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Research Article

Using Modified BCS to Explain High-Temperature Superconductivity in YBCO Compounds

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Abstract

Superconducting materials pays attention of scientists for its important roles in generation of powerful magnetic field and electric energy beside transportation and magnetic resonance imaging (MRI). In this search, Six cases of the behavior of superconducting materials and Yttrium Barium Copper Oxide (YBCO) compounds were studied. The aim of this work is to explain behavior of superconductors by using quantum, statistical and mechanical laws. The methodology is based on using mathematical derivation based on basic physical laws then comparing the results obtained with experimental foundation. The first case was investigated infinite conductivity, we are assuming that electrical damping serves as a measure of the resistance to the motion of magnetic vortices, which arise due to strong magnetic fields in type-II superconductors. The second and third cases, was compared critical temperature of high-temperature superconductivity on the YBCO compounds between using condensed matter laws and modified BCS theory. Firstly, we calculated the Tc by using the number density and momentum quantization for Fermi level but this concept is failed to explain the Tc for YBCO compounds. Secondly, critical temperature of YBCO compounds was derived by using a modified BCS theory. We assumed that the mechanism of Cooper pair formation in HTSC is due to the magnetic exchange resulting from random spin fluctuations for electrons, this result indicate the Tc is strongly dependent on the Debye temperature. After was taken experiments value for Debye temperature $\theta_D \approx 330$ for YBCO, was calculated the critical temperature was given $T_c \approx 92\ K$, recently we noticed the modified BCS theory its give better expression for Tc on YBCO than condensed matter.

Keywords

YBCO, Critical Temperature, Spin Fluctuations, Debye Temperate

1. Introduction

Superconductivity is one of the major breakthrough in the history of Physics. In the year 1911 just after the refrigeration technique via liquid helium has emerged [1], H. K. Onnes discovered something capable of carrying current without any resistance when it is cooled down below a certain critical temperature of the order of few Kelvin. Initially it was done with Mercury. Later people started tend metals beyond certain low temperature are capable of being superconductor. The year 1933 came with another surprise; Meissner and Oschenfeld observed superconductors are capable of expelling magnetic fields [2]. Within 1950's and 1960's a complete and satisfac-

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tory theory of classical superconductor had been revolutionized with the emergence Landau-Gingburg [3] effective theory and most celebrated microscopic theory of superconductor given by Bardeen, Cooper, Schreifer (BCS) in 1957 [4].

Superconductivity is a phenomenon occurring in certain materials at extremely low temperatures, characterized by exactly zero electrical resistance and the exclusion of the interior magnetic field (the Meissner Effect).

"Conventional" superconductivity is described by Bardeen-Cooper- Schrieffer (BCS) theory: in normal metals the electrons behave as fermions, while in superconducting state they form "Cooper pairs" and behave like bosons [5, 6].

High temperature superconductivity is a property of some doped cuprates, obtained by the introduction of charge carriers into the highly correlated antiferromagnetic insulating state chemically. Actually, understanding of the origin of high temperature superconductivity and that of the nature of the doped antiferromagnetic Mott insulators are closely associated. The strong correlations play important role in understanding the high *Tc* superconductivity.

YBCO Cuprate superconductors, Yttrium is a transition element symbolized by the letter Y and located in the 3rd group of the periodic table with atomic number of 39 and atomic mass number of 89,906. Many compounds (or alloys) formed by yttrium have superconducting properties. Although superconducting parameters for yttrium element could not be determined in normal state, superconductivity is more under high pressure or in thin films of compounds it forms.

Superconducting properties of yttrium are not affected much by means of its replacement with various rare surfaced elements with high moments. On the other hand, partial relocation of copper in YBCO alloys with third transition metal ions has a substantial effect on its transition properties.

Transition temperature is less affected by the replacement of 1-2-3 ceramic superconductors of cadmium. This temperature is arranged form 89 K to 93, 5 K. [7, 8].

YBCO compounds consist of yttrium, barium, copper and oxygen, and are called (123) compounds in short. In the crystal structure of $YBa_2Cu_3O_{7-\delta}$, which is one of these types of compounds, there are CuO_2 planes formed by copper (Cu) and oxygen (O) atoms and it is observed that these planes play an important role in the conductivity of superconducting materials [9, 10]. Electric resistance of that kind of an alloy is anisotropic. Alteration of δ in $YBa_2Cu_3O_{7-\delta}$ compound changes the amount of oxygen. The change in oxygen amount also changes the properties of the compound.

When δ value in $YBa_2Cu_3O_{7-\delta}$ equals to 0,6, the material is an antiferromagnetic insulator. If oxygen is added to that kind of an alloy, the tetragonal symmetry of crystal structure turns into orthorhombic structure as the value of δ closes to 0,6 and an insulating metal transition occurs [11].

It is a well known fact that maximum values of super flows are high on copper-oxygen planes and very low at vertical directions to these planes. In fact, considering 10^{10} A/m² critical flows on the copper oxygen plane in $YBa_2Cu_3O_{7-\delta}$ thin films, flows in the c- direction are quite low. This means that the flow should be two dimensional. Unfortunately, flow density of voluminous ceramics is much lower due to factors like boundary effects. For instance, the critical flow density of $YBa_2Cu_3O_{7-\delta}$ samples with multi-crystal structure is between 10^5 and 10^7 A/m² [7].

The main problem of this study is to understand the behavior of high-temperature superconductors (HTS) [12, 13].

The second issue lies in identifying an appropriate theory, which is essential for studying critical temperature for YBCO.

To develop suitable modifications of physic BCS theory for copper pair. The study will also calculate the critical temperature for YBCO compounds.

2. Result

2.1. Condensed Matter Model for YBCO

The number density is given by:

$$n_f = \frac{N}{V},\ \varepsilon_f = \frac{P_f^2}{2m} \tag{1}$$

Using momentum quantization:

$$p = \frac{2\pi\hbar}{L} \tag{2}$$

Quantum state volume:

$$V = (\frac{2\pi\hbar}{L})^3 \tag{3}$$

Volume of Fermi ball in momentum space half of dimater P_f

$$V_0 = \left(\frac{3}{4}\pi\right) P_f^{\,3} \tag{4}$$

$$N = \frac{\left(\frac{3}{4}\pi\right)P_f^{\,3}}{(\frac{2\pi\hbar}{L})^3},\ n_f = \frac{N}{V} = \frac{\left(\frac{3}{4}\pi\right)P_f^{\,3}}{(\frac{2\pi\hbar}{L})^3/L^3} = \frac{\left(\frac{3}{4}\pi\right)P_f^{\,3}}{(2\pi\hbar)^3} \tag{5}$$

$$n_f = 4\times\left(\frac{\left(\frac{3}{4}\pi\right)P_f^{\,3}}{(2\pi\hbar)^3}\right),\ P_f^{\,3} = n_f(2\hbar)^3\,\pi^2\left(\frac{3}{16}\right) \tag{6}$$

$$P_f^{\,2} = n_f^{\frac{2}{3}}(\hbar)^2(\pi^2)^{\frac{2}{3}}(\frac{3}{2})^{\frac{2}{3}} = (\hbar)^2(\frac{3}{2}n\pi^2)^{\frac{2}{3}} \tag{7}$$

$$\varepsilon_f = \frac{(\hbar)^2(3n\pi^2)^{\frac{2}{3}}}{2m},but\ v_f = \frac{(\hbar)^2(3n\pi^2)^{\frac{1}{3}}}{m} \tag{8}$$

$$\varepsilon_f = 1.01\hbar v_f n_f^{\frac{1}{3}}, if\ \varepsilon_f = K_B T_c \tag{9}$$

$$T_c = \frac{1.01\hbar v_f n_f^{\frac{1}{3}}}{K_B} \tag{10}$$

Equation (10) it found also in [5], and we used to calculate T_c for YBCO with take the experiments values $v_f \approx 10^5\ m/s, n_f \approx 10^{21}\ electron\ /m^3$we given $T_C \approx 76\ K$

2.2. BCS Modified Model for YBCO

The pairon of Energy gap in BCS

$$1 = v(0)N(0)\int_0^{\hbar\omega_D} dE\ \frac{1}{\sqrt{(E^2+\Delta^2)}} tanh\left[\frac{\sqrt{(E^2+\Delta^2)}}{2k_BT}\right] \quad (11)$$

If we consider the copper pairs production near the Fermi level hence the

$$\Delta = 1.7k_BT,\ E = \varepsilon_f = k_BT_c \text{ and } T=T_c \quad (12)$$

We can consider

$$\tanh\left[\frac{\sqrt{(E^2+\Delta^2)}}{2k_BT}\right] \approx .08 \quad (13)$$

$$1 = v(0)N\frac{1}{2.7k_BT_c}\int_0^{\hbar\omega_D} dE, 1 = v(0)N(0)\frac{\hbar\omega_D}{2.7k_BT_c} \quad (14)$$

$$\frac{1}{v(0)N(0)} = 0{,}28\frac{\theta_D}{T_c} \quad (15)$$

if we change the interaction between electron-phonon by exchange spin magnetic J, the equation become

$$e^{T_c} = .28\frac{e^{\theta_D}}{e^{\frac{1}{JN(0)}}} = .28e^{\left[\theta_D - \frac{1}{JN(0)}\right]} \quad (16)$$

iftheN(0) ⋙ Jso$e^{\frac{1}{JN(0)}} \approx 1$if the $N(0)J$ ⋙ $1, so\ e^{\frac{1}{JN(0)}} \approx 1$ (17)

The critical of temperature become

$$T_c = 0{,}28\theta_D \quad (18)$$

We used (25) to calculate T_c for YBCO compounds with take average of the experiments value [14] $\theta_D \approx 330$ we given $T_c \approx 92\ K$.

2.3. Fluid Model

The equation of motion is given by:

$$m\frac{dv}{dt} = eE - \Upsilon V + BeV - KX \quad (19)$$

Where the velocity v and x is

$$v = v_0e^{-iwt},\ x = \int vdt\ = v_0\frac{e^{-iwt}}{-iw} \quad (20)$$

Thus, the derivative becomes:

$$\frac{dv}{dt} = -iwv_0e^{-iwt} = -iwv \quad (21)$$

$$X = \frac{-v}{iw} = i\frac{v}{w} \quad (22)$$

Therefore, the equation simplifies to:

$$-imwv = eE - \Upsilon V + BeV - i\frac{kv}{w} \quad (23)$$

$$eE = [(Be - \Upsilon)v + i\left(\frac{k}{w} - mw\right)v \quad (24)$$

Solving for v, we have

$$v = \frac{[(Be-\Upsilon)-i\left(\frac{k}{w}-mw\right)eE}{[(Be-\Upsilon)^2+\left(\frac{k}{w}-mw\right)^2} \quad (25)$$

Current density J is:

$$J = nev, J = \sigma E,\ J = (\sigma_1 + i\sigma_2)E \quad (26)$$

$$J = \frac{[(Be-\Upsilon)-i\left(\frac{k}{w}-mw\right)ne^2E}{[(Be-\Upsilon)^2+\left(\frac{k}{w}-mw\right)^2} \quad (27)$$

The real part of conductivity, σ_1 is:

$$\sigma_1 = \frac{[(Be-\Upsilon)ne^2}{[(Be-\Upsilon)^2+\left(\frac{k}{w}-mw\right)^2} \quad (28)$$

$$Let\ k = mw^2,\ when\ Be = \Upsilon = 0,\ \sigma_1 = \infty \quad (29)$$

$$\sigma_1 = \infty \quad (30)$$

3. Discussion

The concept of particle density and the occupied quantum states at the Fermi surface we were used to find the critical temperature, when substituting the known experimental values of the v_f and the n_f of electrons in the Fermi level for the compound YBCO the temperature becomes inaccurate for T_C in equation (10). On another hand, when we used the equation (11) as the concept, the energy gap equation in the BCS theory, and the interaction mechanism between electrons and phonons was replaced by the magnetic exchange constant. The equation was obtained and given correct value for the critical temperature of the compound YBCO (18).

Were studied the specific case for equation (29), it is possible to achieve an infinite conductivity value (30) in the superconducting state because there is no damping factor, type-II superconductors, such as in the YBCO where vortices or magnetic flux loops play a key role in the material's dynamics. In this context, the damping coefficient is a measure of the resistance that the vortices face when they move, if we consider the vortices it formed as the result to found strong magnetic into material. It can be neglected in this case if the vortices are pinned and prevented from moving.

4. Conclusion

The fundamental parameters (infinite conductivity) of superconductors and the critical temperature of YBCO compounds have been studied using severl different physical laws and concepts that are closely related and analogous, under certain conditions, to the idea of superconductivity.

The theory BCS and laws of condensed matter explains the behavior of type-I superconductors but fails to explain materials like the compound YBCO and type-II superconductors, but when we modified the BCS theory and attempted to explain the pairing mechanism by the concept strong magnetic fluctuations that occur in ceramics, we were found clear expression YBCO compounds.

Abbreviations

YBCO	*Yttrium Barium Copper Oxide*
Tc	Critical Temperature
HTS	High-Temperature Superconductor
BCS	Bardeen–Cooper–Schrieffer Theory
GL theory	Ginzburg–Landau Theory

Author Contributions

Alaa Faisal Bashir: Conceptualization, Investigation, Methodology, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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