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Mathematical Study of the Optimal Control of COVID-19 Transmission Amongst the Vaccinated Individuals

Christopher Chukwuma Asogwa*, Stephen Ekwueme Aniaku, Emmanuel Chukwudi Mbah

Department of Mathematics, University of Nigeria, Nsukka, Nigeria

Email address:
chukwuma.asogwa@unn.edo.ng (C. C. Asogwa), stephen.aniaku@unn.edu.ng (S. E. Aniaku), emmanuel.mbah@unn.edu.ng (E. C. Mbah)
*Corresponding author

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Abstract: Center for Disease control, informs that it takes two weeks after one is fully vaccinated for the body to build protection (immunity) against the virus that causes COVID-19. Moreover, no vaccine is hundred percent effective and that includes the COVID-19 vaccines. This implies that one can still contact and spread the virus for some days after getting vaccinated. In this paper, we formulated a model for COVID-19 transmission dynamics amongst the vaccinated individuals using differential equations. We analyzed all the parameters that are responsible for the disease spread and showed the effect of other social control measures, like the use of face masks in the public, on the spread of the virus. Numerical values of these parameters were derived from some acknowledged literatures, some calculated with the data from other literatures and others judiciously estimated. The disease reproduction number R_0 was obtained and found that the disease will only spread if its value exceeds one. Numerical simulation was carried out on the model, using MATLAB to show the dynamics in the different compartments and the effect of these other social control measures on the disease spread among the vaccinated individuals. The result showed that in the absence of other social control measures, almost all the vaccinated persons will be infected and will be able to infect others especially within few days of receiving the COVID-19 vaccine.

Keywords: Mathematical Modelling, COVD-19 Transmission, COVID-19 Vaccines, Numerical Simulation, Reproduction Number

1. Introduction

The world has been rampaged by the novel corona virus disease 2019 (COVID-19), a pandemic disease that has spread over almost all parts of the world. COVID-19 emerged from Wuhan city, China in December 2019. It has recorded about 4 million confirmed cases with 280,000 confirmed deaths world wide as at April, 2020 [4]. Currently, (as at 13 February 2022), over 409 million confirmed cases of covid-19 and over 5.8 million deaths have been reported globally [15]. This has stalled many economic activities worldwide.

COVID-19 is spread by human transmission through contacts or exchange of body fluids with an infected person. Recent study shows that it can also be spread through aerosol. Ability to spread the disease by infected person can last up to 14 days. The symptoms of the disease range from mild to severe fever, consistent dry cough, shortness of breath. The symptoms resemble that of influenza or even malaria. This close resemblance to malaria fever which is common in Africa has led many to doubt the existence of the virus in Africa.

Health experts have advocated social distancing, hand washing with soap and water, the use of face mask and hand sanitizer to check the spread of the virus. Recently, some vaccines have been developed though most of them are still in their trial stages. World leaders have advocated for mass vaccination to check the spread of the virus. However, many people have been skeptical in the use of the vaccines because most of the side effects are yet to be known.

According to Dr. Kristen Marks, an infection disease specialist in New York- Presbyteria/Weill Cornel medical center, Pfizer and Morderna vaccines require 2 shots and were found to be 95% effective in the early months after the vaccines. They require a buster dose 21 to 28 days after the first dose and the full protection takes at least two weeks to be achieved. The one made by Johnson and Johnson's, Janssen which require one shot, has 75% effective rate against severe illness of COVID-19 in US [9]. Thus, one needs both doses of the two-dose vaccines to archive immunity. Center for Disease control, CDC in her newsletter dated May 27, 2021 retreated that it typically *takes two weeks after one is fully vaccinated for the body to build protection (immunity)* against the virus that causes COVID-19 [3].

Also, in her morbidity and mortality weekly report, CDC in her report titled 'Monitoring incidence of COVID-19 cases, Hospitalization, and death by vaccination statue-13 U.S. jurisdictions, April 4-July 17, 2021'; released on September 10, 2021, noted that fully vaccinated account for 5% of cases, 7% of hospitalization, and 8% of deaths of COVID-19 infected in those researched areas [2]. Therefore, it is possible that a person could be infected with the virus just before or after vaccination, get sick and transmit the virus to another person, because the vaccine did not have enough time to provide protection. Those vaccines seem to reduce the likelihood a person will develop symptoms if they are infected as well as the severity of the illness. Dr. Raman Gangakhedkar, Former director in-charge of ICMR-NARI, on vaccine misuse, said that the recent outbreak of COVID-19 in Israel include many who had been fully vaccinated. Those are called break through infections and they are typically mild.

Experts do not yet know how long the immunity provided by the vaccines will last since they are still on their trial stages. It remained unknown how long vaccine immunity against COVID-19 will last, or how the vaccines would perform against B.1.1.529 (Omicron) and other potential variants of the virus in the future. Immunity wanes as antibody levels drop, so the need for annual shots and boosters to maintain immunity is likely [1].

Long term protection will depend on the 'memory response' developed by our immune systems [12]. WHO advocates that one continues to take precautionary measures like social distancing, use of face masks and hand sanitizing even when one has got the full dose of the vaccines [14]. It is noteworthy that even highly vaccinated Isreal is seeing a dramatic surge in New COVID-19 cases. This is because Immunity from the vaccine dips overtime, delta variant broke through the vaccine's waning protection. Vaccinations are key, but they are not enough [5]

Based on these, we produce a mathematical model for the optimal control of COVID-19 transmission amongst the vaccinated individuals.

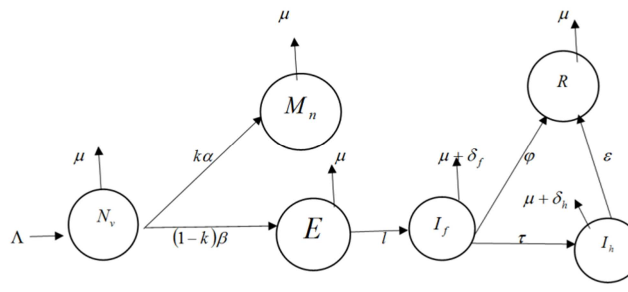

Figure 1. *Flow diagram of the population in various compartments.*

2. Mathematical Model

The total human population size in the community at time t, is divided into six compartments. They are; the Newly vaccinated, the Exposed class, the vaccinated but infected class, the Immune class, isolated or hospitalized vaccinated infected class, and Recovered individuals.

The variables and parameters used in the model formulation are:

1. $N_v(t)$: the Newly vaccinated COVID-19 class at time t. This includes the newly vaccinated individuals that are yet to build immunity and those that failed to complete their vaccine dosage.
2. $E(t)$: the vaccinated exposed class of individuals that are still incubating the virus.
3. $M_n(t)$: the class of Vaccine immuned and those with full compliance to other control measures. These are fully vaccinated individuals that with other control measures, have had enough time to build the required immunity.
4. $I_f(t)$: the infected class. This is a class of individuals, though vaccinated, but contacted the virus before the body could build the required immunity to the virus.
5. $I_h(t)$: the vaccinated infected but isolated or hospitalized class of individuals.
6. $R(t)$: the class of individuals that have recovered from COVID-19 infection.

Model parameters

Λ: rate at which individuals are being vaccinated.

k: rate/level of compliance to other COVID-19 control measures like social distancing, appropriate use of face masks, hand washing and use of sanitizers. $0 \le k \le 1$, thus full compliance implies $k = 1$

β: Force of infection of COVID-19 amongst the vaccinated group. Here, $\beta = \dfrac{n_1\lambda_1 I_f + n_2\lambda_2 I_h}{N}$

n_1, n_2: Infective rates of individuals in I_f *and* I_h respectively

λ_1, λ_2: rate of transmission of COVID-19 by the individuals in I_f *and* I_h respectively to the Newly Vaccinated class of individuals.

α: rate at which the Newly vaccinated progress to the immune class.

l: the rate of progression of the exposed class of individuals to the infected class.

τ: the rate at which the infected individuals are being isolated or hospitalized.

ϕ: the rate at which the infected vaccinated individuals are recovering from the disease.

ε: the rate at which the isolated or hospitalized vaccinated infected individuals recover from the disease.

μ: natural death rate of people.

δ_f, δ_h: the respective COVID-19 induced death rate of individuals that are in I_f *and* I_h classes. Reduced because of vaccine.

N : the total population of the individuals under study.

Thus, base on the facts above and the diagram in figure 1, the basic model for the optimal control of COVID-19 transmission amongst the vaccinated individuals is given by the following deterministic system of nonlinear differential equations with respect to time, t :

$$\frac{dN_v}{dt} = \Lambda - \left(k\alpha + (1-k)\beta + \mu\right)N_v$$

$$\frac{dE}{dt} = (1-k)\beta N_v - (l+\mu)E$$

$$\frac{dM_n}{dt} = k\alpha N_v - \mu M_n$$

$$\frac{dI_f}{dt} = lE - \left(\phi + \tau + \mu + \delta_f\right)I_f$$

$$\frac{dI_h}{dt} = \tau I_f - (\varepsilon + \mu + \delta_h)I_h$$

$$\frac{dR}{dt} = \varepsilon I_h + \phi I_f - \mu R$$

The force of infection, $\beta = \frac{n_1\lambda_1 I_f + n_2\lambda_2 I_h}{N}$, infects the proportion $(1-k)$ of the vaccinated class of individuals that are not observing full compliance to other control measures of COVID-19 like social distancing, appropriate use of face masks, hand washing and use of sanitizers. $0 \le k \le 1$, thus full compliance implies $k = 1$

It is assumed that the vaccines are 100% effective on those that fully complied to this social control measures.

$$N = N_v + E + M_n + I_f + I_h + R$$

$$\frac{dN}{dt} = \Lambda - \mu N - \delta_f I_f - \delta_h I_h$$

$$\Rightarrow \frac{dN}{dt} \le \Lambda - \mu N$$

This implies that $\lim_{t\to\infty} \sup N \le \frac{\Lambda}{\mu}$

Thus, the feasible region for the system is

$$\Lambda = \left\{ \begin{array}{l} \left(N_v, E, M_n, I_f, I_h, R\right): \\ \left(N_v + E + M_n + I_f + I_h + R\right) \le \frac{\Lambda}{\mu}, \\ N_v > 0,\ E > 0,\ M_n > 0,\ I_f > 0,\ I_h > 0,\ R > 0 \end{array} \right\}$$

Let $\varepsilon_0\left(\overset{*}{N_v}, \overset{*}{E}, \overset{*}{M_n}, \overset{*}{I_f}, \overset{*}{I_h}, \overset{*}{R}\right)$ be the equilibrium point of the system (2.1).

Since the recruitment term, Λ , can never be zero and the population cannot vanish. Thus, there is no trivial equilibrium point like $\varepsilon_0\left(\overset{*}{N_v}, \overset{*}{E}, \overset{*}{M_n}, \overset{*}{I_f}, \overset{*}{I_h}, \overset{*}{R}\right) = (0,0,0,0,0,0)$. So, let $\varepsilon_0\left(\overset{*}{N_v}, \overset{*}{E}, \overset{*}{M_n}, \overset{*}{I_f}, \overset{*}{I_h}, \overset{*}{R}\right) = \left(\overset{*}{N_v}, 0, \overset{*}{M_n}, 0,0,0\right)$. Then the system of equations at this point gives $\overset{*}{N_v} = \frac{\Lambda}{\mu}$ and $\overset{*}{M_n} = \frac{k\alpha\Lambda}{\mu^2}$, which gives the disease free equilibrium point as $\varepsilon_0\left(\overset{*}{N_v}, \overset{*}{E}, \overset{*}{M_n}, \overset{*}{I_f}, \overset{*}{I_h}, \overset{*}{R}\right) = \left(\frac{\Lambda}{\mu}, 0, \frac{k\alpha\Lambda}{\mu^2}, 0,0,0\right)$

3. The Basic Reproduction Number, R_0

The basic reproduction number, R_0 , in this case is the average number of secondary cases of the disease made by a typical infectious vaccinated person in the Newly vaccinated population where other control practices (k), like use of face masks and social distancing, exist, if $R_0 < 1$, the disease will be eradicated with time but if $R_0 > 1$, then the disease is likely going to persist in the population. We use the next generation approach as described by Diekmann and Heesterbeek (2000), [6], to obtain the reproduction number.

Let

$X' = \varepsilon\left(E, M_n, I_f, I_h, R, N_v\right)^T$. Therefore, $X' = F(X) - V(X)$

Where $F(X)$ and $V(X)$ are column matrices given by;

$$F(X) = \begin{bmatrix} \frac{(1-k)\left(n_1\lambda_1 I_f + n_2\lambda_2 I_h\right)N_v}{N} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V(X) = \begin{bmatrix} (l+\mu)E \\ -k\alpha N_v + \mu M_n \\ -lE + \left(\phi + \tau + \mu + \delta_f\right)I_f \\ -\tau I_f + (\varepsilon + \mu + \delta_h)I_h \\ -\varepsilon I_h - \phi I_f + \mu R \\ -\Lambda + \left(k\alpha + \frac{(1-k)\left(n_1\lambda_1 I_f + n_2\lambda_2 I_h\right)}{N} + \mu\right)N_v \end{bmatrix}$$

The derivatives $DF(\varepsilon_0)$ and $DV(\varepsilon_0)$ at disease free equilibrium point ε_0, are partitioned as $DF(\varepsilon_0)=\begin{bmatrix} F & 0 \\ 0 & 0\end{bmatrix}$ and $DV(\varepsilon_0)=\begin{bmatrix} V & 0 \\ J_1 & J_2\end{bmatrix}$ where F and V are 3×3 matrices, since E, I_f, I_h are the only groups that have the virus, given by;

$$F=\begin{bmatrix} 0 & \frac{(1-k)n_1\lambda_1\Lambda}{N\mu} & \frac{(1-k)n_2\lambda_2\Lambda}{N\mu} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } V=\begin{bmatrix} (l+\mu) & 0 & 0 \\ -l & (\phi+\tau+\mu+\delta_f) & 0 \\ 0 & -\tau & (\varepsilon+\mu+\delta_h) \end{bmatrix}$$

The effective basic reproduction number R_0, which is the spectral radius of FV^{-1}, is $\rho\left(FV^{-1}\right)$.

Thus,

$$R_0=\frac{\Lambda l(1-k)\left(n_1\lambda_1(\varepsilon+\mu+\delta_h)+n_2\lambda_2\tau\right)}{N\mu(l+\mu)\left(\phi+\tau+\mu+\delta_f\right)(\varepsilon+\mu+\delta_h)}$$

4. Stability of Disease Free Equilibrium

The disease free equilibrium is stable if all the eigenvalues of the Jacobian matrix of the system, (2.1), have negative real parts.

The Jacobian of the model equation sat the disease-free equilibrium point is given as;

$$J=\begin{bmatrix} -\mu & 0 & 0 & -\frac{(1-k)n_1\lambda_1\Lambda}{N\mu} & -\frac{(1-k)n_2\lambda_2\Lambda}{N\mu} & 0 \\ 0 & -(l+\mu) & 0 & \frac{(1-k)n_1\lambda_1\Lambda}{N\mu} & \frac{(1-k)n_2\lambda_2\Lambda}{N\mu} & 0 \\ k\alpha & 0 & -\mu & 0 & 0 & 0 \\ 0 & l & 0 & -\left(\phi+\tau+\mu+\delta_f\right) & 0 & 0 \\ 0 & 0 & 0 & \tau & -(\varepsilon+\mu+\delta_h) & 0 \\ 0 & 0 & 0 & \phi & \varepsilon & -\mu \end{bmatrix}$$

It can be seen that trace(J) is negative.

Solving for the determinate of J, $\det(J)$, we have;

$$\det(J)=-\frac{\mu^3 l\tau\left((1-k)n_2\lambda_2\Lambda\right)}{N\mu}+\mu^3(\varepsilon+\mu+\delta_h)\left(\phi+\tau+\mu+\delta_f\right)(l+\mu)-\frac{\mu^3(\varepsilon+\mu+\delta_h)l\left((1-k)n_1\lambda_1\Lambda\right)}{N\mu}$$

For $\det(J)>0$, we have;

$$\begin{pmatrix} -\frac{\mu^3 l\tau\left((1-k)n_2\lambda_2\Lambda\right)}{N\mu}+\mu^3(\varepsilon+\mu+\delta_h)\left(\phi+\tau+\mu+\delta_f\right)(l+\mu) \\ -\frac{\mu^3(\varepsilon+\mu+\delta_h)l\left((1-k)n_1\lambda_1\Lambda\right)}{N\mu} \end{pmatrix}>0$$

Which gives,

$$1>\frac{\Lambda l(1-k)\left[n_1\lambda_1(\varepsilon+\mu+\delta_h)+n_2\lambda_2\tau\right]}{N\mu(\varepsilon+\mu+\delta_h)\left(\phi+\tau+\mu+\delta_f\right)(l+\mu)} \quad \Rightarrow R_0<1$$

This implies that the disease-free equilibrium is locally asymptotically stable if $R_0<1$ and unstable if otherwise.

5. Sensitivity Analysis

The impact of each of the parameter on the reproduction number R_0, can be ascertained by their sensitivity indices. This helps to determine which parameters contribute in making the disease endemic. It also helps to know the parameters that should be kept in check in order to control COVID-19 pandemic.

The parametric values listed below where as observed from COVID-19 data in US and presented by the corresponding researchers. The data may differ from country to country. However, some of them were judiciously estimated.

Table 1. *Parametric values used in the model.*

parameters	Definition	Values	Source
μ	Natural death rate	0.02	Nita H. Shah, et al (2014) [11]
l	Progression rate from E to I	0.196 per day	Iboi et al (2020) [8], Ngonghala et al (2020) [10]
ϕ	The rate of recovering of the infected individuals	0.9 per day	Estimated
ε	Recovery rate of the hospitalized infected group	0.925 per day	,,
δ_f	COVID-19 induced death rate of individuals in I_f	(8% of 0.1) = 0.008 per day	Calculated from CDC (2021) and Iboi et al (2020) data. [10]
δ_h	COVID-19 induced death rate of individuals in I_h	8% of 0.015) = 0.0012 per day	,,
α	Progression rate of the Newly vaccinated to the immune class.	(95%-75% effective) 0.85	Dr. Kristen Marks, New York-presbytarian (2021) [9]
n_1, n_2	Infective rates of individuals in I_f and I_h classes	0.1925, 0.0167 per day respectively	Gumel A. B. Et al (2021) [7]
τ	The rate of isolating or hospitalizing the infected individuals	0.0221 per day	Gumel A. B. et al (2021) [7]
k	Rate of compliance to other control measures. Full compliance is $k=1$	0.4	Assumed
λ_1, λ_2	Infection transmission rate of I_f, I_h respectively to the Newly vaccinated class.	0.4768, 0.016 per day	Estimated from Abba B. Gumel et al (2021) data. [7]
β	Force of infection of COVID-19 amongst the vaccinated group. Here, $\beta = \frac{n_1\lambda_1 I_f + n_2\lambda_2 I_h}{N}$	0.055 per day	Simon A. Rella, et al (2021) [13]

A. B. Gumel et al (2021) gave the contact (Transmission) rate of the asymptomatic and symptomatic COVID-19 infected as 0.7611 and 0.1925 per day respectively. Thus, we take the mean value, 0.4768 as the contact rate of the infected, I_f. They also had the value, 0.016, for the hospitalized infected individuals.

We calculate the normalized forward sensitivity indices for those parameters in which the value of basic reproduction number depends using.

$$\zeta_p^{R_0} = \frac{\partial R_0}{\partial P} \times \frac{P}{R_0},$$

where P is any of the parameter in R_0 and obtained the data as shown in the table below;

Table 2. *Sensitivity indices of R_0 to the parameters.*

Parameters	Sign	Values
Λ	+	1
l	+	0.0926
k	-	0.667
λ_1	+	0.9791
ε	-	0.00031026
ϕ	-	0.41305
λ_2	+	0.000397
τ	-	0.0294

This implies that those parameters with positive indices indicate that the value of the reproduction number increases as the value of the parameters increase and decreases if the values of the parameters are decreased. Those with negative indices imply that the infection will decrease as those parameters are increased in value.

Thus, effort should be geared toward increasing the value of the parameters; k, ϕ, ε *and* τ even among the COVID-19 vaccinated individuals and decrease the parametric values of l, λ_1 *and* λ_2.

6. Numerical Simulation

Numerical simulation indicates the future trend of the epidemic. It helps to determine the parameters that should be checked, for the effective control of the disease. Here, we simulate with a sample population of 5,000 of the newly vaccinated individuals in a COVID-19 endemic area for a period of 20 days. Results of the simulations where shown in the diagrams below.

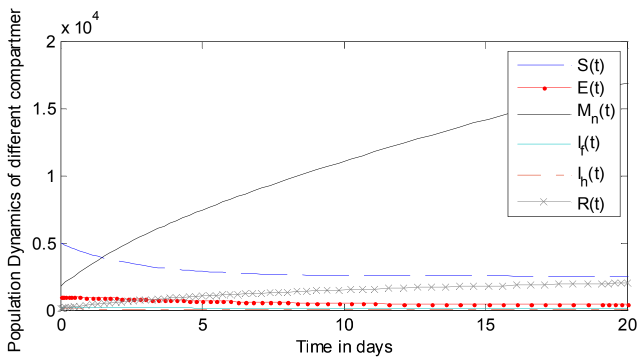

Figure 2. *Population Dynamics of different compartments of the model.*

This shows the population dynamics of the various compartment when 40% of the newly vaccinated, observe other social control measures like the use of face mask.

From the graphs, the population of the Exposed group continues to increase at the rate of 84 persons per day while that of the infected group increased at the rate of 17 per day. Virtually, without these social control measures like face mask and good hygiene, all the vaccinated will eventually get exposed to the virus.

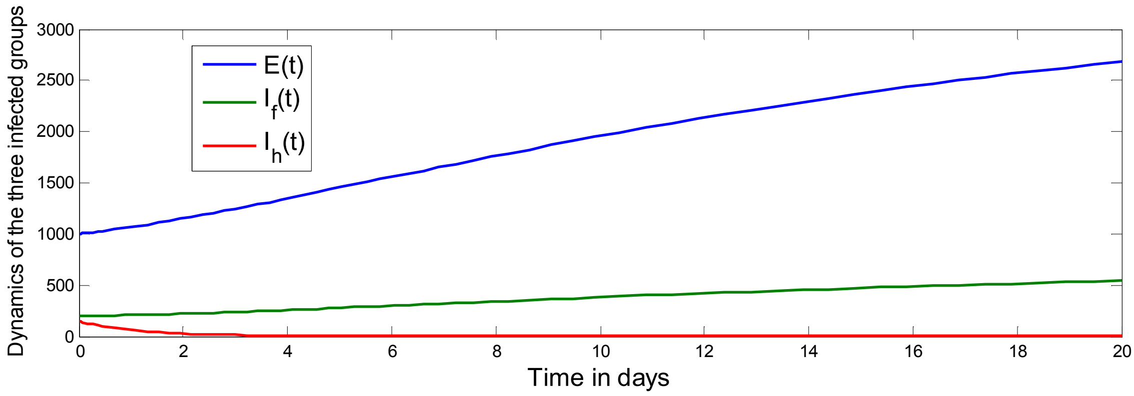

Figure 3. Population dynamics of the infected groups in the absence of other social control measures.

In the following figures, the exposed, Infected and Hospitalized compartments where simulated varying the degree of the social control measures (k), 20%, 60% and 90%.

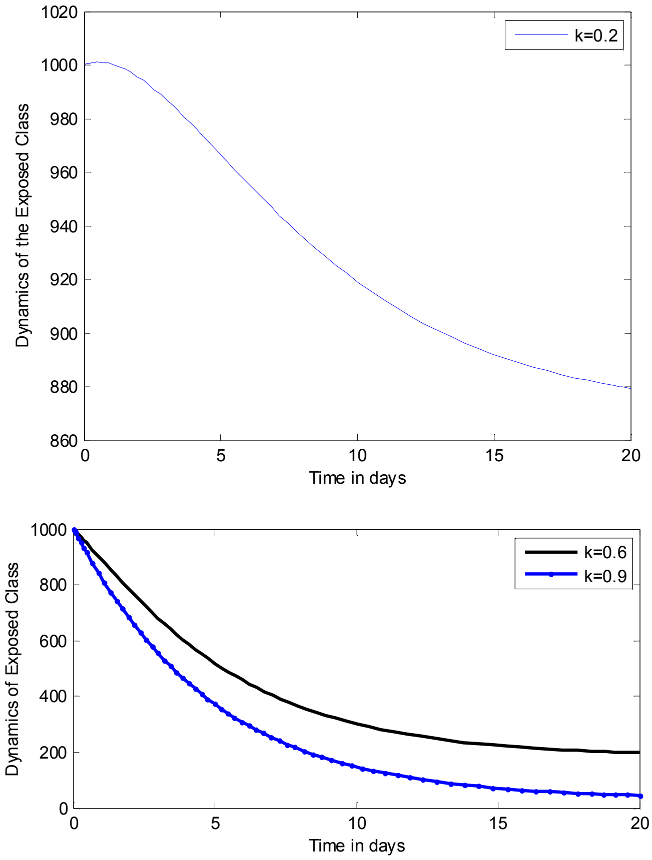

Figure 4. Population Dynamics of Exposed Class as the rate of social control compliance is varied.

For the Exposed class, when the social control measures compliance is 20%, the population of the Exposed class decreases on the average of 6 per day. Also further decrease of 40 and 48 persons per day is observed in the population when the controlled measures are raised to 60% and 90% respectively.

For the Infected class, when the social control measures compliance is 20% the population of the Infected class is observed to decreased on the average of 1 per day. When the rate of social control compliance is raised to 60% and 90%, then the population of the infected persons is seen to be decreasing at the rate of 8 and 10 per day respectively.

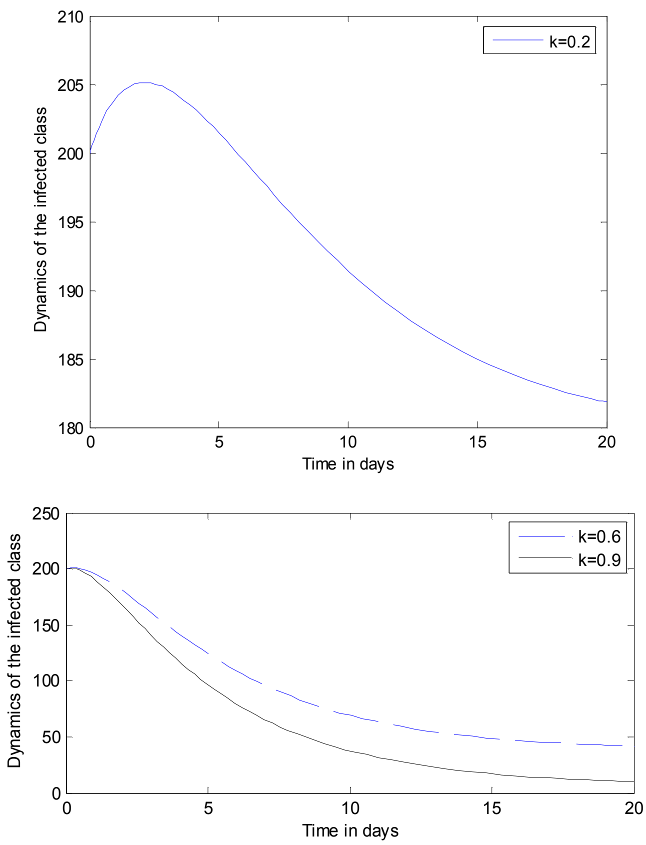

Figure 5. Population Dynamics of Infected Class as the rate of social control compliance is varied.

Also, for the Hospitalized class, we observe that the variation of the rate of social control measures has a little decreasing effect on the population of the infected persons that are isolating or being Hospitalized.

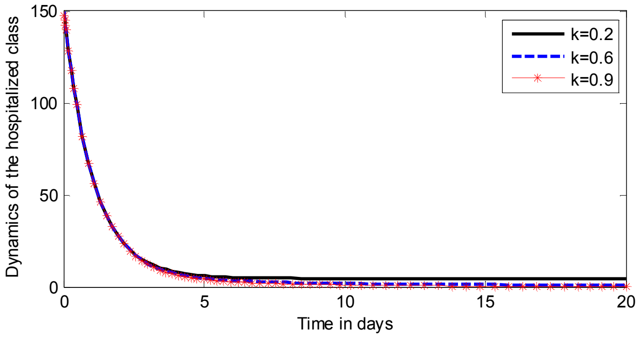

Figure 6. *Population Dynamics of Hospitalized or isolated Class as the rate of social control compliance is varied.*

7. Conclusion

We presented compartmental, deterministic model of the COVID-19 spread to gain insight on the effect of social control measures (social distancing, use of face masks in public places and adequate maintenance of good hygiene) on the spread of the virus amongst the vaccinated. The disease-free equilibrium is found to be asymptotically stable whenever the reproduction number is less than one. Assessing the parameters involved in the disease spread shows that when the parameters like k, ε, τ *and* ϕ are allowed to decrease in value, then the disease spread will be rapid irrespective of the fact that they have received COVID-19 vaccine.

In fact, from the numerical simulation carried on the model, it is noted that in the absence of social control measures (k), almost all the vaccinated persons will be infected and will be able to infect others especially within few days of receiving the COVID-19 vaccine.

Therefore, apart from advocating for mass vaccination of the people, efforts should be made to optimally increase other social control measure compliance among the vaccinated individuals.

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