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Features of Modeling of Reinforced Concrete Structures in Calculations by the Finite Element Method

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Abstract: Finite elements and solution methods of nonlinear equations in combination with high-performance computing technology make it possible to perform analysis of large and complex systems taking into account nonlinear deformation of the material. However, the applicable standard methods of calculation are presented on the cross-section of the element. To fill this gap, the authors propose a method of modeling of concrete finite element, based on existing norms and state standards. The authors propose to take into account the destruction of elements in the treatment process. In this case, the nodal reaction of destroyed elements are applied as external load. Modeling of damage to concrete cube press. A comparison of different theories of strength for the calculation of the equivalent stress. As a skeletal chart selected state diagram with explicit separation of plastic and elastic deformations. Diagrams of deformation of the cross section of the cube in the process of loading the Deformation model. Identified areas of damage to concrete cube when reaching the ultimate strain. Defined theory of strength and the model that best reflects actual stress - deformation of concrete during the treatment process. The proposed method can be applied for numerical analysis of reinforced concrete structures by finite element method.

Keywords: Concrete, Strength Theory, Finite Element Method, Equivalent Stress, Energy, Deformation, The Physical Nonlinearity

1. Introduction

The relationship between static loads and displacements of a building structure in the general case of small displacements can be expressed by the equation [1]:

$$\int_0^y \frac{d\mathrm{F}(y)}{dy} dy = \mathrm{P} , \tag{1}$$

where y is the displacement of nodes, P is the vector of nodal loads, and F is the vector of nodal reactions.

It is easy to see that with the linear reaction — displacement relationship $F(y)=R\ y$, equation (1) degenerates into the well-known static equation. Equation (1) is adequate if there is no brittle destruction of the elements during the action. If the material of the structure is destroyed during loading, then the nodal reactions of the destroyed elements are transmitted as an external load and equation (1) is converted to the form:

$$\int_0^y \frac{d\mathrm{F}(y)}{dy} dy = \mathrm{P} + \mathrm{F_p} , \tag{2}$$

where F_p is the nodal reactions of the elements at the moment of destruction.

2. Methods

To integrate equations (1) and (2), it is necessary to explicitly represent the function F(y). This function can be obtained from the function σ(ε) by the finite element method. The strain-stress ratio is constructed based on the state diagram of the uniaxial stress state. The most commonly used diagram is one in which elastic and plastic deformations are clearly separated [2-4].

$$\varepsilon = \frac{\sigma}{E_b}\left(1+\eta\left(\frac{\sigma}{R_b}\right)^m\right), \tag{3}$$

where η is the fraction of plastic deformation, m is the nonlinearity parameter. The principles for determining these parameters are described in [2].

To calculate the equivalent stresses corresponding to the uniaxial stress state, various models corresponding to a particular strength theory are used. On the basis of these models, various stress surfaces can be constructed. Young and Burzynski [5, 6] proposed a dependence describing all known stress surfaces satisfying the Drukker postulate [7].

$$\sigma_e = I_1(\gamma_1+\gamma_2)/2 \pm \sqrt{I_1^2(\gamma_1-\gamma_2)^2/4+3(1-\gamma_1)(1-\gamma_2)J_2}\,, \qquad (4)$$

where I_1 is the first invariant of the stress tensor, J_2 is the second invariant of the stress deviator, γ_1, $\gamma_2 \in [0, 1]$ is parameters that depend on the applied strength theory, σ_e is the stress of the uniaxial state is equivalent to the volumetric stress state.

All existing models (strength theories) do not take into account the nature of the nonlinearity of the state diagram. At best, different compression and extension limits are taken into account. We propose to determine the equivalent voltage from the condition of equality of the energies of the uniaxial and triaxial stress states. When using the dependence (3), the equivalent voltage can be determined from the condition:

$$W_3 = \frac{\sigma_e^2}{E}\left(\frac{1}{2}+\frac{\eta m+\eta}{m+\eta}\left(\frac{\sigma_e}{R_b}\right)^m\right), \qquad (5)$$

where W_3 is the deformation energy of the triaxial stress state, calculated during the impact. The expression on the right side is the value of the total strain energy of the uniaxial stress state.

3. Results and Discussions

To compare different approaches, a numerical experiment of concrete cube destruction was performed. The cube was modeled by tetrahedra with a linear shape function. The advantages of a tetrahedron with a linear function over other forms are in two aspects. First, the stresses and deformations are constant within the finite element, which makes it very easy to determine the plastic deformations and the moment of failure. The second aspect is that the values of the coefficients of the shape function can be quite simply obtained in the global system of geometric coordinates from the matrix equation:

$$XA=E$$

where A is the matrix of coefficients of the shape functions, X is the coordinates of the nodes in the global coordinate system and the unit first column, E is the unit matrix. The dimension of all the matrices is 4x4.

The disadvantage of four nodal volume elements is their large number to obtain acceptable accuracy.

The following models (strength theories) are considered.

1. Huber-Mises flow condition [8]: $\gamma_1=\gamma_2=0$ in formula (4).
2. The Balandin model [9]: $\gamma_1=1-\chi$, $\gamma_2=0$.
3. The Mirolyubov / Drukker-Prager model [10]: $\gamma_1=\gamma_2=(1-\chi)/2$.
4. Model of the total strain energy.
5. The model proposed by the authors.

In the Balandin and Mirolyubov models $\chi=R_{bt}/R_b$.

To conduct numerical experiments, a program was developed that implements the described algorithm. The article considers the deformation loading that simulates the destruction of a concrete cube in a press according to GOST 10180-2012 [11]. Material characteristics: Class B30 concrete E=32500 МПа, R_b=30 MPa, R_{bt}=3 MPa, maximum total strain under short-term loading ε=0.0035 [12]. The loading process was carried out in 20 steps. When the final element reached the maximum strain, its rigidity was reset to zero. The number of elements in the numerical experiment was 24000.

Two variants of the calculation were performed in accordance with equations (1) and (2). For each variant, a diagram of the deformation of the cross-section of the cube was constructed. The stress is calculated as the sum of the reactions of the surface nodes related to the cross-sectional area of the cube. Deformations correspond to the movements of the press plate, stresses correspond to the pressure of the press.

Figure 1 shows a diagram of the deformation of the cube section in accordance with equation (1). During the action, when the limit deformations of the final element are reached, no destruction occurs. In this case, an unlimited plastic flow occurs at a constant voltage for all the considered models. For models 1 and 5, the diagrams are almost the same. For all the models considered, the finite element deformation diagrams strictly correspond to the dependence (3).

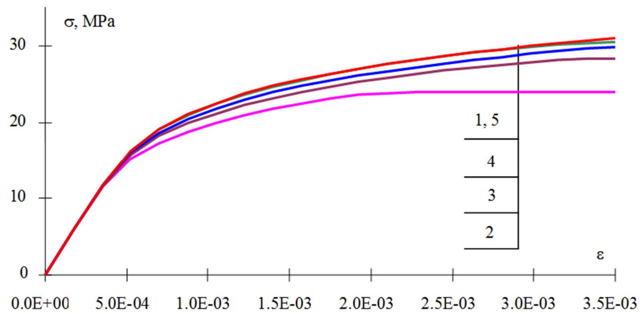

Figure 1. *State diagram (1) for unlimited plastic flow.*

Figure 2 shows a diagram of the deformation of the cube section in accordance with equation (2). When a finite element is destroyed, its nodal reactions are added to the right side of equation (2). As some finite elements are destroyed, the unloading of adjacent elements is observed. In the deformation diagram, this circumstance is manifested by the appearance of a descending branch for all the models considered without exception. In Figures 1 and 2, the numbers indicate the models in accordance with the order of their description at the beginning of the article.

The maximum stresses in the cross section are shown in the table.

Maximum stresses achieved under strain loading, MPa.

Table 1. *Maximum stresses in cross section.*

Model	1. Mises	2. Balandin	3. Mirolyubov	4. The total ener.	5. Authors
Plastic (1)	30.44	23.99	28.37	29.77	30.90
Destruction (2)	29.48	22.07	26.22	28.30	29.73

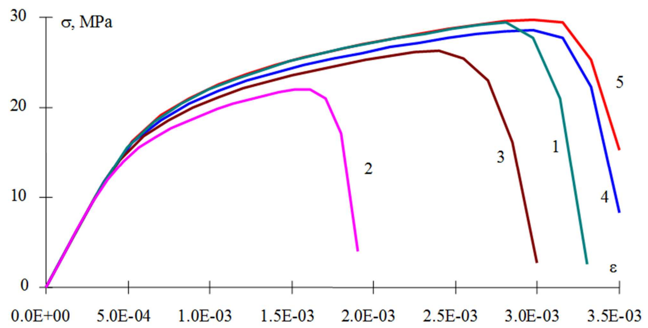

Figure 2. *Diagram of the state at the destruction of the CE in accordance with equation (2).*

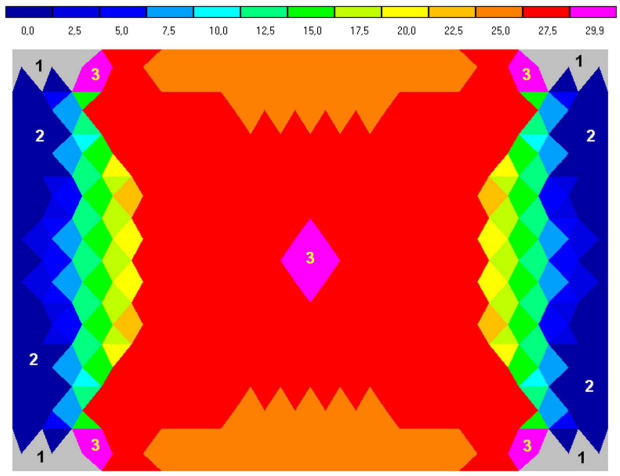

Figure 3. *Distribution of equivalent stresses over the cube cross-section under deformation of 0.0025. 1 — areas of failure, 2 — areas of discharge, 3 — areas of maximum stresses.*

Figure 3 shows the stress distribution in the diagonal section of the cube when the strain reaches 0.0025 — the beginning of the fracture. The distribution is presented for the model proposed by the authors. In models 1, 4, and 5, the destruction starts at the corners of the cube. In models 2 and 3, the destruction starts in the center. As the deformations increase, the development of the maximum stress regions occurs from the corners to the center for models 1, 4, 5, or from the center to the corners for models 2 and 3.

Figure 4 shows the deformation diagrams of the finite elements in the characteristic regions in accordance with equation (2). In the process of loading until the moment of destruction, the deformation diagrams for all FE coincide. When a part of the FE is destroyed, the under-stressed areas adjacent to them are partially unloaded. When applying the model proposed by the authors, the unloaded elements are loaded with the reverse sign. As a result, the elements adjacent to the vertical faces are destroyed by tensile stresses. Figure 5 shows the stress distributions at the moment of complete failure when the cube deformation reaches 0.0035.

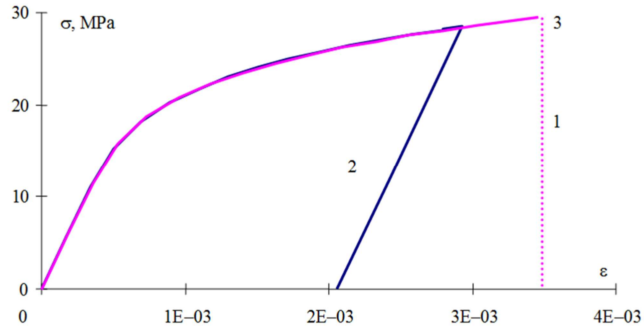

Figure 4. *Finite element deformation diagrams: 1 — destroyed element, 2 — unloaded element, 3 — maximum stresses at the moment of destruction.*

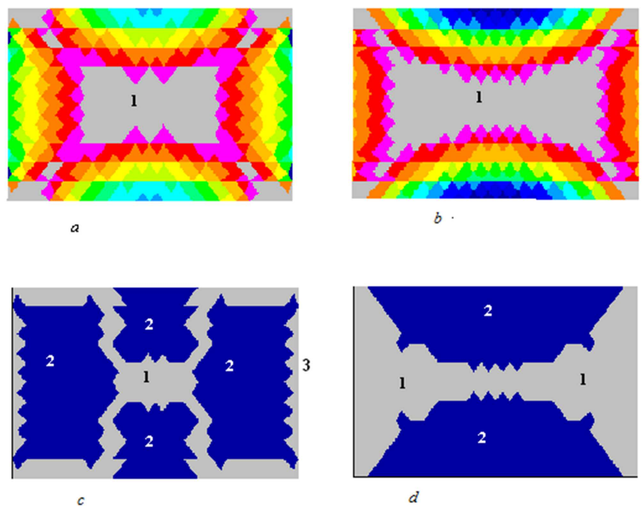

Figure 5. *The fracture under deformation is 0.0035. a, b — equation (1), c, d — equation (2), a, c — the model proposed by the authors, b, d — the Huber – Mises theory. 1 — zone of plastic flow (a, b) / destruction of compressed concrete (c, d), 2 – zone of unloading, 3 — zone of destruction of stretched concrete.*

4. Conclusion

1. The models of Mises, Mirolyubov and Balandin give underestimated results on stresses and deformations during failure. At the same time, the Mirolyubov and Balandin models significantly depend on the tension/compression ratio.
2. The descending branch of the deformation diagram for the section as a whole appears as a result of partial destruction of the total volume during loading.
3. When using equation (2), when modeling concrete in the skeletal diagram (3), it is necessary to use the cubic, rather than prismatic, strength of the concrete.
4. For modeling concrete by the finite element method, the most adequate models are the total energy models and the ones proposed by the authors-4 and 5.

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