Estimation of value-at-risk measures in the Islamic stock market: Approach based on Extreme Value Theory (EVT)

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Abstract: In this paper, we have combined the Extreme Value Approach with GARCH model which is called conditional EVT. We have used their approach on the Islamic stock price index to measure the conditional VaR and the related risk statistic expected shortfall (ES). The dynamic risk measures have been estimated for different percentiles for negative and positive returns. The empirical results show a strong stability across of the selected threshold, implying the accuracy and reliability of the estimated quantile based risk measures. Interested islamic index fund managers could employ these techniques as a means of risk management.

Keywords: Extreme Value Theory (EVT), Value-at-Risk (VaR), Peak over Threshold Method (POT),
Expected Shortfall (ES)

1. Introduction

The Value-at Risk (VaR) answers the question of how much we can lose, with a given probability, over a certain horizon. From a mathematical viewpoint, VaR is simply a quantile of the Profit & Loss (P&L) distribution of a given portfolio over a prescribed holding period.

The VaR technique has undergone a significant refinement since it originally appeared more than two decades ago and now the existing approaches for estimating VaR can be divided into three groups: i) the non-parametric historical simulation (HS) method; ii) fully parametric methods based on an econometric model for volatility dynamics and the assumption of conditional normality (most model from the ARCH/GARCH family) and Risk metrics) and iii) methods based on extreme value theory (EVT).

The extreme value theory (EVT) relies on extreme observations to derive the distribution of the tails random variable. By doing so, risk is measured more efficiently than by modeling the entire distribution of the random variable itself. The link between the extreme value theory and risk management is that EVT methods fit extreme quantiles better than the conventional approaches for heavy-tailed data. The EVT method needs to choose a threshold and only uses the data which exceed this threshold (namely extreme value) to estimate the generalized Pareto distribution (GPD) parameters. There are several advantages about the EVT-based method: (1) risk is measured more efficiently than by modeling the entire distribution of the random variable itself. The reason is that the extreme value method focuses on extreme events, and the event risk is explicitly taken into account; (2) with the normal distribution or any given distribution of returns, the distribution tails may be badly fitted. As the extreme value method does not assume a particular model for returns but lets the data speak for themselves to fit the distribution tails, the model risk is considerably reduced; (3) the EVT allows for a separate treatment of two tails of a distribution considering the fact that most financial returns are asymmetric.

Hence, the motivation of this paper is to propose a method of value at risk measurement by integrating the advantage of the power of EVT in accounting asymmetric fat-tails of a return distribution separately.

The rest of this paper is organized as follows. We introduce the sample data and discuss how daily returns are constructed in Section2. In Section 3, we present an overview of the theoretical framework of EVT, describe the measures of extremes risks - VaR and ES and then explain how conditional EVT is applied on VaR and ES. We discuss

in Section 4 the tail modeling of the Islamic market return series, assess the outcomes and provide the estimates of the risk measures. Finally, we conclude the study in section 5.

2. Date

The data set in our empirical study are daily Dow *Jones Islamic Market index (DJIM[1]),* Stock index in compliance with the Sharia. This index so includes assets of several countries of the Middle-East and North Africa as Tunisia, the newcomer from 01 January, 1999 to 03 February, 2011. The daily price series and the daily returns are shown in Fig.1. The daily returns measured as differences in the natural logarithm $(r_t = \ln p_t - \ln p_{t-1})$[2],
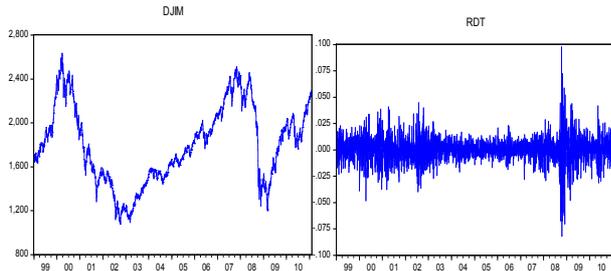

Figure 1. *The daily price series of DJIM and tje daily returns*

Table.1 reports the descriptive statistics for daily returns:

Table 1. *Descriptive statistics of returns.*

Statistics	Value
Mean	0.000104
Median	0.000558
Maximum	0.097753
Minimun	-0.081855
Std.Dev.	0.011280
Skewness	-0.252243
Kurtosis	-0.252243
Jaque-Bera	9.668571
Q(16)	45.224 (0.000)
Q²(16)	3671.5 (0.000)
Sum	0.327635
Sum Sq.Dev.	0.401308
ADF test	-3.41
P-P test	-3.41
Observation	3179

* ADF & P-P are statistics for the Augmented Dickey-Fuller and Phillips-Perron unit root tests based on the least AICcriterion,respectly. Denote significance at 5% level.

- The mean daily return is positive showing an upward movement of share price.
- The negative skewness and negative kurtosis clearly indicate the non-normality of the distribution which is confirmed by the Jarque-Bera statistics.
- The Ljung-Box Q(16) and Q²(16) statistics indicate the presence of serial correlation, as well as volatility.

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[1] http://www.djindexes.com/mdsidx/downloads/rulebooks/Dow_Jones_Islamic_Market_Indices_Rulebook.pdf
[2] p_t Price of closure of *DJIM* in the date t.

- Moreover, the results of augmented Dickey-Fuller and Phillips-Perron unit root tests reject the null hypothesis of a unit root in this series, indicating that they are stationary and may be modeled directly without further transformation. Fig.1 and Table.1 demonstrate the defining characteristics of the stock market: high volatility, occasional extreme movements, and volatility clustering and fat tailed distributions. These findings support the need for the AR-GARCH model to filter the data series and then to apply the EVT to it.

3. GARCH-Type Models

Financial returns are often modeled as autoregressive time series with random disturbances having conditional heteroscedastic variances, especially with GARCH type processes. GARCH processes have been intensely studied in financial and econometric literature as risk models of many financial time series. In order to analyse two data sets of stock prices, we try to fit AR(1) processes with GARCH or EGARCH errors to the log returns. Moreover, hyperbolic or generalized error distributions occur to be good models of white noise distributions.

Here we use the simplest possible AR-GARCH model with the mean return modeled as an AR(1) process and the conditional variance of the return as a GARCH(1,1) model:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \varepsilon_t = \alpha_0 + \alpha_1 r_{t-1} + \sqrt{h_t}\eta_t,$$
$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1} \qquad (1)$$

With $\varepsilon_t / \Omega_{t-1} \to$ Student's t distribution with mean = 0, variance = h_t and degree of freedom parameter, v, and where Ω_t is the information set of all information at time t.

Table.2 presents the estimated parameters of the AR-GARCH model with t distributed innovations applied daily return series. Both the constant term and the AR(1) coefficient in the mean equation are found to be significant Similarly, the parameters in the volatility equation: the constant, the ARCH(1) coefficient and the GARCH(1,1) coefficient, are all found to be significant.

All of the reported parameter estimates are statistically significant at the 5% level and, based on the Ljung-Box Q-staistics, there is no evidence of serial correlation in the standardised residuals or the squared standardised residuals. Consequently, the GARCH(1,1) model, reported in Table2,appears adequate

Table 2. *GARCH (1,1) model*

Parameter	Value
Mean equation	
α_0	0.000595
α_1	0.139380
Variance equation	
β_0	7.80E-07
β_1	0.072618
β_2	0.921574

Table.3 presents diagnostic statistics of returns and standardized residuals. The significant value of ljung-Box Q(16) statistic of the first column indicates that raw returns are serially correlated and hence are not iid as required by EVT.

In contrast, the standardized residuals are as their Q(16) statistic not significant. Thus the filtering producing iid residuals on which EVT can be implemented. The Q^2 (16) statistic of standardized residuals also suggests that the

AR(1)-GARCH(1,1) model is well specified. However, it appears from the table that skewness and excess kurtosis remain in the standardized residuals, it is also noted that neither the return series nor the standardized residual series are normally distributed as suggested by Jaque-Bera statistics. All these findings motivate the second stage of McNeil and Frey's (2000) EVT implementation, where fat tails of the standardized residuals are explicitly modeled.

Table 3. statistics of returns and standardized residuals

Q(16) =45.224(0.000)
Q²(16) =3671.5 (0.000)

Q(16) =18.354 (0.245)
Q²(16) =17.518 (0.289)
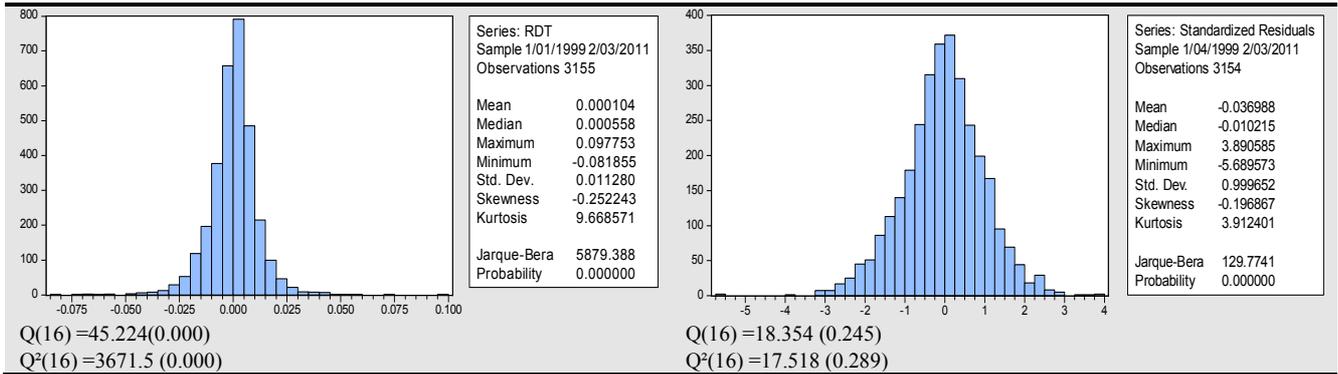
4. Extreme Value Theory

The EVT relates to the asymptotic behavior of extreme observations of a random variable. It provides the fundamentals for the statistical modeling of rare events and is used to compute tail-related risk measures. There are two different but related ways of identifying extremes in real data over a certain time horizon.

The first approach divides the time horizon into blocks or periods and considers the maximum the variable takes in successive periods, for example months or years. These selected observations constitute the extreme events, also called block maxima (BM). In this case, the generalized extreme value (GEV) distribution is used to fit the BM. On the other hand, the peak-over-threshold (POT) approach focuses only on the observations that exceed a given threshold. In this paper, we adopt the POT model to identify the extreme observations that exceed a high threshold u.

4.1. The Peak over Threshold Model (POT)

4.1.1. Theory

Assume that the $X_1, X_2, ...X_n$ are iid random variables representing risks or losses with $F(x) = \Pr(X_t \le x)$. Let u denote a high threshold beyond which observations of X_t are considered exceedence. The magnitude of the erxceedence is given by $y_i = x_i - u$, for i = 1,...k, where k is the total number of exceedences in the sample.

Given a high threshold u, the probability distribution of excess value of X over threshold u is defined by

$$F_u(y) - \Pr(X - u \le y / X > u) - \frac{F(y+u) - F(u)}{1 - F(u)} \quad (2)$$

For a sufficiently high threshold u, the distribution function of the excess may be approximated by the generalized Pareto distribution (GPD) because as the threshold gets large, the excess distribution $F_u(y)$ converges to the GPD Balkema and De Haan (1974) and Pickands (1975).

The GPD in general is defined as

$$G_{\xi,\beta}(y) = \begin{cases} 1 - (1 + \frac{\xi y}{\beta})^{-1/\xi}, if \, \xi \ne 0 \\ -1 - \exp^{-y/\beta}, if \, \xi = 0 \end{cases} \quad (3)$$

Where ξ is the shape parameter, α is the tail index, and β is the parameter scale parameter.

Acknowledging that $F(u)$ can be written as (n-k)/n, where n is the total number of observations, and k is the number of observations above the threshold u, and that $F_u(y)$ can be replaced by $G_{\xi,\beta}(y)$. Eq (4) can be simplified to

$$F(X) = 1 - \frac{k}{n}(1 + \xi^{(x-\mu)}\beta)^{-1} \quad (4)$$

For X > u, where ξ and β can be estimated by the method of maximum likelihood. For a given probability q > $F(u)$, the tail quantile can be obtained by inverting the tail estimation formula above to get.

$$x_q = u + \frac{\beta}{\xi}\left[\left(\frac{1-q}{k/n}\right)^{-\xi} - 1\right] \quad (5)$$

4.1.2. Determination of Thresholds

As mentioned earlier, we employ the POT method using GPD for tail estimation of the standardized residual series. The first step in this modeling is to estimate the threshold for identifying the relevant tail region. Several techniques are available for threshold determination. In this study, we apply two approaches: the first one is to use exploratory tools prior to model estimation; the second one is to assess the stability of the estimates of parameters, based on fitting the model across a range of different thresholds.

- The first approach for threshold selection utilizes the empirical mean excess function (MEF). An MFE is the sum of the excesses over the threshold u divided by the number of data point which exceed the threshold u and is expressed by

$$e(u) = \frac{\sum_{i=1}^{n}(X_t - u)}{\sum_{i=1}^{n} I_{(X_t > u)}} / X > u \qquad (6)$$

Where I is an indicator function. It is an estimate of the mean excess function which describes the expected overshoot of a threshold once an exceedance occurs. If the empirical MEF is a positively sloped straight line above a certain threshold u, it is an indication that the data follows the GPD with a positive shape parameter ξ. On the other hand, exponentially distributed data would show a horizontal MEF while short-tailed data would have a negatively sloped line.

4.1.3. Exploratory Data Analysis

The QQ plot is a graphical technique which allows to compare the quantiles of the empirical distribution to those of a reference distribution. In our case, we are interested in investigating whether our sample follows the normal law or not. The graph of QQ plot presented by fig.2 shows that the time series deviates widely from the normal distribution and exhibits thicker than the latter tails
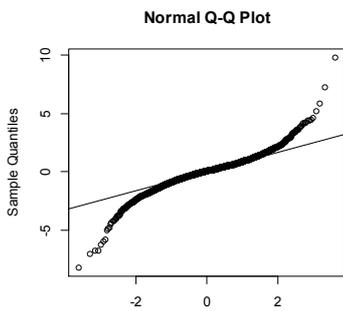

Figure 2. *The graph of QQ plot*

The thickness of the tails of the distribution of returns is in favor of our choice of EVT modeling tails and more precisely the choice of the POT method for modeling the distribution tails.

The MEF (Fig.3) of both positive and negative returns is estimated to choose thresholds. From the MEF, the

thresholds can be chosen from range of 1.01 to 1.52 for the left tail and from the range of 1.09 to 1.8 for the right tail based on the criterion of linearity in the MEF plots.
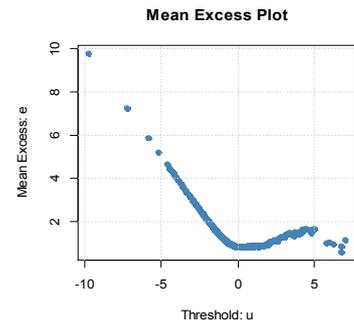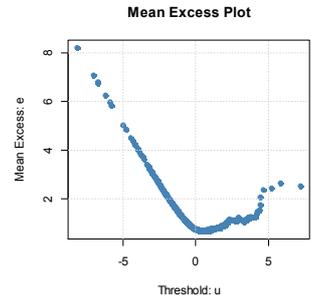

Figure 3. *The MEF plots*

4.1.4. Estimation of HILL
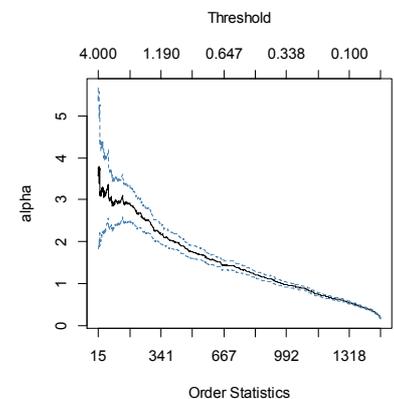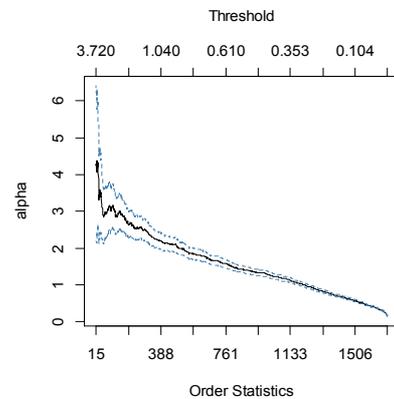
Figure 4. *The variation of the Hill estimator*

Figures depict the variation of the Hill estimator of the tail index ξ compared to the number of exceptions and thresholds are correspondents. The choice of the tail index begins by the region where the graph becomes relatively stable. This corresponds to a number equal to overflow Nu = 380 and a threshold for the u = 1.2 and a straight tail number exceeded equal to Nu = 380 and a threshold u = 1.04 for the left tail DJIM yields.

4.1.5. Estimation of GPD Parameters

The next step is the estimation of shape (ξ) and scale (β) parameter by fitting the GPD in Eq. (5) to the standardized residuals. The estimated shape parameter and scale parameter as well as their related statistics under different thresholds are listed in table 3 for negative and positive returns.

4.1.6. Value-at-Risk ((VaR) and Expected Shortfall (ES)

A popular model of market risk is the VaR, which is generally defined as the maximum potential losses in the market value, say, a financial portfolio with a given level of probability over a specific period. For example, if the given period of time is one day and the given probability is 1%, the VaR measure would be an estimate of the decline in the portfolio value that could occur with a 1% probability over the next trading day.

For a given probability q, VaR can be defined as the qth quantile of the distribution F

$$VaR_q = F^{-1}(1-q) \tag{8}$$

Where F^{-1} is the so-called quantile function defined as the inverse of the distribution function F. Since VaR is an extreme quantile, it can be estimated using the quantile formula given in Eq. (7) by

$$V\hat{a}R_q = x_q = u + \frac{\beta}{\xi}\left[\left(\frac{1-q}{k/n}\right)^{-\xi} - 1\right] \tag{9}$$

Another measure of risk is the expected shortfall (ES) which is defined as the expected size of a loss that exceeds VaR. Where VaR addresses the question: "How bad can things get?" The ES addresses the question: "If things go bad, what is the expected loss?" Mathematically the ES for risk X at given probability level q is expressed as

$$ES_q = E(X / X > VaR_q)$$

The ES is estimated by the following equation

$$E\hat{S}_q = \frac{V\hat{a}R_q}{1-\hat{\xi}} + \frac{\hat{\beta} - \hat{\xi}u}{(1-\hat{\xi})} \tag{10}$$

Is based on the estimated values (ξ, β) in Table 4 the VaR quantiles for negative as well as positive returns are obtained from Eq.(8)

And the expressions of ES are obtained from Eq (10) are also reported in Table 4.

Table 4. *Parameter estimates for the AR-GARCH(1,1) model under different thresholds*

	+ returns	- returns
	u= 1.2	u= 1.04
Total in-simple observation T	3179	3179
Number of exceedences k	380	380
% of exceedences in-simple k/T	11.95%	11.95%
GPD shape parameter ξ	0.11894	0.1487
GPD scale parameter β	0.74965	0.6304
VaR quantile:		
$V\hat{a}R(Z)_{0.95}$	1.801915	1.65862
$V\hat{a}R(Z)_{0.99}$	3.141606	2.931907
Expected shortfall:		
$E\hat{S}_{0.95}$	2.072543	2.489570
$E\hat{S}_{0.99}$	4.368787	4.0281137

4.1.7. Extreme Risk Measure

We now calculate the dynamic risk measure. Table 4 reports the conditional VaR and conditional ES for positive and negative returns. For a one-day horizon, an estimate of the conditional VaR is:

$$VaR_q^{t+1} = \hat{\mu}_{t+1} + \sqrt{\hat{h}_{t+1}}\,V\hat{a}R(Z)_q.$$

Similarly for a 1-day horizon, an estimate of conditional ES is:

$$ES_q^{t+1} = \hat{\mu}_{t+1} + \sqrt{\hat{h}_{t+1}}\,E\hat{S}_q$$

Where $V\hat{a}R_q$ is given by Eq.9 and $E\hat{S}_q$ is given by Eq.10.

Table 5. *Conditional VaR and Conditional ES*

	- returns	+returns
	u= 1.2	u=1.04
Conditional VaR:		
$VaR_{0.95}^{t+1}$	0.02037526	0.01876333
$VaR_{0.99}^{t+1}$	0.03544554	0.03308662
Conditional ES:		
$ES_{0.95}^{t+1}$	0.0234196	0.02802077
$ES_{0.99}^{t+1}$	0.04925021	0.04541795

The conditional VaR is estimated as 0.0187633 at the 5th percentile for the right tail. This implies that, for the lower 5% positive daily returns, the worst daily loss in the Islamic Market value may exceed 2.03% in expectation. ie, if we invest in market portfolio, we are 95% confident that our daily loss at worst will not exceed 2.03 during one trading day.

On the other hand, VaR is estimated as 0.02037526 at the 95th percentile for the left tail. We expect that a daily change in the market portfolio would not increase by more than 2.03%. Put differently, we are 95% confident that our daily loss will not exceed 2.03 % if we take short position of market portfolio.(similarly at a lower quantile of 99-level.)

We can say that under different thresholds, the estimates of VaR exhibit strong stability.

The other interesting observation is that, for any given threshold and quantile level, the corresponding VaR estimate in the left tail is larger than that in the right tail.

Similarly, the estimates of conditional ES exhibit similar characteristics to those observed from the Conditional VaR. the ES are stable for any given confidence level and this is true for both the tails.

Moreover, we find the corresponding ES estimates in the left tail are larger than that in the right tail.

5. Conclusion

The high degree of volatility seen in financial markets in recent year has been regarded as complex and nonlinear dynamic systems. EVT is a powerful tool to estimate the effects of extreme events in risky markets based on sound statistical methodology.

This paper exhibits how EVT can be used to model tail-related risk measure such as VaR and ES by applying it to the daily returns of market portfolio of Dow Jones Islamic Market. A conditional approach is favored as the return series exhibit stochastic volatility and are non-iid. We calculate the daily VaR for the islamic returns by combining the EVT with GARCH models. The objective was to get standardized residuals that are close to iid so that EVT models can be applied. In the context of applying conditional EVT, the POT method provides a simple and effective means to choose thresholds and estimate parameters. By assessing the empirical excess distribution functions and survival functions with associated theoretical distribution simulations, we find the goodness-of-fit in tail modeling.

The point and interval estimates of conditional VaR and conditional ES computed under different high quantile levels exhibit strong stability through the selected thresholds, implying the accuracy and reliability of the estimated quantile-based risk measures.

The VaR and ES measures based on conditional EVT model provide quantitative information for analyzing the extent of potential extreme risks in the market portfolio of DJIM. Interested index fund managers could employ these techniques as a means of risk management.

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