski and Ignaczak [1] examined five generalizations of the coupled theory of thermoelasticity. The first generalization formulates the generalized thermo-elasticity theory involving one thermal relaxation time by Lord and Shulman [2]. Green and Lindsay [3] developed the temperature rate-dependent thermoelasticity, where includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction, when the body under consideration has a center of symmetry. One can review and presentation of generalized theories of thermo-elasticity by Hetnarski and Ignaczak [4]. The third generalization of the coupled theory of thermoelasticity is developed by Hetnarski and Ignaczak and is known as low-temperature thermoelasticity.

The fourth generalization to the coupled theory of thermo-elasticity introduced by Green and Naghdi and this theory is concerned with the thermoelasticity theory without energy dissipation, referred to as (G-N) theory of type II in which the classical Fourier law is replaced by a heat flux rate-temperature gradient relation and Green and Naghdi with energy dissipation referred to as (G-N) theory of type III.

The fifth generalization of the coupled theory of thermo-elasticity is referred to the dual-phase-lag thermoelasticity by Tzau [5] and Chandrasekhariah [6]. Disturbance due to internal heat source in thermoelastic solid using dual phase lag model has studied by Ailawalia and Singla [7].

The stability of the three-phase-lag, the heat conduction equation was discussed by Quintanilla and Racke [8]. The vibration analysis of wave motion in micropolar thermo-visco-elastic plate was investigated by Kumar and Partap [9].

Some researcher in the past investigated different problems of rotating media. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic. Abo-Dahab *et al.* [10] discussed the rotational effect on Rayleigh, Love and Stoneley waves in non-homogeneous fibre-

reinforced anisotropic general viscoelastic media of higher order. Wave propagation in fibre-reinforced anisotropic thermoelastic medium subjected to gravity field was studied by Abd–Alla *et al.* [11]. The electro-magneto-thermoelastic surface waves in non-homogeneous orthotropic granular half space have studied by Kakar and Kakar [12].

The two-temperature theory of thermoelasticity was introduced by Chen and Gurtin [13], in which the classical Clausius-Duhem inequality was replaced by another one depending on two-temperature; the conductive temperature and the thermodynamic temperature, the first is due to the thermal processes, and the second is due to the mechanical processes inherent between the particles and the layers of elastic material, this theory was also investigated by Ieşan [14]. The two-temperature model was underrated and unnoticed for many years thereafter. Only in the last decade the theory has noticed, developed in many searches, and find its applications, mainly in the problems in which the discontinuities of stresses have no physical interpretations.

The initial stress present in the medium also has a considerable effect on the propagation of waves by Biot (1965). Initial stress in solids has the significant influence on the mechanical response of the material from an initially stressed configuration and has applications in geophysics, engineering structures, and in the behavior of soft biological tissues. Initial stress arises from processes, such as manufacturing or growth and is present in the absence of applied loads. The generalized thermoelastic interaction in a fiber-reinforced anisotropic half-space under hydrostatic initial stress was studied by Abbas and Othman [15]. The effect of rotation on piezo-thermoelastic medium, using different theories have studied by Othman *et al.* [16]. The wave propagation in a two-temperature fiber-reinforced magneto-thermo-elastic medium with three-phase-lag model was discussed by Othman *et al.* [17]. The analysis of wave motion in an anisotropic initially stressed fiber-reinforced thermoelastic medium was studied by Gupta and Gupta [18].

In the classical theory of elasticity, the gravity effect is generally neglected. The generalized thermoelastic medium with temperature-dependent properties for different theories under the effect of gravity field has studied by Othman *et al.* [19]. Surface waves under the influence of gravity were studied by De and Sengupta [20].

The aim of this paper is to study the influence of magnetic field, rotation, hydrostatic initial stress and gravity with two-temperature on thermoelastic isotropic medium in the context of the (3PHL) model. Numerical results for the field quantities are obtained and represented graphically. Comparisons are made with the results predicted by three-phase-lag model, (G-N III) and (L-S) theories for different cases.

2. Formulation of the Problem

We consider a homogeneous thermoelastic half-space with two-temperature, under the influence of magnetic field, initial stress and gravity, rotating uniformly with angular velocity $\boldsymbol{\Omega} = \Omega\, \boldsymbol{n}$, where $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation. All quantities are considered functions of the time variable t and of the coordinates x and y. The displacement equation in the rotating frame has two additional terms Schoenberg and Censor (1973): centripetal acceleration $\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{u})$ due to time varying motion only and Coriolis acceleration $2\boldsymbol{\Omega} \wedge \dot{\boldsymbol{u}}$ where $\boldsymbol{u} = (u, v, 0)$ is the dynamic displacement vector and angular velocity is $\boldsymbol{\Omega} = (0, 0, \Omega)$. These terms, do not appear in non-rotating media. A magnetic field with constant intensity $\boldsymbol{H} = (0, 0, H_0)$ acts in the direction of the $z-$axis. Due to the application of initial magnetic field $\boldsymbol{H}$, an induced magnetic field $\boldsymbol{h}$ and an induced electric field $\boldsymbol{E}$ are results. The simplified linear equations of electrodynamics of slowly moving medium for a homogeneous, thermally and electrically conducting elastic solid are:

$$\operatorname{curl} \boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \dot{\boldsymbol{E}} \tag{1}$$

$$\operatorname{curl} \boldsymbol{E} = -\mu_0 \dot{\boldsymbol{h}} \tag{2}$$

$$\operatorname{div} \boldsymbol{h} = 0 \tag{3}$$

$$\boldsymbol{E} = -\mu_0 (\dot{\boldsymbol{u}} \wedge \boldsymbol{H}) \tag{4}$$

The stress-strain relation

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma (T - T_0)]\delta_{ij} - p\,(\omega_{ij} + \delta_{ij}) \tag{5}$$

The displacement components have the following form $\boldsymbol{u} = (u, v, 0)$.

The equations of motion in the absence of body force

$$\sigma_{jx,j} + \rho g v_{,x} + F_x = \rho[\ddot{u} + \{\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{u})\}_x + (2\boldsymbol{\Omega} \wedge \dot{\boldsymbol{u}})_x] \tag{6}$$

$$\sigma_{jy,j} - \rho g u_{,x} + F_y = \rho[\ddot{v} + \{\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{u})\}_y + (2\boldsymbol{\Omega} \wedge \dot{\boldsymbol{u}})_y] \tag{7}$$

The equation of heat conduction under three-phase-lag model

$$K^* \nabla^2 \phi + \tau_v^* \nabla^2 \dot{\phi} + K \tau_t \nabla^2 \ddot{\phi}$$

$$= (1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2!} \frac{\partial^2}{\partial t^2})(\rho c_e \ddot{T} + \gamma T_0 \ddot{e}) \tag{8}$$

Where $\tau_v^* = (K + K^* \tau_v)$, $F_i = \mu_0 (\boldsymbol{J} \wedge \boldsymbol{H})_i$

$$T = (1 - b\nabla^2)\phi \tag{9}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{10}$$

$$\omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}) \tag{11}$$

For a two-dimensional problem in *xy*-plane, Eqs. (6) and (7) can be written as:

$$(\mu-\frac{p}{2})\nabla^2 u+(\lambda+\mu+\frac{p}{2}+\mu_0 H_0^2)\frac{\partial \mathrm{e}}{\partial \mathrm{x}}-\gamma(1-b\nabla^2)\frac{\partial \phi}{\partial \mathrm{x}}-\varepsilon_0\mu_0^2 H_0^2\ddot{u}=\rho[\ddot{u}-u\Omega^2-2\Omega\dot{v}-g\frac{\partial v}{\partial x}] \tag{12}$$

$$(\mu-\frac{p}{2})\nabla^2 v+(\lambda+\mu+\frac{p}{2}+\mu_0 H_0^2)\frac{\partial e}{\partial y}-\gamma(1-b\nabla^2)\frac{\partial \phi}{\partial \mathrm{y}}-\varepsilon_0\mu_0^2 H_0^2\ddot{v}=\rho[\ddot{v}-v\Omega^2+2\Omega\dot{u}+g\frac{\partial u}{\partial x}] \tag{13}$$

$$K^*\nabla^2\phi+\tau_v^*\nabla^2\dot{\phi}+K\tau_t\nabla^2\ddot{\phi}=(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2})[(\rho c_e(1-b\nabla^2)\ddot{\phi}+\gamma T_0\ddot{e}] \tag{14}$$

Where $h=-H_0 e$

For the purpose of numerical evaluation, we introduce dimensionless variables:

$$(x',y')=\frac{\omega^*}{c_0}(x,y),\ (u',v')=\frac{\rho c_0\omega^*}{\gamma T_0}(u,v),\ \{\sigma_{ij}',p'\}=\frac{1}{\gamma T_0}\{\sigma_{ij},p\},\ (T',\phi')=\frac{1}{T_0}(T,\phi),$$

$$(t',\tau_v',\tau_q',\tau_t')=\omega^*(t,\tau_v,\tau_q,\tau_t),\ c_0^2=\frac{\lambda+2\mu}{\rho},\ e=\frac{\gamma T_0}{\rho c_0^2}e',\ \omega^*=\frac{\rho c_e c_0^2}{K},$$

$$\nabla^2=\frac{\omega^{*2}}{c_0^2}\nabla'^2,\ h'=\frac{h}{H_0},\ \gamma=(3\lambda+2\mu)\alpha_t,\ g'=\frac{g}{c_0\omega^*},\ \Omega'=\frac{\Omega}{\omega^*}.$$

Using the above dimensions quantities, Eqs. (12)-(14) become:

$$a_2\nabla^2 u+[\frac{\mu_0 H_0^2}{\rho c_0^2}+a_1+(1-\frac{2}{\beta^2})]\frac{\partial e}{\partial x}-(1-b^*\nabla^2)\frac{\partial \phi}{\partial x}=[(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho})\ddot{u}-u\Omega^2-2\Omega\dot{v}-g\frac{\partial v}{\partial x}] \tag{15}$$

$$a_2\nabla^2 v+[\frac{\mu_0 H_0^2}{\rho c_0^2}+a_1+(1-\frac{2}{\beta^2})]\frac{\partial e}{\partial y}-(1-b^*\nabla^2)\frac{\partial \phi}{\partial y}=[(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho})\ddot{v}-v\Omega^2+2\Omega\dot{u}+g\frac{\partial u}{\partial x}] \tag{16}$$

$$\varepsilon_1\nabla^2\phi+\varepsilon_2\nabla^2\dot{\phi}+\tau_t\nabla^2\ddot{\phi}=(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2})[(1-b^*\nabla^2)\ddot{\phi}+\varepsilon_3\ddot{e}] \tag{17}$$

Where $\varepsilon_1=\frac{K^*}{\rho c_e c_0^2}$, $\varepsilon_2=1+\varepsilon_1\tau_v$, $\varepsilon_3=\frac{\gamma^2 T_0}{\rho^2 c_e c_0^2}$, $b^*=\frac{b\omega^{*2}}{c_0^2}$, $a_1=\frac{2\mu+\gamma T_0 p}{2\rho c_0^2}$, $a_2=\frac{2\mu-\gamma T_0 p}{2\rho c_0^2}$.

We define displacement potentials q and ψ which relate to displacement components u and v as:

$$u=q_{,x}-\psi_{,y}\quad v=q_{,y}+\psi_{,x} \tag{18}$$

Using Eq. (18) in Eqs. (15)-(17), we obtain:

$$[\frac{\mu_0 H_0^2}{\rho c_0^2}+a_1+a_2+(1-\frac{2}{\beta^2})]\nabla^2 q-(1-b^*\nabla^2)\phi=[(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho})\ddot{q}-q\Omega^2-2\Omega\dot{\psi}-g\psi_{,x}] \tag{19}$$

$$[(-a_2\nabla^2-\Omega^2)]\psi=[-(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho})\ddot{\psi}-2\Omega\dot{q}-g\,q_{,x}] \tag{20}$$

$$\varepsilon_1\nabla^2\phi+\varepsilon_2\nabla^2\dot{\phi}+\tau_t\nabla^2\ddot{\phi}=(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2})[(1-b^*\nabla^2)\ddot{\phi}+\varepsilon_3\nabla^2\ddot{q}] \tag{21}$$

3. Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[u,v,e,T,\phi,q,\psi,\sigma_{ij}](x,y,t)=[u^*,v^*,e^*,T^*,\phi^*,q^*,\psi^*,\sigma_{ij}^*](y)\exp[\mathrm{i}(\omega t+ax)] \tag{22}$$

Where ω is the complex time constant and a is the wave number in x-direction.

Using (22) in Eqs. (19)-(21), we obtain:

$$(\mathrm{A}_1\mathrm{D}^2 - A_2)q^* + (b^*\mathrm{D}^2 - A_3)\phi^* + A_4\psi^* = 0 \tag{23}$$

$$(-a_2\mathrm{D}^2 + A_5)\psi^* + A_4 q^* = 0 \tag{24}$$

$$(A_6\mathrm{D}^2 - A_7)\phi^* + (A_8\mathrm{D}^2 - A_9)q^* = 0 \tag{25}$$

where $A_1 = \dfrac{\mu_0 H_0^2}{\rho c_0^2} + a_1 + a_2 + (1-\dfrac{2}{\beta^2}),$

$$A_2 = \frac{a^2\mu_0 H_0^2}{\rho c_0^2} + a_1 a^2 + a_2 a^2 + a^2(1-\frac{2}{\beta^2}) - [1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}]\,\omega^2 - \Omega^2,$$

$$A_3 = 1 + b^* a^2,\ A_4 = 2i\Omega\omega + iag,$$

$$A_5 = a_2 a^2 - \Omega^2 - [1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}]\,\omega^2,$$

$$A_6 = \varepsilon_1 + i\varepsilon_2\omega - \tau_t\omega^2 - b^*\omega^2[1 + \mathrm{i}\tau_q\omega - \frac{\tau_q^2}{2!}\omega^2],$$

$$A_7 = \varepsilon_1 \mathrm{a}^2 + i\varepsilon_2\omega\,\mathrm{a}^2 - \tau_t\omega^2\mathrm{a}^2 - (1+b^*\mathrm{a}^2)\omega^2[1 + \mathrm{i}\tau_q\omega - \frac{\tau_q^2}{2!}\omega^2],$$

$$A_8 = \varepsilon_3\omega^2[1 + i\tau_q\omega - \frac{\tau_q^2}{2!}\omega^2],\quad A_9 = \varepsilon_3\omega^2 a^2[1 + i\tau_q\omega - \frac{\tau_q^2}{2!}\omega^2].$$

Eliminating ϕ^* and ψ^* between Eqs. (23)-(25), we get:

$$[\mathrm{D}^6 - A\mathrm{D}^4 + B\mathrm{D}^2 - C]\{q^*(y), \phi^*(y), \psi^*(y)\} = 0 \tag{26}$$

Where

$$A = \frac{A_1A_7a_2 + A_1A_5A_6 + A_2A_6a_2 - A_9a_2b^* - A_5A_8b^* - a_2A_3A_8}{a_2A_1A_6 - a_2A_8b^*},$$

$$B = \frac{A_1A_5A_7 + A_2A_7a_2 + A_2A_5A_6 - A_5A_9b^* - A_3A_9a_2 - A_3A_5A_8 + A_4^2A_6}{a_2A_1A_6 - a_2A_8b^*},$$

$$C = \frac{A_2A_5A_7 - A_3A_5A_9 + A_4^2A_7}{a_2A_1A_6 - a_2A_8b^*},\ \mathrm{D} = \frac{d}{dy}.$$

The solution of Eq. (26) has the form:

$$q^* = \sum_{n=1}^{3} M_n e^{-k_n y} \tag{27}$$

$$\phi^* = \sum_{n=1}^{3} H_{1n} M_n e^{-k_n y} \tag{28}$$

$$\psi^* = \sum_{n=1}^{3} H_{2n} M_n e^{-k_n y} \tag{29}$$

$$T^* = \sum_{n=1}^{3} H_{3n} M_n e^{-k_n y} \tag{30}$$

where M_n $(n = 1,2,3)$ are some constants, k_n^2 are the roots of the characteristic equation of Eq. (26).

Dimensionless variables of the stress components yield the following:

$$\sigma_{xx} = u_{,x} + (1-\frac{2}{\beta^2})v_{,y} - T - p \tag{31}$$

$$\sigma_{yy} = v_{,y} + (1-\frac{2}{\beta^2})u_{,x} - T - p \tag{32}$$

$$\sigma_{xy} = a_1 u_{,y} + a_2 v_{,x} \tag{33}$$

$$\sigma_{yx} = a_2 u_{,y} + a_1 v_{,x} \tag{34}$$

Using Eq. (18) and Eqs. (27)-(30) in (31)-(34) we get:

$$u = \sum_{n=1}^{3} H_{4n} M_n e^{\mathrm{i}(\omega t + ax) - k_n y} \tag{35}$$

$$v = \sum_{\mathrm{n}=1}^{3} H_{5n} M_n e^{\mathrm{i}(\omega t + ax) - k_n y} \tag{36}$$

$$\sigma_{xx} = \sum_{n=1}^{3} H_{6n} M_n e^{\mathrm{i}(\omega t + ax) - k_n y} - p \tag{37}$$

$$\sigma_{yy} = \sum_{n=1}^{3} H_{7n} M_n e^{\mathrm{i}(\omega t + ax) - k_n y} - p \tag{38}$$

$$\sigma_{xy} = \sum_{n=1}^{3} H_{8n} M_n e^{\mathrm{i}(\omega t + ax) - k_n y} \tag{39}$$

$$\sigma_{yx} = \sum_{n=1}^{3} H_{9n} M_n e^{\mathrm{i}(\omega t + ax) - k_n y} \tag{40}$$

$$T = \sum_{n=1}^{3} H_{3n} M_n e^{\mathrm{i}(\omega t + ax) - k_n y} \tag{41}$$

Where $H_{1n} = -\dfrac{(A_8k_n^2 - A_9)}{A_6k_n^2 - A_7},\ H_{2n} = -\dfrac{A_4}{A_5 - a_2k_n^2},$

$$H_{3n} = H_{1n}[1 - b^*(k_n^2 - a^2)],\ H_{4n} = (ia + k_n H_{2n}),$$

$$H_{5n} = (-k_n + iaH_{2n}),$$

$$H_{6n} = [i\,a\,H_{4n} - (1-\frac{2}{\beta^2})k_n H_{5n} - H_{3n}],$$

$$H_{7n} = [-k_n H_{5n} + i\,a\,H_{4n}(1-\frac{2}{\beta^2}) - H_{3n}],$$

$$H_{8n} = (-a_1k_nH_{4n} + i\,a\,a_2H_{5n}),\ H_{9n} = (-a_2k_nH_{4n} + iaa_1H_{5n}).$$

4. Boundary Conditions

The boundary conditions on the plane surface $y = 0$ are

$$\sigma_{xx} = -P + P_1 e^{i(\omega t + ax)} \quad T = P_2 e^{i(\omega t + ax)} \quad \sigma_{xy} = 0 \qquad (42)$$

Using Eqs. (37), (39) and (41) in boundary conditions (42), we get three equations in three constants M_n $(n = 1,2,3)$ as:

$$\sum_{n=1}^{3} H_{6n} M_n = P_1 \qquad (43)$$

$$\sum_{n=1}^{3} H_{8n} M_n = 0 \qquad (44)$$

$$\sum_{n=1}^{3} H_{3n} M_n = P_2 \qquad (45)$$

Solving Eqs. (43)-(45) the constants M_n $(n = 1,2,3)$ are defined as follows

$$M_1 = \frac{\Delta_1}{\Delta} \quad M_2 = \frac{\Delta_2}{\Delta} \quad M_3 = \frac{\Delta_3}{\Delta}$$

where

$$\Delta = H_{61}(H_{83}H_{32} - H_{82}H_{33}) - H_{62}(H_{83}H_{31} - H_{81}H_{33}) + H_{63}(H_{82}H_{31} - H_{81}H_{32}),$$

$$\Delta_1 = P_1(H_{83}H_{32} - H_{82}H_{33}) - P_2(H_{83}H_{62} - H_{63}H_{82}),$$

$$\Delta_2 = -P_1(H_{83}H_{31} - H_{81}H_{33}) + P_2(H_{61}H_{83} - H_{63}H_{81}),$$

$$\Delta_3 = P_1(H_{82}H_{31} - H_{81}H_{32}) + P_2(H_{62}H_{81} - H_{61}H_{82}).$$

5. Numerical Results

To study the effect of gravity, rotation, initial stress, magnetic field and two-temperature, we now present four cases and some numerical results. For this purpose, copper is taken as the thermoelastic material for which we take the following values of the different physical constants.

$$\lambda = 7.7\times10^{10}\, N.M^{-2}, \quad \mu = 3.86\times10^{10}\, Kg.m^{-1}.s^{-2},$$
$$K = 300\ w.m^{-1}.K^{-1}, \quad K^* = 2.97\times10\text{^}13,$$
$$\alpha_t = 1.78\times10^{-5} K^{-1},$$

$$\rho = 8954 Kg.m^{-3}, \quad c_e = 383.1\, J.Kg^{-1}K^{-1}, \quad T_0 = 293K,$$
$$\omega = \omega_0 + i\xi, \quad P_1 = 0.1, \quad P_2 = 0.2$$

The numerical technique, outlined above, was used for the distribution of the real part of the displacement component u, the temperature T, and the stress components σ_{xy}, σ_{xx} of the problem.

Case 1: Figures 1-4 show the variation of the physical quantities based on (L-S), (G-N III) and (3PHL) in the case of $p = 0,1$, when $b = 0.1$, $\Omega = 0.1$, $H_0 = 0$, $g = 0$, $x = 0.5$, $t = 0.3$, $a = 0.5$, $\omega_0 = -0.2$, $\xi = 0.6$, $\tau_\upsilon = 0.05$, $\tau_t = 0.2$, $\tau_q = 0.8$

Figure 1 depicts that the displacement component u increases with the decrease of initial stress in the three theories. In the absence of initial stress (i.e. $p = 0$), u begins to increase, then smooth decreases again to decay zero at infinity and in the presence of initial stress (i.e. $p = 1$), u decreases and increases until it develops to zero. Figure 2 demonstrates that the temperature T decays meaning that the temperature decrease for $p = 0,1$ and take the form of a wave until it develop to zero. Figure 3 represents that the stress component σ_{xy} increases with the increase of initial stress P in the three theories and take the form of the wave until it decays to zero, and decreasing with the decrease of the initial stress in the three theories and takes the form of the wave until it decays to zero. Figure 4 depicts that the stress component σ_{xx} increases with the decrease of initial stress and decays to zero.

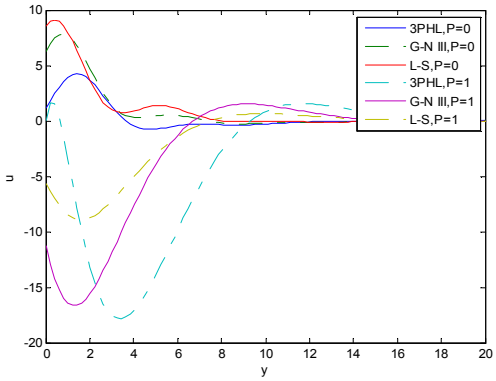


Figure 1. *Variation of the displacement component u in the absence and presence of p.*

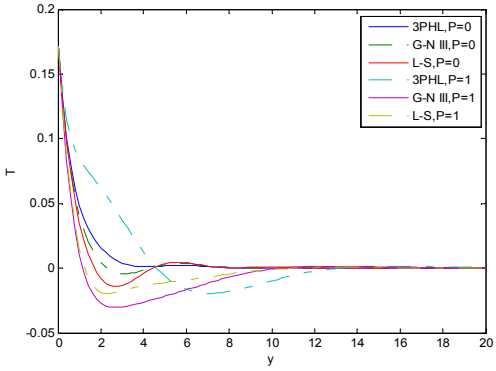


Figure 2. *Variation of the temperature T in the absence and presence of p.*

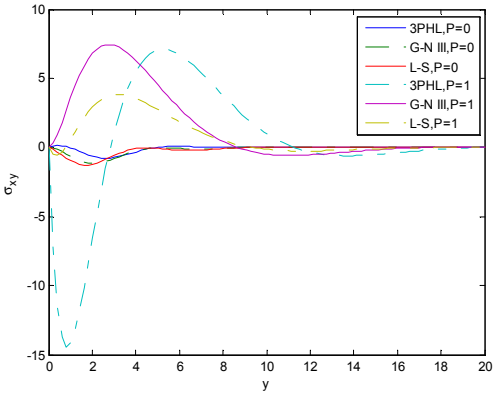


Figure 3. *Variation of the stress component σ_{xy} in the absence and presence of p.*

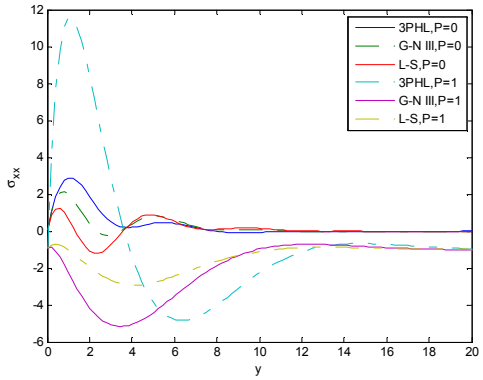


Figure 4. *Variation of the stress component σ_{xx} in the absence and presence of p.*

Case 2: Figures 5-8 exhibit the variation of the physical quantities based on (L-S), (G-N III) and (3PHL) in the case of $\Omega = 0, 0.1,$ when $b = 0.1$, $P = 1$, $H_0 = 0$, $g = 0,$ $x = 0.5$, $t = 0.3$, $a = 0.5$, $\omega_0 = -0.2$, $\xi = 0.6$, $\tau_\upsilon = 0.05$, $\tau_t = 0.2$, $\tau_q = 0.8$

Figure 5 depicts that the displacement component u increases with the decrease of rotation and decays to zero at infinity. Figure 6 demonstrates that the temperature satisfies the boundary condition at $y = 0$ and decreases, in the three theories at $\Omega = 0, 0.1,$ to a minimum value in the range $0 \le y \le 2$ and increasing in the range $2 \le y \le 4,$ until it decays to zero. Figure 7 explains that in the absence of rotation the stress component σ_{xy} decreases in the range $0 \le y \le 1,$ in three theories, and increases in the range $1 \le y \le 3.$ While in the presence of rotation, σ_{xy} decreases in the range $0 \le y \le 1$, then, increases in the range $2 \le y \le 5$ and takes the form of the wave until it develops to zero in (L-S) and (G-N III) and (3PHL) theories. Figure 8 shows that at $\Omega = 0,$ the stress component σ_{xx} satisfies the boundary condition and decreasing to a minimum value in the range $0 \le y \le 1,$ while, increases in the range $1 \le y \le 6$ and decays to zero in the context of three theories. However, at $\Omega = 0.1,$ it increases in the range $0 \le y \le 2,$ then, decreases in the range $2 \le y \le 8$ and decays to zero in the three theories.

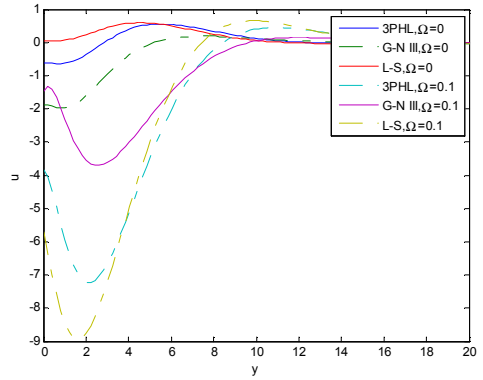


Figure 5. *Variation of the displacement component u in the absence and presence of rotation.*

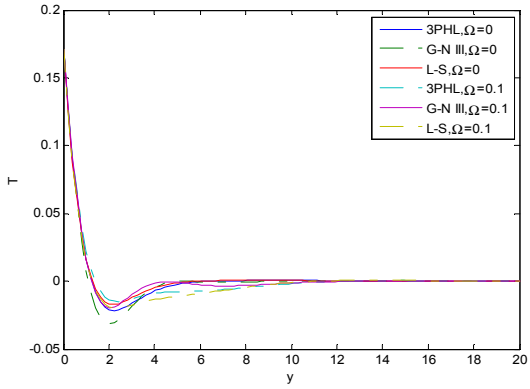


Figure 6. *Variation of the temperature T in the absence and presence of rotation.*

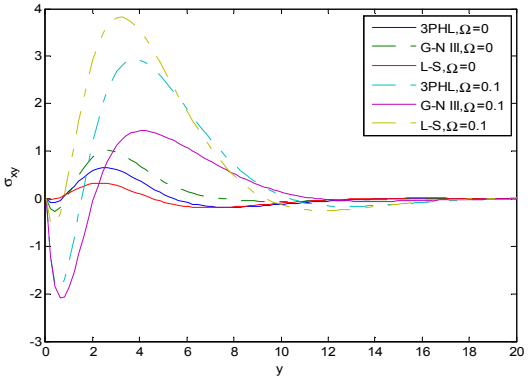


Figure 7. *Variation of the stress component σ_{xy} in the absence and presence of rotation.*

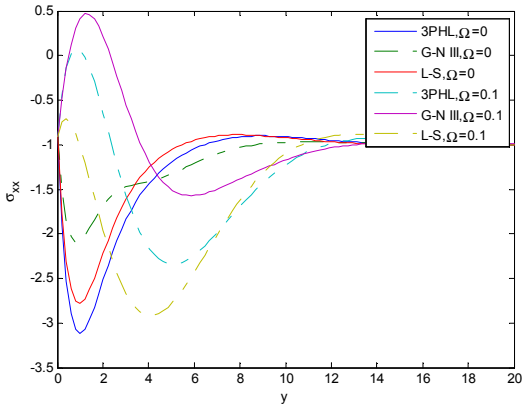
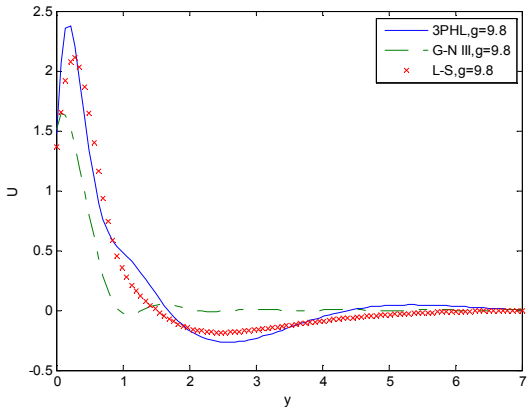


Figure 8. *Variation of the stress component σ_{xx} in the absence and presence of rotation.*

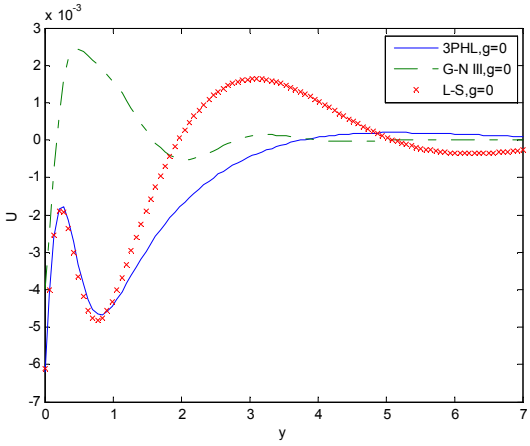
Case 3: Figures 9 (a, b) -12 (a, b) show the variation of the physical quantities based on (L-S), (G-N III) and (3PHL) in the case of $g = 0,9.8$ respectively, when $b = 0.1,$ $P = 0,$ $H_0 = 0$, $\Omega = 0$, $x = 0.5$, $t = 0.3,$ $a = 1.5$, $\omega_0 = -2.5$, $\xi = 3$, $\tau_\upsilon = 0.05$, $\tau_t = 0.2$, $\tau_q = 0.8$

Figure 9(a, b) depicts that the displacement component u increases with the increase of gravity in the three theories. In the absence and presence of gravity (i.e. $g = 0,9.8$), u begins to increase then smooth decreases and takes the form of wave and try to return to zero. Figure 10(a, b) demonstrate

that the temperature T decreases for $g = 0,9.8$ and takes the form of a wave until it develop to zero. Figure 11(a, b) show that the stress component σ_{xy} decreases for $g = 0,9.8$ and takes the form of a wave until it develop to zero. Figure 12(a, b) depict that the stress component σ_{xx} decreases with the increase of gravity in (L-S) and (G-N III), and increases in (3PHL) at $g = 9.8$. But it decreases at $g = 0$ in three theories.

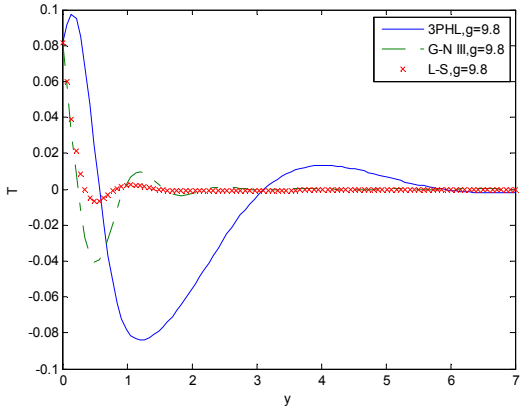


(a).

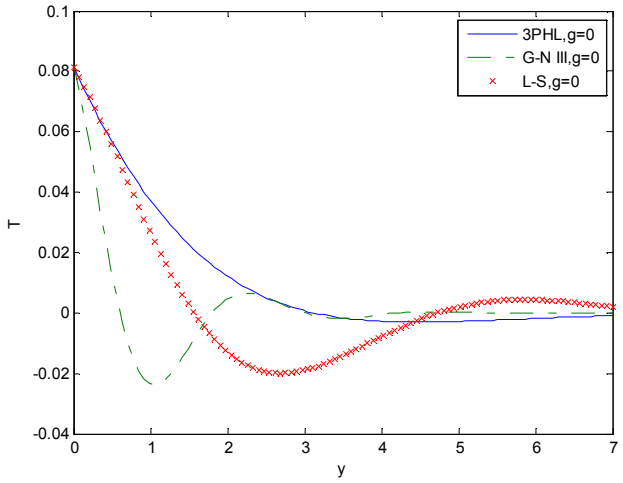


(b)

Figure 9. *(a,b) Variation of displacement component u in the presence and absence of g.*

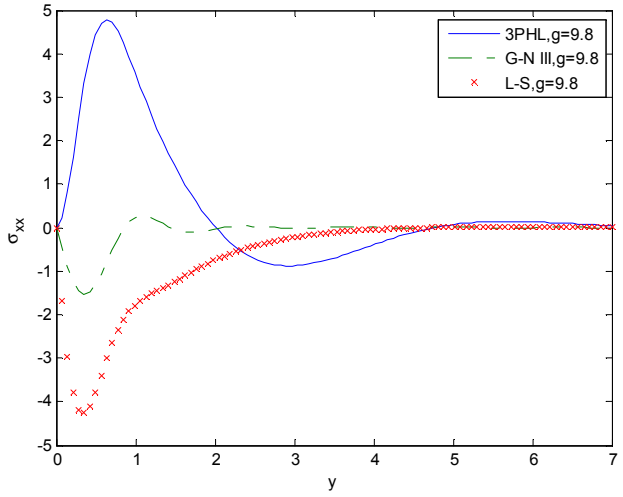


(a)

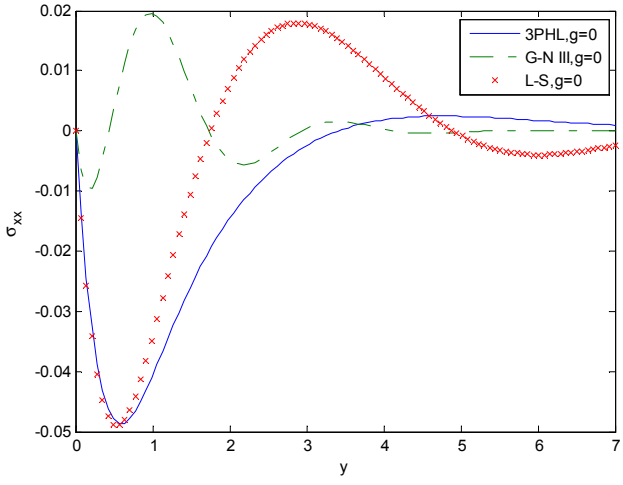


(b)

Figure 10. *(a,b) Variation of the temperature T in the presence and absence of g*



(a)



(b)

Figure 11. *(a,b) Variation of the stress component σ_{xx} in the presence and absence of g .*

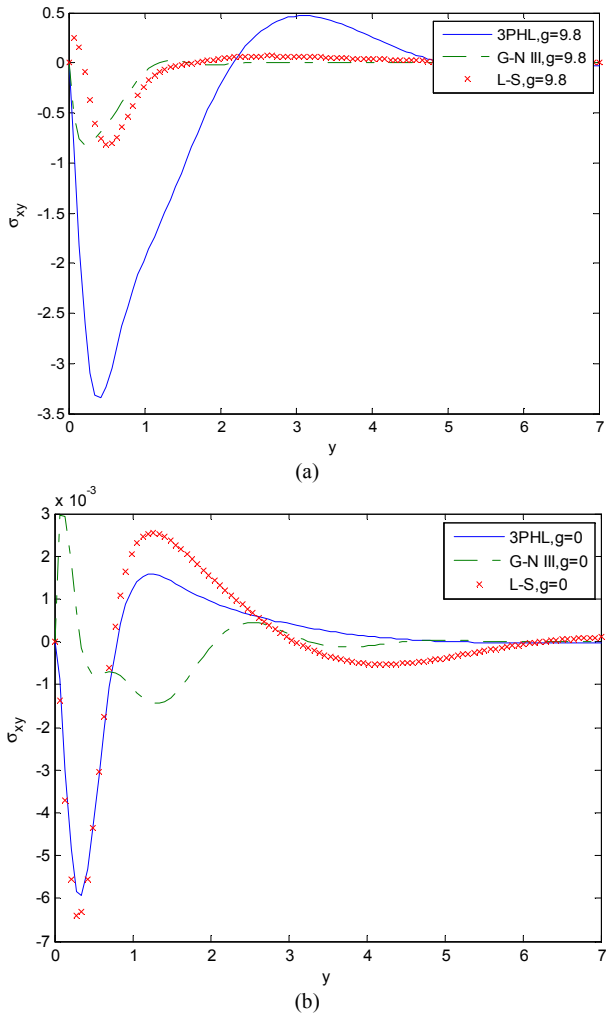


Figure 12. *(a,b) Variation of the stress component σ_{xy} in the presence and absence of g .*

Case 4: Figures 13-16 show the variation of the physical quantities based on (L-S), (G-N III) and (3PHL) theories, in the case of $b=0,0.1$ and $H_0=10^8$, when $g=0$, $P=0$, $\Omega=0$, $x=0.1$, $t=0.3$, $a=2.5$, $\omega_0=-1.2$, $\tau_\upsilon=1.1$, $\tau_t=1.5$, $\tau_q=3.5121$

Figure 13 depicts that the displacement component u decreases to the minimum point in the range $0\le y\le 1$, and increasing in the range $1\le y\le 3.5$, in the three theories for $H_0=10^8$ and decays to zero. Figure 14 demonstrates that the temperature T begins from the value 0.2 and satisfies the boundary condition at $y=0$ in the three theories. The temperature T decreases to the minimum point between $1\le y\le 2$, in the three theories, and decays to zero. Figure 15 depicts that the stress component σ_{xy} satisfies the boundary condition and increases in the three theories at $b=0,0.1$. In the absence of magnetic field (i.e. $H_0=10^8$) σ_{xy} begins to increase then smooth decreases and takes the form of wave and try to return to zero. Figure 16 explains that the stress component σ_{xx} begins from the value 0.1 and satisfies the boundary condition at $y=0$ in three theories, then it decreases to the minimum point between $1\le y\le 2$, in the three theories, in the absence of a magnetic field (i.e. $H_0=10^8$) and decays to zero.

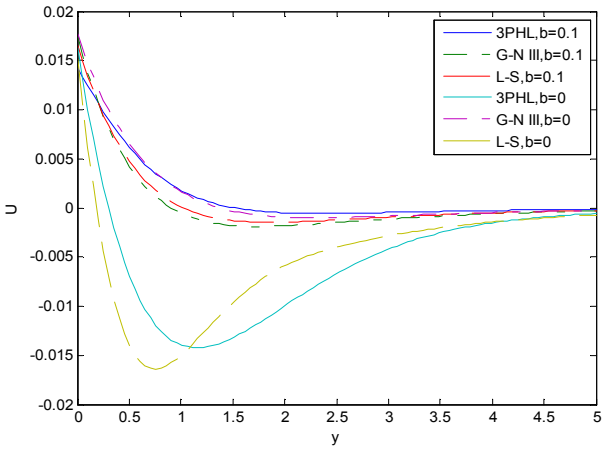


Figure 13. *Variation of the displacement component u.*

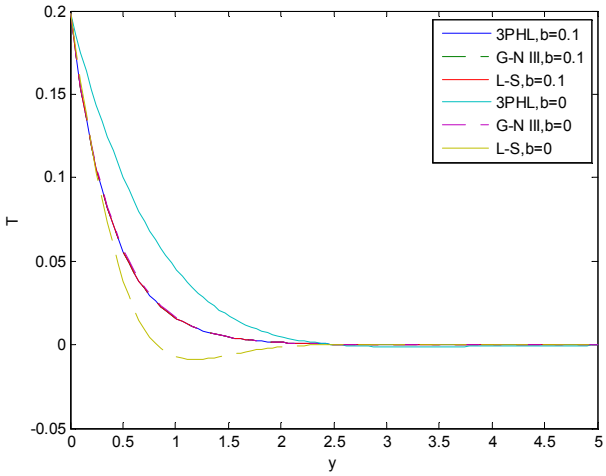


Figure 14. *Variation of the temperature T*

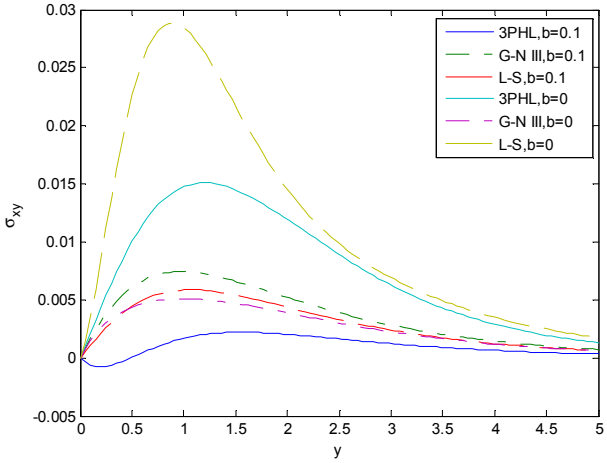


Figure 15. *Variation of the stress component σ_{xy}.*

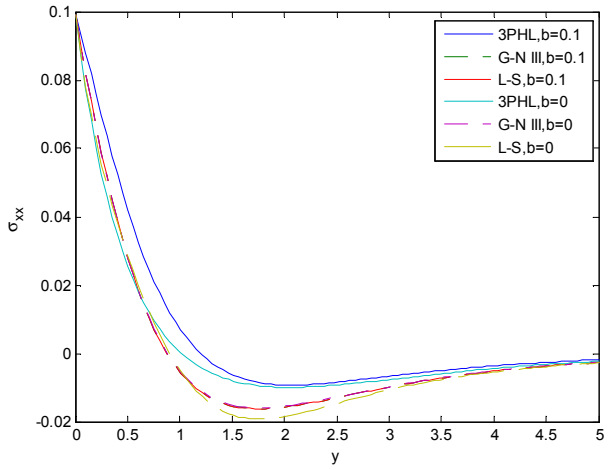


Figure 16. *Variation of the stress component* σ_{xx}.

6. Conclusion

By comparing the figures obtained under the three theories, important phenomena are observed:

1. Analytical solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed. The method that is used in the present article is applicable to a wide range of problems in hydro-dynamics and thermoelasticity.
2. There are significant differences in the field quantities under (GN-III), (3PHL) and (L-S) theories.
3. The presence of the gravitational field, magnetic field, initial stress, rotation and two-temperature plays a significant role on all physical quantities.
4. The comparison of the three theories of thermoelasticity, (L-S), (3PHL) and (G-N III) is carried out.
5. The value of all the physical quantities converges to zero, and all the functions are continuous.

Nomenclature

λ, μ Lame' constants

h the induced magnetic field Vector

E the induced electric field Vector

ε_0 the electric permeability for free space

a the volume coefficient of thermal expansion

K^* the material characteristic of the theory

T the thermodynamic Temperature

τ_v the phase lag of thermal displacement gradient.

τ_t the phase lag of temperature gradient

u_i the displacement vector

ρ the mass density

p the initial stress

c_e specific heat at constant strain

K (≥ 0) the thermal conductivity

T_0 the reference temperature

ϕ the conductive temperature

τ_q the phase lag of heat flux

δ_{ij} the Kronecker delta

b the parameter of two temperature

H_0 the initial uniform Magnetic field

μ_0 the magnetic permeability for free space

J the current density vector

F_i the Lorentz force

g the gravity

ω_{ij} the skew symmetric tensor called the rotation tensor

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