

Research Article

# Guaranteed Open-loop Channel Estimation for mmWave Hybrid MIMO Communications, Using Orthogonal Matching Pursuit (OMP) Algorithm

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## Abstract

Channel estimation for millimeter wave (mmWave) hybrid MIMO communications, is challenging because of the complexities associated with the large antenna arrays at the transceivers and with the higher propagation loss of the mmWaves. However, with open-loop training and exploiting the inherent sparse nature of the mmWave channel, it becomes easier by formulating the channel estimation problem in compressive sensing (CS) theory, and solving the problem using orthogonal matching pursuit (OMP) algorithm. In the CS theory, coherence and restricted isometry property (RIP) of sensing matrices, and restricted isometry constant (RIC) based k-sparse signal recovery exactly in k iterations, are significant conditions for guaranteed recoverability. Most of the earlier works are focused on coherence only, because of the impracticality of computation of RICs for the larger dimensional mmWave channel. In this paper, a novel technique, for the first time different from the earlier works, is devised to achieve guaranteed open-loop training based channel estimation. As there is hurdle for computation of RIC for the channel, smaller dimensional sensing (DFT) matrices are synthesized and are subjected for guaranteed recoverability conditions. From the simulation results of recoverability with synthesized and channel matrices, guarantee of the mmWave channel estimation is achieved.

## Keywords

mmWaves, Channel Estimation, Hybrid MIMO Communications, Open-loop Methods, Compressive Sensing, OMP, Coherence and RIP, Sparse Signal Recovery in K-iterations

## 1. Introduction

Millimeter wave (mmWave) MIMO cellular systems will enable gigabit-per-second data rates, which is required to cope up with the dramatic proliferation of data traffic of 5G and beyond, future wireless mobile communications [1-5]. These systems will employ large antenna arrays at the transceivers to overcome the large propagation losses of the mmWaves [6]. To realize sufficient link margin, high gain

directional beamforming is required in between the transeiver antenna arrays. Hybrid (analog and digital) beamforming is proposed for the mmwave MIMO systems, as this reduces the hardware cost and power consumption, when compared to those of either analog or digital beamforming [7-12].

Channel state information (CSI) is required at the transceivers to perform adaptive beamforming techniques for ef-

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efficient utilisation of the channel. CSI can be accumulated in channel training and estimation phases. Channel training and estimation, is of two methods: closed loop and Open loop methods. Closed loop methods are disadvantageous because of large training overhead and increase of overhead with the number of users [13-15]. In contrast to these, open loop methods are more practical, as these are with less training overhead, which does not change with the number of users [9, 16]. Also these are with low complexity, by performing explicit channel estimation: the transmitter emits pilot vectors for channel estimation, and the receiver estimates the parameters of the dominant channel paths from the received pilot signals.

Channel estimation for mmWave hybrid MIMO system, is challenging because of the complexities associated with the architecture of the system [17, 19, 20]. Eventhough channel matrix is of large dimensional, mmWave channel is inherently sparse (rank deficient) in nature, as it is experimentally observed that only a few dominant paths in angular domain are contributing effectively for mmWave communications [21-24]. Exploiting this sparse nature, it becomes easier, when sparse channel estimation problem is formulated and solved it using compressive sensing (CS) methods. One of the greedy CS methods, is orthogonal matching pursuit (OMP), which gained popularity, because of its simplicity, efficiency, and lower execution time [32].

In the CS theory, coherence and restricted isometry property (RIP) of sensing matrices, and k-sparse signal recovery exactly in k iterations, are significant conditions of guaranteed recoverability. In [25], channel is estimated using inverse discrete Fourier transform (IDFT) based training sequence (TS) and optimization of the auto-coherence and cross-coherence of the blocks of the sensing matrix. Authors in [26] designed pilot beam patterns to minimize the overall coherence of the equivalent sensing matrix. It is observed that reduced coherence of the system enabled better estimation accuracy. The authors in [16] proposed an open-loop training design, to lower mutual coherence and a better CSI estimation accuracy. In [13] an open-loop hybrid analog-digital beam-training framework is proposed to improve the recoverability guarantee. The proposed training method achieved a lower mutual coherence and an improved channel estimation accuracy than the methods of [9, 16]. Thus most of these works, are focused on coherence only, because of the impracticality of computation of RIP and RIC for the larger dimensional mmWave channel.

In this paper a novel technique, for the first time, is devised to achieve guaranteed open-loop training channel estimation based on signal recoverability conditions. As there is hurdle for computation of RIP for the channel, smaller dimensional sensing (DFT) matrices are synthesized. Coherence and RIP parameters of these matrices are computed for evaluation of

the conditions for guaranteed recoverability. Many authors proposed conditions based on RIC for k-spares signal recovery exactly in k-iterations using OMP algorithm [35-40]. In the present paper, from the simulation results of (i) signal recovery from both the synthesized and the channel sensing matrices, (ii) recovery success rate of more than 98%, and (iii) a linear relation in between smaller and larger dimensional matrices, guarantee of the channel estimation is achieved.

This paper is organized as follows: A single user, uplink hybrid MIMO System is presented in Section-2. Formulation of CS based sparse channel estimation and conditions for guaranteed recoverability are presented in Section-3. In the same section, synthesized matrices, and their analysis are presented. Simulation results demonstrating the guaranteed mmWave channel estimation are presented in Section-4 and finally, conclusions are drawn in the subsequent Section-5.

## 2. System Model and Channel Estimation

### 2.1. System Model

A single user uplink mmWave massive hybrid MIMO system [16, 18] is shown in Figure 1. In the system, the transmitter is equipped with a digital baseband processor (denoted by  $F_{BB}$ ) and an analog RF precoder (denoted by  $F_{RF}$ ) with  $N_{RF}$  chains, connected to  $N_t$  transmitting antennas, and the receiver is equipped with  $N_r$  receiving antennas connected to an analog RF combiner (denoted by  $W_{RF}$ ) with  $N_{RF}$  chains and a digital baseband processor (denoted by  $W_{BB}$ ), for communication of  $N_s$  data streams, such that  $N_s \leq N_{RF} \leq N_t$  at the transmitter and  $N_s \leq N_{RF} \leq N_r$  at the receiver respectively.

At the transmitter,  $N_{RF}$  chains are capable of generating  $N_{Tx}^{Beam}$  ( $N_{Tx}^{Beam} \leq N_t$ ), denoted by  $f_m$   $m \in (1, \dots, N_{Tx}^{Beam})$ , and at the receiver, the  $N_{RF}$  chains are capable of receiving  $N_{Rx}^{Beam}$ , denoted by  $w_n$   $n \in (1, \dots, N_{Rx}^{Beam})$ . During the training period,  $N_{Tx}^{Beam}$  are sent successively, one after the other, and in the receiver each transmitted beam is received as  $N_{Rx}^{Beam}$  simultaneously through  $N_r$  antennas.  $N_{Tx}^{Beam}$  and  $N_{Rx}^{Beam}$  are chosen as multiples of  $N_{RF}$  chains and hence rf chains generate transmitter blocks as  $N_{Tx}^{Block} = \frac{N_{Tx}^{Beam}}{N_{RF}}$  and receiver blocks as,  $N_{Rx}^{Block} = \frac{N_{Rx}^{Beam}}{N_{RF}}$ .

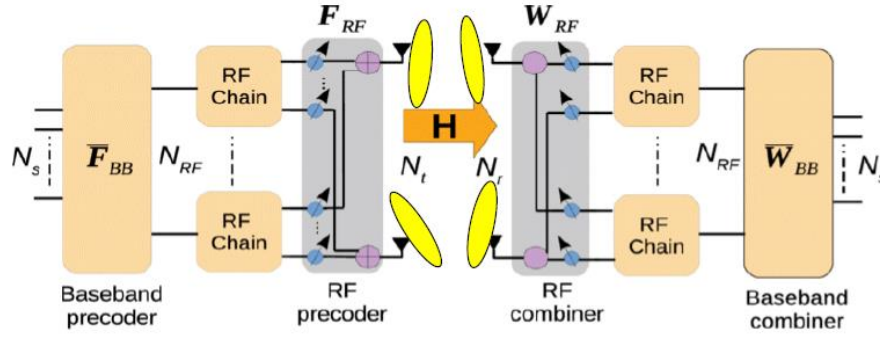


Figure 1. A Block diagram of single user mmWave Hybrid MIMO System.

The received vector for a single ( $m^{\text{th}}$ ) transmitted beam and for a single ( $n^{\text{th}}$ ) received block is denoted as  $y_{n,m}$ . Collecting all  $N_{Rx}^{\text{Beam}}$  the received vector for a single ( $m^{\text{th}}$ ) transmitted beam is denoted as  $y_m$ . With the training pilot symbols,  $X$ , representing the received signal vector in matrix form for all the transmitted beams,  $m \in (1, \dots, N_{Tx}^{\text{Beam}})$ , it is given as

$$Y = W^H H F X + N_{ns} \quad (1)$$

Where hybrid precoding ( $F = F_{RF} F_{BB}$ ), combining matrices ( $W = W_{RF} W_{BB}$ ), channel, ( $H$ ), and noise ( $N_{ns}$ ) are parameters of  $Y$ , in (1). When,  $X$  is assumed as  $X = \sqrt{P} I_{N_{Tx}^{\text{Beam}}}$ , where  $P$  is the pilot power, the received signal vector is represented as

$$Y = \sqrt{P} (W_{RF} W_{BB})^H H (F_{RF} F_{BB}) + N_{ns} \quad (2)$$

## 2.2. Sparse Channel formulation and estimation

Adopting channel model from [16], the channel  $H$  is given as

$$H = \sqrt{\frac{N_t N_r}{L}} \sum_{l=1}^L \alpha_l a_R(\theta_{R,l}) a_T^H(\theta_{T,l}) \quad (3)$$

Where  $(\alpha_1, \dots, \alpha_L)$  are complex gains and  $L$  are propagation paths. And assuming uniform linear array (ULA) at both the transmitter and receiver, the antenna steering vectors,  $a(\theta)$  are represented as

$$a(\theta) = [1, e^{-j\frac{2\pi}{\lambda}d \cos \theta}, \dots, e^{-j\frac{2\pi}{\lambda}(N-1)d \cos \theta}]^T \quad (4)$$

where,  $N = N_t$  and  $a_T(\theta_{T,l}) \in C^{N_t \times 1}$  is the steering vector, at the transmitter and,  $N = N_r$ , and  $a_R(\theta_{R,l}) \in C^{N_r \times 1}$  is the steering vector at the receiving antenna.

To apply CS techniques to the channel estimation, with virtual channel representation, when angular domain (AOA, AOD) is discretized into points called as grid, such that the azimuth angle ( $\Theta$ ) in the range 0 to  $\pi$ , is divided into  $G$  grid points, then the channel in matrix form is given as

$$H = A_R H_b A_T^H \quad (5)$$

where the AOD and AOA steering matrices are represented in terms of grid points as

$$A_T = [a_T(\theta_{T,1}), a_T(\theta_{T,2}), \dots, a_T(\theta_{T,G})] \in C^{N_t \times G} \quad (6)$$

$$A_R = [a_R(\theta_{R,1}), a_R(\theta_{R,2}), \dots, a_R(\theta_{R,G})] \in C^{N_r \times G} \quad (7)$$

Vectorizing the channel,  $H$ , and using the identity,

$$\text{vec}(ABC) = (C^T \otimes A) \cdot \text{vec}(B), \quad (8)$$

equation (5) is rewritten as

$$\text{vec}(H) = A_D \cdot \text{vec}(H_b) = A_D \cdot h_b, \quad (9)$$

where  $A_D = A_T^* \otimes A_R$  and  $h_b = \text{vec}(H_b)$ .

To formulate the sparse channel estimation, vectorizing the received signal,  $Y$ , in (2), and using equation (8), it is given as

$$\text{vec}(Y) = y_v = \sqrt{P} (F_{BB}^T F_{RF}^T) \otimes (W_{BB}^H W_{RF}^H) \cdot \text{vec}(H) + n_Q$$

using equation (9),  $\text{vec}(Y)$  is represented as

$$\begin{aligned} \text{vec}(Y) &= \sqrt{P} (F_{BB}^T F_{RF}^T) \otimes (W_{BB}^H W_{RF}^H) \cdot A_D h_b + n_Q \\ &= Q h_b + n_Q \end{aligned} \quad (10)$$

where  $n_Q$  is resultant noise term after vectorization, and

$$Q = \sqrt{P}(F_{BB}^T F_{RF}^T) \otimes (W_{BB}^H W_{RF}^H). A_D \quad (11)$$

Rest of the paper, it is assumed that  $Q$  as the channel sensing matrix given by (11). In the CS theory under noiseless case, equation (10) is of the form  $y = A.x$ , where  $y$  ( $\text{vec}(Y)$ ) is measuring vector,  $A$  ( $Q$ ) is sensing matrix, and  $x$  ( $h_b$ ), is the sparse signal (channel) to be recovered.

As it is acknowledged that the minimum coherence of the sensing matrix results, guaranteed recoverability, components of,  $Q$ , viz., rf beamformer/combiner, baseband processor, and antenna steering dictionaries are accordingly designed for training and estimation. Since properly designed DFT matrices are nearly orthonormal bases and as suggested by [16], rf beamformer/combiner, and baseband processor are designed in DFT matrices. And in ULA, because of the array response matrices, are fixed for a fixed spacing in between the antenna elements, for minimum coherence, grid resolution,  $G$ , is chosen as equal to  $N_r$  and ad nearly equal to  $N_t$  to reduce the redundant dictionaries ( $A_R, A_T$ ) to approximately orthogonal dictionaries, as shown by [16]. With these dictionaries, using ray tracing method, a channel,  $H$ , is simulated and measurements,  $y$ , from this channel are obtained. Then, applying,  $y$ ,  $Q$ , and channel sparsity,  $L$ , as inputs to the OMP algorithm, dominant paths of the mmWave channel are estimated, as shown in simulations section.

### 3. Guaranteed Channel Estimation from K-sparse Signal Recovery Using OMP Algorithm

#### 3.1. Conditions for Sparse Signal Recovery

In the compressive sampling theory [27], coherence ( $\mu$ ) and restricted isometry property (RIP) of any sensing matrix, are significant parameters for sparse signal recovery. Lower coherence, and fulfilling RIP are wellness conditions of matrices, for guaranteed recoverability.

Definition-1: Coherence index of sensing matrix  $A$  ( $\mu(A)$ )

The coherence index  $\mu(A)$ , of matrix  $A$ , is defined as the largest absolute inner product between any two columns,  $A_i$ ,  $A_j$ , and is given as

$$\mu(A) = \max_{1 \leq i < j \leq N} \left| \frac{\langle A_i, A_j \rangle}{\|A_i\|_2 \|A_j\|_2} \right| \quad (12)$$

Where  $\|\cdot\|_p$ , is  $l_p$ -norm for respective  $p$  ( $p=0,1,2, \infty$ ). Let a matrix,  $A$ , of size  $M \times N$ , and  $M < N$ , ( $N > 2$ ), whose columns are normalized, i.e.  $\|A_i\|_2 = 1, \forall i$ , then the bounds of the co-

herence index of  $A$ , satisfies,

$$\sqrt{\frac{N-M}{M(N-1)}} \leq \mu(A) \leq 1 \quad (13)$$

Lower bound is called as Welch bound.

Definition-2: Restricted isometry Property (RIP) and Restricted isometry constants (RICs)

A matrix  $A \in C^{M \times N}$ , is said to satisfy the RIP of order  $k$  with a constant  $\delta_k \in (0,1)$  if the following holds:

$$(1 - \delta_k) \|x\|_2^2 \leq \|Ax\|_2^2 \leq (1 + \delta_k) \|x\|_2^2 \quad (14)$$

for every  $x \in \sum_k$ , where  $\sum_k = \{x \in C^N : \|x\|_0 \leq k\}$  is the set of all  $k$ -sparse vectors in  $C^N$ . Constants  $\delta_k \in (0,1)$  satisfying the above equation for sparsity levels  $k$  are called as restricted isometry constants (RICs). In particular, the minimum of all constants  $\delta$  satisfying the above equation is called the  $s$ -order Restricted Isometry Constant (RIC) and denoted by  $\delta_s$ .

For any sparsity order,  $k$ , the constant ( $\delta_k$ ) satisfy ( $0 \leq \delta_k < 1$ ) and for sparsity levels,  $k=1,2,3,\dots$  it satisfy  $\delta_1 \leq \delta_2 \leq \dots \leq \delta_k \leq \dots \leq \delta_N$ .

A sensing matrix,  $A$ , ( $M \times N$ ,  $M < N$ ), that satisfies RIP acts almost like an orthonormal system for sparse linear combinations of its columns [28]. Random matrices with i.i.d. Gaussian or Bernoulli entries and matrices randomly selected from the discrete Fourier transform were shown to satisfy the RIP with high probabilities, when they satisfy some specific conditions on  $K$ ,  $M$  and  $N$  [29, 30]. In the present analysis, a set of DFT matrices, are synthesized and are analysed in the following steps:

Step-1: Each synthesized matrix,  $A$ , ( $M \times N$ ,  $M < N$ ) is normalized using Gram-Schmidt procedure.

Step-2: Coherence index, ( $\mu$ ), and restricted isometry constants (RICs), are computed. From the RICs, ( $0 \leq \delta_k < 1$ ), sparsity level,  $k$  is obtained.

Step-3: Matrices, which satisfy, signal recovery in exactly  $k$ -iterations, and recovery success rate of more than 98%, are only subjected for further graphical analysis, which is shown in the subsequent simulations section.

Also in the present paper, compression ratio ( $\rho$ ) for a matrix,  $A$  of size, ( $M \times N$ ) is taken as ratio of  $M$  to  $N$ , i.e. ( $\rho = M/N$ ), for representation of specific properties of sensing matrices.

#### 3.2. K-Sparse Signal Recovery, Exactly in k Iterations Based on RIC, Using OMP Algorithm

RIP has been utilized for proving theoretical guarantees of

exact recovery for many algorithms in the CS literature. One of the algorithms is orthogonal matching pursuit (OMP). It is a simple, yet empirically competitive algorithm for sparse recovery. It aims at finding the support, i.e. the set of nonzero indices, of  $x$  one by one. At each iteration, OMP identifies the index corresponding to the column of sensing matrix,  $A$ , which has maximum correlation to the residue of  $y$ . OMP and its variants have been frequently used in sparse recovery and approximation problems, due to their simplicity and empirically competitive performance, [31-33]. Initial contributions on the theoretical analysis of OMP have concentrated on coherence [33] or probability analysis [32, 34]. Davenport and Wakin [35] have presented a straight forward K-step analysis of OMP based on RIP. Their work states that OMP guarantees exact recovery of any K-sparse signal from noise-free measurements in K iterations if  $A$ , fulfills RIP with RIC satisfying

$$\delta_{k+1} < 1/3\sqrt{k} \quad (15)$$

Wang and Shim [36] have proven a less restricted bound for OMP, which perfectly recovers any K-sparse signal from noise-free measurements in K iterations if matrix-  $A$ , satisfies RIP and RIC with

$$\delta_{k+1} < 1/(\sqrt{k} + 1) \quad (16)$$

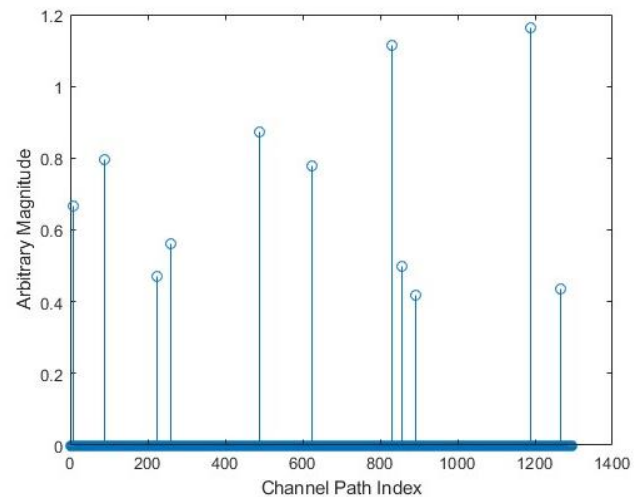
Later, the conditions have been improved to in [37]. Moreover, there are a few works concerning sufficient conditions for recovering restricted classes of K-sparse signals with a more relaxed bounds on RIC [38, 39]. A further improvement over all these is given by Liu et.al., [40]. The author proves that under some constraints ( $R(x) < (\sqrt{1 - \delta_{k+1}^2} / \delta_{k+1})$ , where  $R(x) = \|x_s\|_1 / \|x_s\|_2$ ), on the signal,  $x$ , OMP can also exactly recover the signal if RIC, satisfies

$$\delta_{k+1} < (\sqrt{2}/2) = 0.707 \quad (17)$$

In the present analysis, from the computational results, RIC value of a matrix,  $A_1$  (14x18), is obtained as 0.387, which is sufficiently far less than the given limit of 0.707 [40] and thus satisfying the condition for recovery exactly in k-iterations. A notable feature of both the sensing matrices,  $A$  and  $Q$ , is sparse signal recovery exactly in k-iterations. This recovery indicates the wellness condition of the sensing matrix, which satisfy RIP and RIC. Thus even though RIC is not available for  $Q$ , recovery is guaranteed on par with,  $A$ , whose RIP and RICs are satisfying the conditions for guaranteed recovery, which is shown in the following simulations section.

## 4. Numerical Simulation Results

It is assumed that the transmitter and receiver antennas, are as,  $N_t = 28$ , and  $N_r = 36$  respectively, for a single uplink mmwave hybrid MIMO communication. For input,  $N_s = 4$ , data streams, Rf chains at the transmitter and receiver are as:  $N_{RF} = 4$ . As described in section-II, for each transmitted beam, the number of receive beams and the blocks developed by the rf chains, are with the selection of  $N_{Tx}^{Beam} = N_t$ ,  $N_{Rx}^{Beam} = N_r$ . The number of propagation paths,  $L = 11$ , (sufficiently more than the experimentally observed sparsity, (3 to 5), of the mmWave channel) and the angular space grid resolution,  $G$ , is taken as 36. With these parameters, as described in section-II, precoder, and combiner matrices are designed in DFT matrices and by formulating the channel estimation problem in CS theory, the resulted sensing matrix,  $Q$ , (eq.9) of dimension (1008x1296), is obtained. As described at the end of Section-II, channel is estimated for sparsity level,  $k=11$ , using OMP algorithm and the estimation of dominant paths of the mmWave channel, is shown in Figure 2.



**Figure 2.** Recovered channel dominant paths ( $k=11$ ) using OMP algorithm.

Guarantee of the channel estimation is achieved from the following results of the graphical analysis of the selected matrices (specified in section-3).

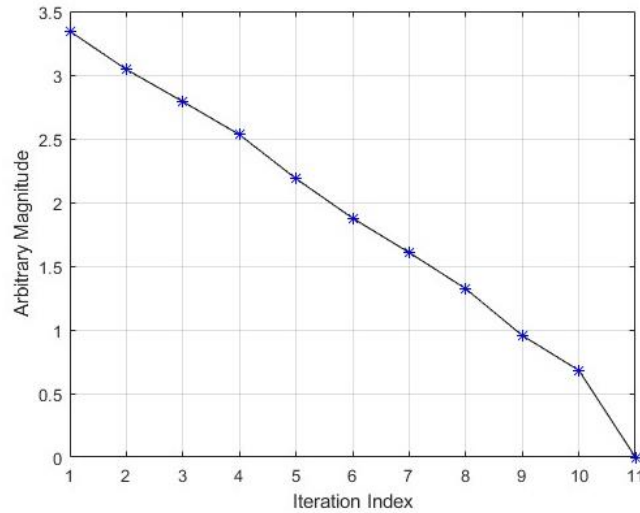
### 4.1. K-Sparse Signal Recovery, Exactly in k Iterations

Based on conditions of RIC and  $k$  of the sensing matrix, some earlier works [35-39] showed conditions for guaranteed sparse signal recovery exactly in k-iterations using OMP algorithm. An improvement over these bounds is given by Liu et. al., [40], i.e., that RIC ( $\delta_{k+1}$ ) should be less than  $(\sqrt{2}/2)$ ,

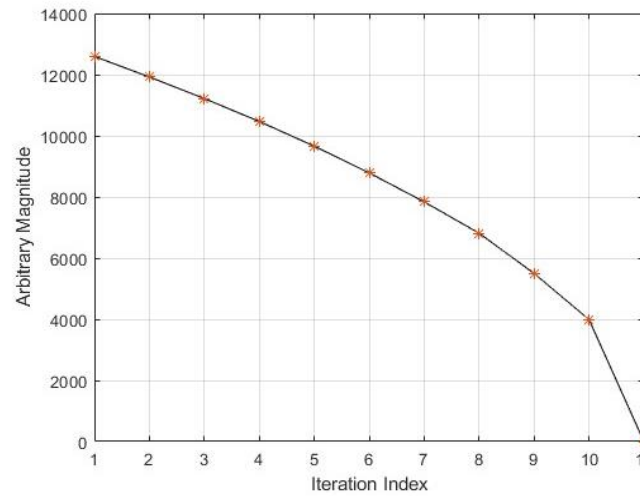


i.e 0.707. In our analysis, for a matrix,  $A_1$ , of size, (14x18), having a sparsity,  $k=11$ ,  $\rho=0.7777$ , and RIC value obtained is 0.387. It satisfies the limit given by [40] for guaranteed recovery condition. This recovery indicates the wellness condition of the sensing matrix, which satisfies RIP and RIC.

Eventhough RIC is not available for,  $Q$  (with normalized columns), sparse channel is recovered exactly in  $k$ -iterations. Evolution of recovery for both the sensing matrices ( $A$ , and  $Q$ ) is shown in Figure 3. Also it is observed that the recovery success rate is more than 98% in both the cases.



(a)



(b)

**Figure 3.** Recovered  $k$ -sparse signal in  $k$ -iterations. (a) with matrix  $A_1$  (b) with channel matrix,  $Q$ .

In this context, it is to be mentioned that if we consider number of measurements ( $M$ ) required for recovery as another evaluation parameter, it is found that in our analysis,  $M$  (related to  $k$ , and  $N$ ), also satisfies the guaranteed recovery, for both the matrices,  $A$  and  $Q$ . This result is not incorporated here, since it will also support same guarantee, conveyed by the already presented (three) results in the current simulations section.

#### 4.2. Relation in Between a Primitive Size ( $A_1$ ) and Larger Size ( $Q$ ) Matrices

A set of matrices,  $A$ , of sizes,  $A_1$  (6x8) to  $A_5$  (18x24), which are having sparsity level,  $k=4$ , are selected. This set is selected, because of the possibility for computation of RIP and RIC, and coherence parameters for these matrices. Notable features observed common to all these matrices are:  $k=4$ ,

$\rho = 0.75$ , satisfying conditions for recovery in  $k$ -iterations and success rate of more than 98%.

A plot of row index ( $M$ ) versus column index ( $N$ ), of these matrices is shown in the Figure 4. The plot shows, that a linear regression in between  $A1$  and  $A5$ . From this regression, it can reasonably be argued that the curve can be extended further,

beyond  $A5$ , to any large size matrix, which is a multiple of primitive size,  $A1$ , since  $A3$  and  $A5$ , are multiples of  $A1$ . All these matrices lying on this regression will satisfy, same conditions:  $k=4$ ,  $\rho = 0.75$ , success rate and guaranteed recovery in  $k$ -iterations.

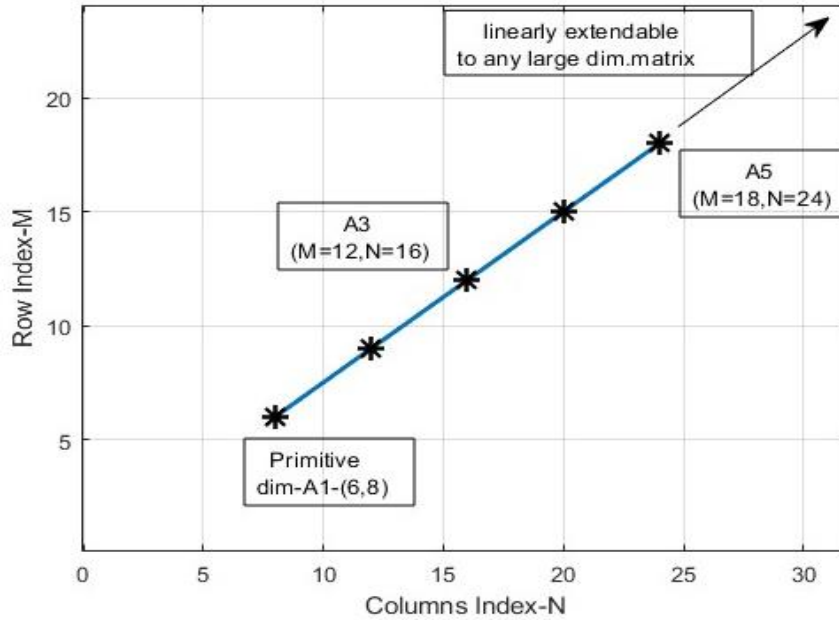


Figure 4. Relation between smaller ( $A$ ) and larger dimensional ( $Q$ ) size matrices.

The above argument can be applied for matrices in another set with primitive size,  $A1$ , ( $14 \times 18$ ), having sparsity level of  $k=11$ , when regression curve is extended from  $A1$  to any larger size matrix,  $Q$  ( $1008 \times 1296$ ). Since  $Q$  is a multiple of  $A1$ , both  $A1$ ,  $Q$  are having same:  $k=11$ ,  $\rho = .7777$ , recovery success rate and sparse recovery in  $k$ -iterations, the specified guaranteed conditions, are applicable for both  $A1$  and  $Q$  in this set. Thus from Figure 3, and Figure 4, guarantee of the mmWave channel is achieved.

## 5. Conclusions

Open-loop training based mmWave hybrid MIMO channel is estimated using OMP algorithm. From the simulation results of  $k$ -sparse signal recovery exactly in  $k$ -iterations, and recovery success rate of more than 98%, with both synthesized and channel sensing matrices, it can be concluded that, even though RICs are not available for  $Q$ , and since the specified sparse recovery conditions are satisfied by both the matrices ( $A$  and  $Q$ ), recovery with  $Q$  is guaranteed, on par with that of  $A$ . Thus the guaranteed mmWave channel estimation is achieved. Applicability of the result, for broadband wireless systems, audio, and image and data processing systems, is proposed for future research work.

## Abbreviations

MIMO	Multiple Input and Multiple Output
DFT	Discrete Fourier Transform

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## Author Contributions

Anjaneyulu Pattem is the sole author. The author read and approved the final manuscript.

## Conflicts of Interest

The author declares no conflicts of interest.

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