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Applications of B-transform to Some Impulsive Control Problems

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Abstract: In this paper, B-transform is applied to some impulsive control models and closed solution forms for the models obtained. The problems solved via the B-transform are the third order linear impulsive control systems with bang-bang control, Impulsive delay control systems, Impulsive heat control systems, the Impulsive diffusion problem and the impulsive Gross berg control model. Simulation for the bang bang model show that the solutions are negative and positive in some for given time interval. The solutions also exhibit non-periodic and non-oscillatory behaviour in the given interval. The solutions of impulsive diffusion model possess singularities in given interval of simulation.

Keywords: Impulsive, Control Systems, Bang-Bang, B-transform

1. Introduction

Impulses are small perturbations acting on a system for short moment of times and can take the form of "jumps", rapid changes in the system or "shocks", short-time mechanical impacts on two or more moving systems etc ([3, 12]).

Many impulsive physical and biological processes occur in nature; the thermionic current in a vacuum tube is not smooth flow of electrons but is subject to fluctuation due to the random emission of electron from the cathode. This phenomenon is called the small-short effect and was discovered by Schotty [7]. This effect can be explained with the help of impulsive theory [9].

Impulsive control systems (ICS) are control systems characterized by short time jumps and shocks that act on the system rapidly. The impulses may act on the state variables describing the systems or on the control variables regulating the systems ([4, 6, 15, 17-18]).

The solutions of ICS are often discontinuous and are not integrable in the ordinary sense of the word as most hypotheses in the control systems normally assumed. This peculiarity makes ICS not easily accessible to most existing concepts and theorems in the control systems ([9, 16]). Therefore the existing concepts, theories in CS need to be strengthened or new ones developed before applying to the ICS ([9]).

Impulsive control systems are found in biological systems, for examples (See,[4, 9-10, 19]) and in engineering systems (See[9,15-18]) and also have applications in economics wherein the state of an economy of a country can be regulated in a desired way by implementing some policies which has impulsive attributes.

Impulsive control systems are useful in biomedicine where control is strictly needed to regulate biochemical substance in the cells and tissues. ICS also useful in the design of an automatic temperature controlled swimming pool, incubator, nuclear reactors, or heating and cooling system in biological and physical systems that heat or temperature is required to be impulsive [9]. The use of impulsive haematopoiesis control model to measure the replacement of the blood by new blood cells as a result of use of drug, or food supplement and have been obtained together with controllability criteria for the model (See for example [15]).

It is worthy to note that the B-transform was developed by Oyelami and Ale (See [7, 9, 14]) for finding closed solution forms to the fixed moment impulsive systems. B-transform has been applied to solve problems on sickle cell anaemia, HIV/AIDs, fish-hyacinth problem, heart and gas enclosure problems and problem of regulation of cerebrospinal fluid in children with swollen heads (see [4, 7, 8, 11]). Also related to B-transform is the B-stability which was developed and applied to solve some impulsive control system problems ([4, 9]).

In the recent times several models are evolving in attempt to some challenging problems in science and technology. Some of these problem are from the control systems that of impulsive family. We will mention in particular solutions of the KdV-Burgers equation, Bagley-Torvik and Painlevé equations [1-2]. Varieties of methods have been developed to find solutions to the above equations. The motivation in this paper is make use of the B-transform method to find closed solution forms to the following problems: bang-bang control, model for a third order linear impulsive control systems, impulsive delay control systems, Impulsive heat control systems, the impulsive diffusion problem and the impulsive Gross berg control model. Moreover, we propose to study the approximate solutions of the impulsive diffusion problem control model.

It is worthy to note that the B -transform has a lot of potential applications unexplored yet. It is, therefore, recommended that more concerted efforts be devoted to the theory.

2. Preliminary Definitions and Notations

Throughout the paper we will use of the following notations:

$C(\Re^+,\Re)$ the set of continuous functions defined on $\Re^+=[0,+\infty)$ taking values in $\Re$.

$C^\infty(\Re^+)$ the set of smooth functions defined on $\Re^+$.

We consider the impulsive fixed moments $\{t_k\}$, $k=0,1,2,\ldots$ such that $0<t_0<t_1<t_2<\cdots<t_k<a$, a is positive constant.

We define the functions

$$f:(0\ a]\times k\to\Re$$
$$g:(0\ a]\times k\to\Re$$

and let $J=(0,a]$ and $J_{imp}=\{t_k\}_{i=1}^{k}$, we introduce the following piecewise continues function $PC(\Re^+,\Re)$ as follows:

Definition 1:

We say that the function $x:\Re^+\to\Re$ belongs to class $PC(\Re^+,\Re)$ if:

i. The restriction of x to $J/_{Jimp}$ most be continuous function;
ii. There exist two limits such that

$$\lim_{\substack{s\to t_k\\ s<t_k}} x(s)=x\left(t_k^-\right)$$
$$\lim_{\substack{s\to t_k\\ s>t_k}} x(s)=x\left(t_k^+\right).$$

The left and right limits of $x(t)$ at t_k respectively exist such that $x\left(t_k\right)=x\left(t_k^-\right)$. That is, the function $x(t)$ is left continuous at $t=t_k$.

3. Methods

3.1. B -Transform

B-transform can be apply to a fixed moment impulsive differential equation of the form

$$\left.\begin{aligned} &\dot{x}(t)=f(t,x(t)),t\neq t_k,k=0,1,2,...\\ &\Delta x(t_k)=I(x(t_k))\\ &0<t_0<t_1<t_2<...<t_k,\lim_{k\to\infty}t_k=+\infty \end{aligned}\right\} \tag{1}$$

Where f and I in the eq. (1) are assumed to continuous and satisfy all the conditions the guarantee the existence and the uniqueness of the solution of the eq. (1) (See [3, 9, 16]). In [7] we introduced the B -transform of a function $x(t)$ with impulses occurring at some fixed moments during the evolutionary process as

$$B_n' x(t)=x_c(q)+x_I(q) \tag{2}$$

where $x_c(q)$ and $x_I(q)$ are the components of the B -transform and are defined as

$$x_c(q)=L_c x(t)=\int_0^\infty e^{-t/q^{n'}}x(t)dt,t\neq t_k,k=0,1,2,\ldots \tag{3}$$

and

$$x_I(q)=L_I x(t)=\sum_{t_0<t_k<t} e^{-t_k/q^{n'}}I(x(t_k)) \tag{4}$$

$n'=0,1,2,\ldots$ is the order of the transform. For sake of simplicity, we often choose $n'=1$. The advantage of taking $n'=1$ lies in the derivation of the inverse transform.

The inverse transforms for components of $x_c(q)$ and $x_I(q)$ can be obtained as follows:

$$x_c(t)=\int_{v-i\infty}^{v+i\infty}x_c(q)e^{sq}dq \tag{5}$$

$$x(t)=\sum_{t_o<t_k<t}{}' \psi(t_k,q)I(x(t_k)) \tag{6}$$

$$\psi(t_k,q)=\int_{v-i\infty}^{v+i\infty}dq\ e^{-t_k/q^{n'}+sq}. \tag{7}$$

The B -transform is valid in some sets. In [14] symbolic programming method in Maple software was introduced to find the inverse B-transform and the solutions to the impulsive diffusion model and the Von-Foerster – Makendrich model were found.

Our first application of B- transform is to find closed solution form to the third order linear impulsive systems with bang-bang control. The bang-bang control models are found extensively in many engineering applications. We will make use of the B-transform to find solutions to some classes of the third order linear impulsive control systems with bang-bang control.

3.2. Third Order Linear Impulsive Control Systems

Consider a third order linear impulsive control system

$$\begin{gathered} \dddot{x}(t)+\dot{x}(t)=\mu(t), \quad t\neq t_k \\ \Delta x(t_k)=B_k x(t_k) \end{gathered} \tag{8}$$

For $t,T\in[0,+\infty)$ assuming $\mu(t)$ is bang-bang control such that $|\mu(t)|\leq 1$ for all $t\geq 0$ and $x(T)=\dot{x}(T)=\ddot{x}(T)=0$. $x(0)=x_0$, $\dot{x}(0)=\dot{x}_0$ and $\ddot{x}(0)=\ddot{x}_0$.

In our next application, we consider the algebraic impulsive delay control systems with impulsive variable being regarded as linear combination of some impulsive variables and the control variable regulating the system contains impulsive delay variables.

3.3. Impulsive Delay Control System

Consider an Impulsive delay control system

$$\left.\begin{array}{l} \dot{x}(t)=A_{i,j}x(t)+\sum_{i=1}^{n}\sum_{j=1}^{n}B_{ij}x(t-c_ih_i)+Du(t), t\neq t_k, k=0,1,2,\ldots \\ \Delta x(t=t_k)=\sum_{k=1}^{N}a_k x(t_k) \\ u(t):=\sum_{k=1}^{N}k_i x(t-t_k) \end{array}\right\} \tag{9}$$

For strictly increasing impulsive times $\{t_k\}$ such that

$$0<t_0<t_1<t_2<\cdots<t_k, \lim_{k\to\infty}t_k=+\infty$$

where (A_{ij}) and (B_{ij}) are $n\times n$ constant matrices of dimension N, $x(t)\in R^N$, $t\in\Re^+=[0,+\infty)$ (A_{ij}) is the growth rate matrix for the state vector $x(t)$. (B_{ij}), a_k, c_k, h_i, D and k_i are some real life parameter describing the impulsive delay control system. The sequence $\{B_ih_i\}_{i=1}^{\infty}\subset l^2(\Re^+)$ is assume to be convergent such that $|B_ih_i|_{l^2(\Re^+)}\to Bh$ as $i\to\infty$, $\theta:=\min_{i,j,k}[|B_ih_i|,|\gamma_k|,|\tau_k|]$, $k_i=(k_1,k_2,?,k_n)^T$.

3.4. Impulsive Heat Control Systems

Consider the impulsive heat control systems given by

$$\left.\begin{array}{l} \dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+F(t,v), t\geq 0, 0\leq x\leq 1 \\ u(t=t_k,x)=B_k u(t,x)=\displaystyle\sum_{k=0}^{N}\beta_k\phi(t_k)u(t_k,x) \end{array}\right\} \tag{10}$$

Subject to

$$\left.\begin{array}{l} u(x,0)=\phi(x) \\ u(0,t)=a_0(t) \\ u(l,t)=a_1(t) \end{array}\right\} \tag{11}$$

where $u\in PC(\Re,\Re^n)$ the control variable $v, a_i\in C(\Re^+,\Re), i=0,1; \beta_k,\phi\in C^\infty(\Re^+)$.

Our next application is impulsive diffusion problem; such problems are extensively found in the molecular biology, neural network. Impulsive diffusion models also have a lot of applications in real life, especially in water and sanitation problems, and in population dynamics ([14]) and complex chemical reaction systems.

3.5. Impulsive Diffusion Problem

We consider the application of B-transform to the following impulsive diffusion problem described the following differential equations

$$\left.\begin{array}{l} \dfrac{du}{dt}=\displaystyle\sum_{k=0}^{N}\sum_{j=0}^{N-k}a_{kj}u_i^k v_i^k+\frac{Dv}{\delta^2}\frac{\partial^2 u}{\partial x^2}+\beta_1 u, \quad x\neq x_k,\ k=0,1,\cdots \\ \dfrac{dv}{dt}=\displaystyle\sum_{k=0}^{N}\sum_{j=0}^{N-k}b_{kj}u_i^k v_i^k+\frac{Dv}{\delta^2}\frac{\partial^2 u}{\partial y^2}+\beta_2 v, \quad y\neq y_k,\ k=0,1,2,\cdots \\ \Delta u(t=t_k)=I_1(u)=\sum_{i=1}^{n}a_i u_i^k+b_k+f_1(u_k) \\ \Delta v(t=t_k)=I_2(v)=\sum_{i=1}^{n}b_i v_i^k+c_k+f_2(v_k) \end{array}\right\} \tag{12}$$

Then the impulsive system can be approximated by

$$\left.\begin{array}{l} \dfrac{du_i}{dt}=\displaystyle\sum_{k=0}^{N}\sum_{j=0}^{N-k}a_{kj}u_i^k v_i^k+\frac{Dv}{\delta^2}(u_{i-1}-2u_i+u_{i+1})+\beta_1 u_i \\ \dfrac{dv_i}{dt}=\displaystyle\sum_{k=0}^{N}\sum_{j=0}^{N-k}b_{kj}u_i^k v_i^k+\frac{Du}{\delta^2}(v_{i-1}-2v_i+v_{i+1})+\beta_2 v_i \\ \Delta u_i=I_1(u_i)=\displaystyle\sum_{i=1}^{n}a_i u_i^k+b_k+f_1(u_i) \\ \Delta v_i=I_2(v_i)=\displaystyle\sum_{i=1}^{n}b_i v_i^k+c_k+f_2(v_i) \end{array}\right\} \tag{13}$$

The system in the equation (13) is approximation to the system in the equation (12). It is assumed that the behaviour of the solution to the equation (13) will not be significantly different from the parent equation in the equation (12).

3.6. Impulsive Grossberg Model

Most general form of Gross berg model is in perturbed form as follows

$$\frac{dx_i}{dt}=a_i(x_i)\left[b_i(x_i)-\sum_{j=1}^{N}C_{ij}g_i(x_i)\right]+h(t,x_i,u(t)) \tag{14}$$

where $a_i(x_i)$ is the neural output related to internal activity of the neuron; $b(x_i)$ is the neural activity as it jumps from one synapse to another; x_i is the activity of the neuron and C_{ij} the activity of the synapse being in form of excitatory and inhibitory synapses; $g_i(x_i)$ is the neural output. Assume that $a_i(0) = 0$, $b_i(0) = 0$ and g(0) = 0,and h(t, x, u(t)) is a non-linear perturbation function with control variable u(t). For insight impulsive analogue of the Grossberg model (See [9])

The matrix (C_{ij}) and the functions $a_i(x_i), b(x_i)$ and $g_i(x_i)$ are continuous such that

$$f(t,x_i) = a(x_i)\left[b_i(x_i) - \sum_{j=1}^{N} C_{ij}\, g_i(x_i) \right] + h(t,x_i,u(t))$$

If $b_i(x) = x_{i+1} - \beta_i x_i = c =$ constant then the equation becomes neural network describe by an impulsive differential equations of the form:

$$\left.\begin{aligned} &\frac{dx_i}{dt} = ca_i(x_i) - \sum_{j=1}^{N} C_{ij}\, g_i(x_i), t \neq t_k, k = 0,1,2,\cdots \\ &\Delta x_i \,|_{t=t_k} = \beta_i x_i(t_k) \\ &0 < t_0 < t_1 < t_2 < \cdots < t_k, \lim_{k\to\infty} t_k = \infty \end{aligned}\right\} \tag{15}$$

4. Results and Discussion

Applying B-transform to the third order linear impulsive control system we have

$$\left.\begin{aligned} L_C(\dot{x}) &= \int_0^{\infty} \dot{x} e^{-t/q} dt = x_0 + \frac{1}{q} x_C(q) \\ L_C(\dddot{x}) &= -\ddot{x}_0 + \frac{1}{q} x_0 + \frac{1}{q^2} x_C(q) \end{aligned}\right\} \tag{16}$$

Therefore,

$$x_C(q) = \frac{q^3}{q+1}\left[\ddot{x}_0 + \left(1 + \frac{1}{q}\right)\ddot{x}_0 + \frac{\in_1}{q} \right]$$

Therefore,

$$\begin{aligned} x(q) &= x_C(q) + x_I(q) \\ &= x_C(q) + \sum_{t_0<t_k<t} B_k e^{-t_k/q} x(t_k) \\ x(q) &= \frac{q^3}{1+q} \dddot{x}_0 + q^2 \ddot{x}_0 + \frac{q^2}{1+q} \in_{\pm 1} + \sum_{t_0<t_k<1} I(x_k) e^{-t_k/q} \end{aligned} \tag{17}$$

Hence

$$x(t) = B^{-1}(x(q))(t)$$

But

$$B^{-1}\left(\frac{q^3}{1+q}\right) = \int_C \frac{q^3}{1+q} e^{tq} dq$$

By residue theory this is equal to

$$\lim_{q\to -1} (1+q) \frac{q^3}{1+q} e^{tq} = -e^{-t}$$

In the same vein, using the same theory we have

$$\left.\begin{aligned} B^{-1}(q^2) &= \int_C q^3 e^{tq} dq = t \\ B^{-1}\left(\tfrac{q^2}{1+q}\right) &= e^{-t} \end{aligned}\right\}$$

Therefore,

$$\left.\begin{aligned} x(t) &= -e^{-t}\, \dddot{x}_0 + t\ddot{x}_0 + \in_{\pm}\, e^{-t} + \sum \phi(t_k,t) I(x_k) \\ \phi(t_k,t) &= \int_C e^{-t_k/q + tq} dq \end{aligned}\right\} \tag{18}$$

We have taken $\in_{\pm} := \pm 1$. we simulate the solution to the model using the equation (18) for $\in_1 = 0.02, \dot{u}_0 = 1, \ddot{u}_0 = 0.5, \beta = 0.02$ and plotted the graph for $x(t)$ using Maple 17 version and it is the Figure 1 below:

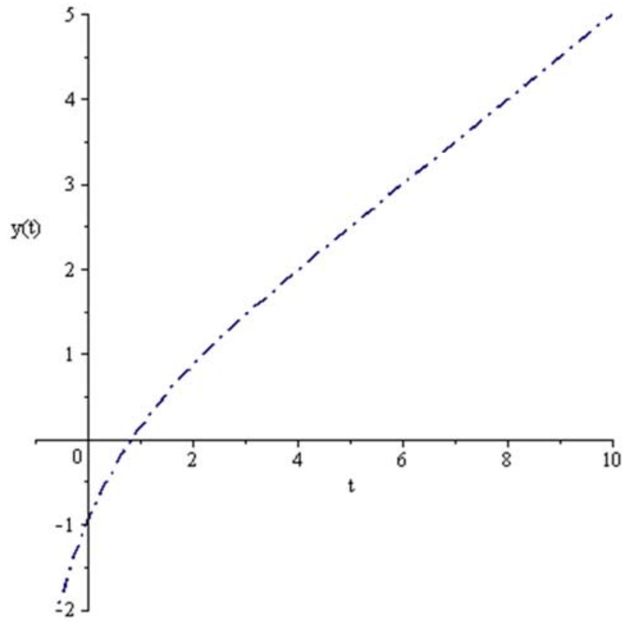

Figure 1. *The plot for solution impulsive bang bang control.*

In the Figure 1 the solution continue increasing for $t \in \Re$, it is non-oscillatory and no-periodic in the given interval and has zero $x(t) = 0$ at t=1.5 seconds. Figure 2 has similar behaviour but zero at about t= 0.5 seconds and simulation period in the interval $0 \le t \le 50$ seconds.

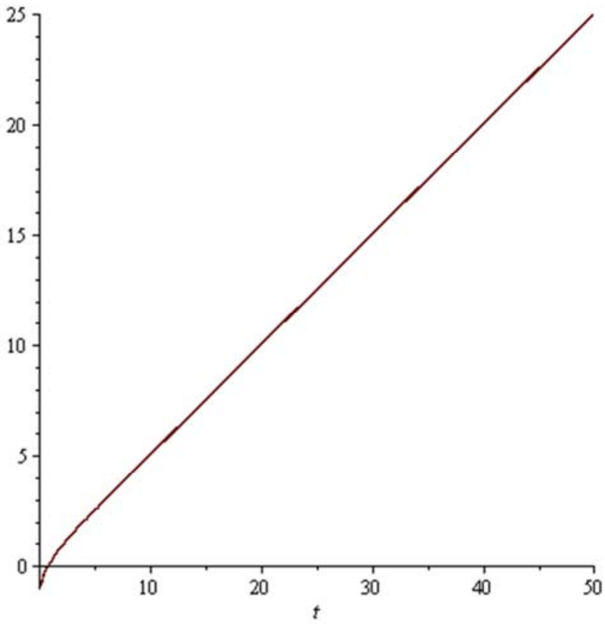

Figure 2. *Plot of* $x(t), \in_1 = 0.03, \dot{u}_0 = 10, \ddot{u}_0 = 0.05, \beta = 0.05$.

Application of B-transform to the impulsive delay control system yields

$$B_c(x(t - B_i h_i)) = \int_0^\infty x(t - B_i h_i) e^{-t/q} dt$$

$$= e^{-B_i h_i / q} \left[x_c(q) + \int_{-B_i h_i}^{0} x(t_i) e^{t_i/q} dt_i \right]$$

$$B_c x(t) = \sum_{j=0}^{r} k_j \int_0^\infty x(t - t_j) e^{-t/q} dt$$

$$= \sum_{j=0}^{r} k_j \left[e^{-\tau_j/q} \left(x_c(q) + \int_{-\tau_0}^{0} x(t_1) e^{-t_i/q} dt_i \right) \right]$$

$$B_c(\dot{x}(t)) = -x_0 + \frac{1}{q} x(q), \quad B_c(A_0 x(t)) = A_0 x(q).$$

Therefore,

$$x_C(q) = \left[A_0 + \left(e^{-B_i h_i / q} - \frac{1}{q} \right) I \right]^{-1} \left[-x_0 - e^{-B_i h_i / q} \int_{-B_i h_i}^{0} x(s) e^{-s/q} ds \right.$$
$$\left. + e^{-t_0/q} u(q) + e^{-\tau_i/q} \int_{-t_i}^{0} u(t_i) e^{-\tau_1/q} dt_i \right]$$

And

$$x_I(q) = \sum_{t_0 < t_i < t} I(x(t_i)) e^{-t_k/q}$$

$$= \sum_{t_0 < t_k < t} \sum_{0 < \gamma_k < t} I(x(t_i)) = \sum_{0 < t_k < t} \sum_{0 < \gamma_k < t} a_k e^{t_k/q} e^{\gamma_k/q} x_I(q).$$

Therefore, $x(q) = x_C(q) + x_I(q)$ and

$$x(t) = \frac{1}{2\pi i} \int_C x(q) e^{tq} dq$$

We observe in the Figure 3 the solutions of the impulsive bang- bang control model for given parameters is negative for $0 < t < 7.8$ and positive otherwise. The solution is found to be non-oscillatory and non- periodic and the Figure 4, the solution is negative for $-\infty < t < 1.8$ and positive otherwise. The solution also exhibits non-periodic and non-oscillatory behaviour in the given interval.

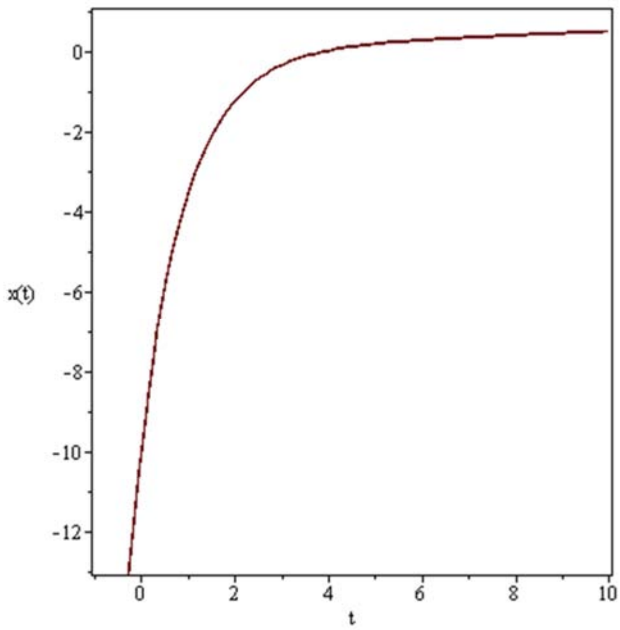

Figure 3. *The solution of the impulsive delay control system.*

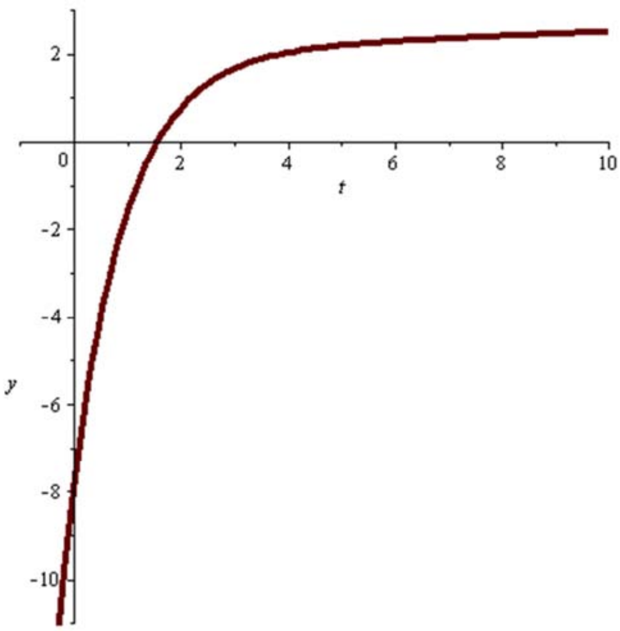

Figure 4. *The plot of* $x(t), \in_1 = 0.02, \dot{u}_0 = 10, \ddot{u}_0 = 0.05, \beta = 0.05, I(x_k) = 2^{-k}$.

Application of B-transform to Impulsive heat control systems:

Let $u(t,x) = u(t)u(x)$ Therefore,

$$\left.\begin{array}{l}\frac{\dot{u}(t)}{u(t)}=\frac{u'(x)}{u(x)}=-k \\ \Rightarrow u^{/}(x)+ku(x)=0 \\ \Delta u(t=t_k,x)=\sum\beta_k\phi(t_k)u(t_k)u(x) \\ \Delta u(t_k)=\sum\beta_k\phi(t_k)u(t_k)\end{array}\right\} \quad (19)$$

Therefore,

$$u(t)=u_0\prod_{t_0<t_k<t}\left(1+\sum B_k\varphi(t_k)\right)e^{-k(t-t_k)} \quad (20)$$

Application of B-transform to $u''(x)$, we set

$$u(q)=\frac{q^2}{kq^2-1}\ddot{u}_0-\frac{q}{kq^2-1}\dot{u}_0$$

Therefore,

$$u(t)=\frac{1}{2\pi i}\int_C u(q)e^{tq}dq$$

Where C is a continuous continuing the poles of $u(q)$ which are $q_+=\frac{1}{\sqrt{k}}$ and $q_-=-\frac{1}{\sqrt{k}}$.

Therefore,

$$B^{-1}\left(\frac{q^2}{kq^2-1}\right)=\begin{cases}\frac{1}{2\sqrt{k}}e^{t/\sqrt{k}}, & q=q_+ \\ -\frac{1}{2\sqrt{k}}e^{-t/\sqrt{k}}, & q=q_-\end{cases}$$

$$B\left(\frac{q}{kq^2-1}\right)=\frac{1}{2}(e^{t/\sqrt{k}}-e^{-t/\sqrt{k}})$$

$$\therefore\ u(x)=\frac{1}{2\sqrt{k}}\left[e^{t/\sqrt{k}}-e^{-t/\sqrt{k}}\right]\ddot{u}_0-\frac{1}{2}\left[e^{t/\sqrt{k}}-e^{-t/\sqrt{k}}\right]\dot{u}_0$$

If we take $k=(n\pi)^2$. Then,

$$\begin{aligned}u_n(x)&=\frac{1}{2n\pi}\left(e^{x/n\pi}-e^{-x/n\pi}\right)\ddot{u}_0\\&-\frac{1}{2}\left(e^{x/n\pi}-e^{-x/n\pi}\right)\ddot{u}_0\\&=\frac{1}{2n\pi}\sinh\frac{x}{n\pi}\ddot{u}_0+\frac{1}{2}\sinh\frac{x}{n\pi}\dot{u}_0\end{aligned}$$

Therefore

$$u(x)=\sum_{n=0}^{\infty}u_n(x)\sin n\pi x$$

And

$$\left.\begin{array}{l}u(x,t)=u(t)u(x)=u_0\Pi_{t_0<t_k<t}\left(1+\sum_{t_0<t_k<t}\beta_k\phi(t_k)\right)\sum_{n=0}^{\infty}u_n(x)e^{-n\pi(t-t_k)}\sin n\pi x \\ u_n(x):=\frac{1}{2\pi n}\sinh\frac{x}{n\pi}\ddot{u}_0+\frac{1}{2}\sinh\frac{x}{n\pi}\dot{u}_0\end{array}\right\} \quad (21)$$

Where $u(x,t)$ is the solution to the impulsive heat model.

We attempted to simulate the solution to the above model, it was found that possess some singularities in the interval $0\le t\le 100$. Applying B-transform to the impulsive diffusion model we have

$$\begin{aligned}L_c\left(\frac{du_i}{dt}\right)&=-u_{0i}+\frac{1}{q}\bar{u}_{ic}(q)\\&=q\sum_{k=0}^{N}\sum_{j=0}^{N-k}a_{kj}\bar{u}_i^k\bar{u}_i^k\\&+\frac{D\mu}{\delta^2}(\bar{u}_{i-1}-2\bar{u}_i+\bar{u}_{i+1})+\beta_1\bar{u}_i\end{aligned}$$

$$\begin{aligned}L_c\left(\frac{dv_i}{dt}\right)&=-v_{0i}+\frac{1}{q}\bar{v}_{ic}(q)\\&=q\sum_{k=0}^{N}\sum_{j=0}^{N-k}a_{kj}\bar{u}_i^k\bar{u}_i^k\\&+\frac{D\mu}{\delta^2}(\bar{v}_{i-1}-2\bar{v}_i+\bar{v}_{i+1})+\beta_2\bar{v}_i\end{aligned}$$

$$L_c(\bar{u}_i^k\bar{v}_i^k)=\int_0^\infty u_i^kv_i^ke^{-t/q}dt=\frac{1}{q}\int\int_0^\infty u_i^kv_i^ke^{-(qt+t/q^2)}dtdt_1$$

We note that $\frac{1}{q}\int_0^\infty e^{-tq}dt=1$.

The system in the equation (12) which was approximate by the system in the equation (13) can be transformed into tridiagonal differential algebraic system of the form:

$$\dot{X}_1=X_1^TAX_1+\alpha AX_1$$

$$\dot{X}_2=X_2^TAX_2+\alpha AX_2$$

$$\Delta X_1=\alpha X_1+b_1+f_1(X_1)$$

$$\Delta X_2=\alpha X_2+b_1+f_2(X_2)$$

Where $\alpha=\frac{Dv}{\delta^2}, A_1=tridiag[1,-2,1]=A_2$

$$f_1(X_1)=f(X_1), f_2(X_2)=f(X_2).$$

Apply the B-transform we have

$$\bar{X}_{1c}-X_0=\overline{X_1}^T A\overline{X_1}+\bar{\alpha}A_1\overline{X_1}$$

$$\overline{X}_{2c}-X_0=\overline{X_2}^TA\overline{X_2}+\alpha A_2\overline{X_2}$$

$$\overline{X}_{1I} = \alpha \overline{X}_{1I} + \bar{b}_1 + f_1(\overline{X}_1)$$

$$\overline{X}_{2I} = \alpha \overline{X}_{2I} + \bar{b}_2 + f_2(\overline{X}_2)$$

Therefore

$$\overline{X}_1(q) = \bar{X}_{1c} + \overline{X}_{1I} = X_{10} + \overline{X}_1^T A\overline{X}_1 + \alpha A_1 \overline{X}_1 + \alpha \overline{X}_{1I} + \bar{b}_1 + f_1(\overline{X}_1)$$
$$\overline{X}_2(q) = \overline{X}_{2c} + \overline{X}_{2I} = X_{20} + \overline{X}_2^T A\overline{X}_2 + \alpha A_2 \overline{X}_2 + \alpha \overline{X}_{2I} + \bar{b}_2 + f_2(\overline{X}_2)$$

Therefore applying the inverse B-transform we have

$$\overline{X}_{ic}(t) = \tfrac{1}{2\pi i}\int_{C_i} \overline{X}_{ic}(q)e^{tq}dq, i = 1, 2.$$

$$\overline{X}_{iI}(t) = \tfrac{1}{2\pi i}\int_{C_i} \overline{X}_{iI}(q)e^{tq}dq, i = 1, 2.$$

$$X(t) = \overline{X}_{ic}(t) + \overline{X}_{iI}(t)$$

C_i are the contours for which the complex integration for the function $\overline{X}_i(q)e^{tq}$ is carried out.

We can use the B-transform to solve the impulsive control Gross berg model,

$$x_i(q) = x_c(q) + x_I(q) = qx_0 + cqa_i(x_i(q)) - \sum_{j=1}^{N} C_{ij}g(x_c(q))$$
$$+ \sum_{0<t_0<t_k<t} \mathrm{e}^{-x_k/q} I(x_I(t_k))$$

Applying the inverse B-transform to the we get

$$x_i(t) = u(t)x_0 + \frac{1}{2\pi i}\int_C c[a_i(x_i(q)) - \sum_{j=1}^{N} C_{ij}g(x_c(q))]e^{tq}dq$$
$$+ \sum_{0<t_k<t} \phi(\mathrm{x_i}, t_k) I(x(t_k)) \tag{22}$$

where

$$\phi(x_i, t_k) = \int_C e^{-t_k/q + tq}dq \tag{23}$$

C is complex domain across which the complex integration is carried out.

For numerical example, take $a_i(x_i(t)) = x_i(t)$ and $g(x_i(t))$ then applying the equations (22)&(23) the solution to the model is $x_i(t) = \frac{1}{1-c}\left[u(t)x_0 - \sum_{i,j}^{n} C_{ij}x_i(t)\right], c \neq 1$.

5. Conclusion

Impulsive control systems offer many interesting features for modelling several life problems. We considered some few of such real life problems. We only considered an impulsive neural network model; such model has potential applications in telecommunication. The impulsive diffusion model has applications in cellular ecology and population dynamics and the impulsive control systems with a lot of applications in the engineering. The study in this paper should be extended to control systems with impulsive delay functions and more other applications in engineering, science and technology.

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