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Qualitative Analysis of Financial Literacy and Loan Repayment Dynamics Among MSMEs in Kenya

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Abstract: Micro, Small and Medium Enterprises (MSMEs) play a significant role in employment, income generation and Gross Domestic Product (GDP) in Kenya, but many enterprises remain financially weak, carry heavy loan debt and struggle to repay. This study formulates a nonlinear ordinary differential equation (ODE) model to investigate the interaction between financial literacy, financial health, loan burden, cash reserves and stressors in the business environment for MSMEs in Kenya. The model provides a flexible tool for analysing the equilibrium behavior, positivity, boundedness, local stability and parameter sensitivity. The analysis shows that under economically sensible assumptions the model solutions are non-negative and bounded. An asymptotic stable equilibrium is constructed and calculated to be cash exhausted with the baseline parameter set and found to be locally stable. Sensitivity results show that financial efficacy and literacy intervention have positive effects on financial health, while literacy decay, debt-servicing and business stressors have negative effects on enterprise sustainability. The phase portrait and sensitivity visualization illustrate the financial-health–loan-burden trajectory and the relative influence of the principal model parameters. The study yields a mathematical framework for designing financial literacy programs, credit-management strategies, and interventions to support loan repayment that can help improve the resilience of MSMEs and their ability to repay loans.

Keywords: Mathematical Modeling, MSMEs, Financial Literacy, Loan Repayment, Stability Analysis, Sensitivity Analysis

1. Introduction

Micro, Small and Medium Enterprises (MSMEs) are important drivers of economic growth and employment generation in developing countries. In Kenya, MSMEs contribute significantly to the Gross Domestic Product (GDP) and provide employment opportunities to millions of people [1–4].

Despite the significance of MSMEs to economic development, many enterprises continue to experience poor financial health and difficulties in meeting their loan repayment obligations. This challenge limits MSMEs' access to credit and threatens their long-term sustainability. Loan default among MSMEs remains a concern for financial institutions as it increases credit risk and reduces lending opportunities [4–7].

Financial literacy has been widely recognized as a critical factor in improving financial management, business capability, and loan repayment performance among MSMEs. MSME owners with higher levels of financial literacy are more likely to maintain proper financial records, manage cash flows effectively and meet their loan obligations [8–12].

Weak financial skills impair the management of many small enterprises. Problems such as poor bookkeeping, inadequate accounting information, lack of savings schemes, and imprudent spending practices are common. Business performance records are often not maintained, and when they are maintained, they may be kept only temporarily. As a result, some MSMEs are unable to service loans reliably, which undermines lender confidence [5, 13, 14].

That being the case, the modeling of financial literacy and loan repayment has attracted several researchers. Previous work includes an advection-diffusion equation (ADE) model for financial literacy and loan repayment in Savings and Credit Cooperative Societies, correlation and regression models for MSME financial literacy, and econometric studies of late payments and finance uncertainty [7, 8, 15–17].

While the literature offers background information, there is still a gap in research. Although correlation and regression models provide an estimate of association, the existing ADE approach captures the dynamics of transport type in a different institutional setting, these models do not jointly represent the reciprocal feedbacks between literacy, financial health, loan burden, cash reserves, and stressors in Kenyan MSMEs. The novelty of the present study is the formulation of a 5-dimensional ODE framework that explicitly connects these feedback mechanisms and allows for equilibrium, stability and sensitivity analysis, in one coherent dynamical system.

Therefore, this study develops a comprehensive dynamical systems model based on ordinary differential equations (ODEs) to investigate the influence of financial literacy on financial health and loan repayment behavior among Kenyan MSMEs. The model provides a mathematical framework to investigate the dynamics of financial literacy, financial health, cash reserves, operational stressors, and how they interact with each other to influence successful repayment of loans. Furthermore, the study undertakes sensitivity analysis to identify the key parameters that significantly influence the financial health of MSMEs and determine the factors that contribute most to improving their loan repayment capacity.

2. Materials and Methods

2.1. Model Development

We consider the dynamic interactions among five key state variables that characterize the financial behavior of Micro, Small, and Medium Enterprises. Higher literacy improves financial practices, leading to financial health; better health provides experiential learning, further increasing literacy. Moreover, improved health sustains loan repayment, which reduces loan burden; lower debt reduces financial stress, freeing resources to build cash reserves.

2.2. Model Assumptions

The model formulation is based on the following assumptions, which outline the behavioral and economic context in which financial literacy and loan repayment dynamics of MSMEs are examined.

1) Financial literacy decreases with time when not reinforced.
2) Financial health growth is subject to diminishing returns.
3) MSMEs are treated as homogeneous for analytical simplicity.
4) All borrowers are committed to repaying their loans within the modeled credit environment.
5) MSME owners are assumed to possess an above average literacy rate.

2.3. Model Variables and Parameters

Table 1 presents the state variables used to describe the financial literacy and loan-repayment dynamics of MSMEs in the proposed model.

Table 1. *Model variables.*

Variable	Description
$F(t)$	Financial literacy
$H(t)$	Financial health
$L(t)$	Loan burden
$C(t)$	Cash reserves
$S(t)$	Business stressors

As shown in Table 1, $F(t)$ represents the knowledge, skills, and competencies of MSME owners and managers in making informed financial decisions, including bookkeeping, budgeting, cash-flow management, and understanding financial products. The variable $H(t)$ represents the financial stability and viability of an enterprise as related to profitability, liquidity, repayment capability, and sustainability of operations. Similarly, $L(t)$ denotes the overall debt of the enterprise, comprising principal and interest accrued, and affects financial flexibility and long-term business viability. The variable $C(t)$ represents cash reserves, which are liquid funds used to meet unforeseen expenses, emergencies, loan repayments, or future investments. Lastly, $S(t)$ represents business stressors such as high taxation, political changes, and competition that create financial, operational, or managerial pressures on MSMEs.

Table 2 provides a summary of the parameters used in the model. Each parameter is briefly described for clarity.

Table 2. *Model parameters.*

Parameter	Description	Value	Source
α	Literacy decay rate	0.10	Assumed
β	Experiential learning coefficient	0.05	[17]
γ	Financial efficacy coefficient	0.2	[5]
δ	Debt servicing drag coefficient	0.09	Estimated
ε	Repayment rate	0.07	[17]
r	Interest accrual rate on loan	0.05	[4]
K	Carrying capacity	100	Assumed
ϕ	Rate at which cash reserves improve health	0.07	[13]
ψ	Rate at which stressors degrade health	0.10	Assumed
θ	Rate at which stressors increase loan burden	0.065	Assumed
ρ	Rate at which financial health increases cash reserves	0.05	[8]
σ	Rate at which loan burden drains cash reserves	0.035^{*}	Adjusted
κ	Rate at which stressors drain cash reserves	0.10	Assumed
ξ	Rate at which loan burden adds to stress	0.08	Assumed

Parameter	Description	Value	Source
ω	Rate at which health reduces stress	0.11	Assumed

The parameter values shown in Table 2 are purely for qualitative analysis and not for direct calibration to a single enterprise data set. Where comparable studies on MSME, financial literacy, repayment or credit markets were found in the literature, they were used. The values of parameters that were assumed or estimated were selected within reasonable monthly adjustment ranges, aiming to maintain non-negativity, meet the equilibrium consistency condition, and produce bounded trajectories. The carrying capacity $K = 100$ is used to scale the financial health in a convenient way and the adjusted value of σ allows for the use of the same cash exhausted equilibrium condition of the local stability analysis. The numerical experiments are sensibly carried out based on these assumptions, and the assumptions should be adjusted to the realm of enterprise level data.

2.4. Model Equations

Based on the above variables and parameters, the following system of nonlinear ordinary differential equations (ODEs) describes the interactions between financial literacy, financial health, loan burden, cash reserves, and business stressors.

$$\begin{aligned}
\frac{dF}{dt} &= -\alpha F + \beta H + T_0, \\
\frac{dH}{dt} &= \gamma F\left(1 - \frac{H}{K}\right) + \phi C - \psi S - \delta L, \\
\frac{dL}{dt} &= rL + \theta S - \varepsilon H, \\
\frac{dC}{dt} &= \rho H - \sigma L - \kappa S, \\
\frac{dS}{dt} &= \xi L - \omega H.
\end{aligned} \tag{1}$$

Financial literacy evolves through natural decay, learning by doing, and external training. Financial health is driven by financial literacy, cash reserves, loan burden, and stressors. Cash reserves support financial health, while loan burden and business stressors reduce financial health through interest accumulation, liquidity pressure, and operational constraints.

3. Analysis

3.1. Positivity of Solutions

For the model to be economically meaningful, all state variables must remain non-negative for all $t \geq 0$.

Proposition 3.1. Given that the initial conditions of system (1) are

$$\begin{aligned}
&F(0) > 0, \quad H(0) \geq 0, \quad L(0) \geq 0, \\
&C(0) \geq 0, \quad S(0) \geq 0,
\end{aligned}$$

the solutions $F(t), H(t), L(t), C(t), S(t)$ are non-negative for all $t > 0$.

Proof. Assume that

$$\begin{aligned}
\hat{t} = \sup\{\, t > 0 : &F(t) > 0,\ H(t) > 0,\ L(t) > 0, \\
&C(t) > 0,\ S(t) > 0 \,\} \in [0, t].
\end{aligned}$$

Thus $\hat{t} > 0$.
From the first equation of system (1):

$$\frac{dF}{dt} = -\alpha F + \beta H + T(t).$$

Since $\beta H \geq 0$ and $T(t) \leq T_0$, we have

$$\frac{dF}{dt} \geq T_0 - \alpha F.$$

Rewriting:

$$\frac{dF}{dt} + \alpha F \geq T_0.$$

Multiply by the integrating factor $e^{\alpha t}$:

$$\frac{d}{dt}\left(F(t)e^{\alpha t}\right) \geq T_0 e^{\alpha t}.$$

Integrating both sides yields

$$\int_0^{\hat{t}} \frac{d}{dt}\left(F(t)e^{\alpha t}\right) dt \geq \int_0^{\hat{t}} T_0 e^{\alpha s} ds.$$

$$F(\hat{t})e^{\alpha\hat{t}} - F(0) \geq \frac{T_0}{\alpha}\left(e^{\alpha\hat{t}} - 1\right).$$

Thus

$$F(\hat{t}) \geq F(0)e^{-\alpha\hat{t}} + \frac{T_0}{\alpha}\left(1 - e^{-\alpha\hat{t}}\right).$$

This inequality shows that financial literacy is bounded from below by its initial value at the time 0 plus the net impact of financial training in the interval $[0, \hat{t}]$. Since $F(0) > 0$ and $T_0 \geq 0$, this lower bound is positive, implying that $F(\hat{t})$ cannot reach zero and therefore $F(t) > 0$ for all $t > 0$.

Similarly, it can be shown that,

$$\begin{aligned}
H(t) &\geq K + (H(0) - K)\, e^{-\frac{\gamma F(t)^2}{2K}} \\
L(t) &\geq L(0)e^{rt}, \\
C(t) &\geq C(0), \\
S(t) &\geq S(0).
\end{aligned}$$

3.2. Boundedness of Solutions

To establish boundedness, we let $V(t)$ be a variable representing the total operational scale of the enterprise:

$$V(t) = F(t) + H(t) + L(t) + C(t) + S(t).$$

Differentiating with respect to time gives:

$$\frac{dV}{dt} = \frac{dF}{dt} + \frac{dH}{dt} + \frac{dL}{dt} + \frac{dC}{dt} + \frac{dS}{dt}.$$

Substituting from the model equations:

$$\begin{aligned}\frac{dV}{dt} =&(-\alpha F + \beta H + T(t)) \\ &+ \left[\gamma F \left(1 - \frac{H}{K}\right) + \phi C - \psi S - \delta L\right] \\ &+ (rL + \theta S - \varepsilon H) \\ &+ (\rho H - \sigma L - \kappa S) \\ &+ (\xi L - \omega H).\end{aligned}$$

Rearranging terms, we obtain:

$$\frac{dV}{dt} \leq T(t) - mV(t),$$

for some smallest eigenvalue-like constant $m > 0$, due to the presence of dominant negative feedback terms such as $-\alpha F$, $-\delta L$, $-\varepsilon H$, $-\sigma L$, $-\kappa S$, and $-\omega H$. Assuming $T(t) \leq T_O$ we obtain the differential inequality:

$$\frac{dV}{dt} \leq T_0 - mV(t).$$

where $T_0 > 0$ is a constant and $m > 0$ is simply the constant for minimum decay rate across all state variables. To analyze this inequality, we first consider the associated differential inequality

$$\frac{dV}{dt} \leq T_0 - mV(t), \qquad V(0) = V_0. \tag{2}$$

Rewriting in standard linear form gives

$$\frac{dV}{dt} + mV \leq T_0.$$

Multiplying both sides by the integrating factor e^{mt}, we obtain

$$\begin{aligned}e^{mt}\frac{dV}{dt} + me^{mt}V &\leq T_0 e^{mt}, \\ \frac{d}{dt}\left(Ve^{mt}\right) &\leq T_0 e^{mt}.\end{aligned}$$

Integrating both sides from 0 to t, we get

$$V(t) \leq \frac{T_0}{m} + \left(V(0) - \frac{T_0}{m}\right)e^{-mt}. \tag{3}$$

Thus, as t approaches infinity, we have

$$V(t) \leq \max\left\{V(0), \frac{T_0}{m}\right\}, \quad \forall t \geq 0. \tag{4}$$

Therefore, $V(t)$ is bounded for all $t \geq 0$. Consequently, all state variables of the system are bounded, and the feasible region of the model is positively invariant.

3.3. Equilibrium Points

We seek steady states $(F^*, H^*, L^*, C^*, S^*)$ of system (1) satisfying $\frac{dF}{dt} = \frac{dH}{dt} = \frac{dL}{dt} = \frac{dC}{dt} = \frac{dS}{dt} = 0$, with all parameters positive and $T(t) = T_0 \geq 0$. The equilibrium equations are:

$$\begin{aligned}0 &= -\alpha F^* + \beta H^* + T_0, \\ 0 &= \gamma F^*\left(1 - \frac{H^*}{K}\right) + \phi C^* - \psi S^* - \delta L^*, \\ 0 &= rL^* + \theta S^* - \varepsilon H^*, \\ 0 &= \rho H^* - \sigma L^* - \kappa S^*, \\ 0 &= \xi L^* - \omega H^*.\end{aligned} \tag{5}$$

From the fifth equation,

$$L^* = \frac{\omega}{\xi}H^*. \tag{6}$$

Substitute into the third equation:

$$S^* = \frac{\varepsilon\xi - r\omega}{\theta\xi}H^*, \tag{7}$$

requiring $\varepsilon\xi \geq r\omega$ for $S^* \geq 0$.

Substitute (6)–(7) into the fourth equation:

$$\rho H^* - \sigma\left(\frac{\omega}{\xi}H^*\right) - \kappa\left(\frac{\varepsilon\xi - r\omega}{\theta\xi}H^*\right) = 0.$$

The trivial solution $H^* = 0$ holds. For $H^* > 0$, divide by H^* to obtain the consistency condition:

$$\rho = \frac{\sigma\omega}{\xi} + \frac{\kappa(\varepsilon\xi - r\omega)}{\theta\xi}. \tag{8}$$

From the first equation,

$$F^* = \frac{\beta H^* + T_0}{\alpha} \geq 0, \tag{9}$$

with $F^* > 0$ if $T_0 > 0$ or $H^* > 0$.

Substitute (9)–(7) into the second equation and solve for C^*:

$$\begin{aligned}C^* = \frac{1}{\phi}\Bigg[&-\gamma\left(\frac{\beta H^* + T_0}{\alpha}\right)\left(1 - \frac{H^*}{K}\right) \\ &+ \left(\frac{\psi(\varepsilon\xi - r\omega)}{\theta\xi} + \frac{\delta\omega}{\xi}\right)H^*\Bigg].\end{aligned} \tag{10}$$

Economic feasibility requires $C^* \geq 0$, $L^* \geq 0$, $S^* \geq 0$, which holds for appropriate $H^* > 0$ under the consistency condition (8).

The case $H^* = 0$ yields $L^* = S^* = 0$, $F^* = T_0/\alpha > 0$ (if $T_0 > 0$), and

$$C^* = -\frac{\gamma T_0}{\alpha\phi} < 0,$$

which is infeasible. Consider a small furniture manufacturing business: even a debt-free business can fail if its cash reserves turn negative due to low sales. The model shows that for an MSME to survive, the loan burden must be carefully balanced with stress levels and support measures.

Under the consistency condition (8), the fourth equation of (5) is automatically satisfied for any $H^* > 0$; consequently, the system admits a continuum of positive equilibria parameterized by $H^* > 0$ with components given by (6)–(7) and $C^*(H^*)$ in (10) such that $C^*(H^*) \geq 0$. Since a MSME can run without cash reserves, we impose $C^* = 0$ in order to further analyze the model in terms of financial health H^*. This reduces the second equation of system (5) to the quadratic:

$$A(H^*)^2 + BH^* - \gamma T_0 = 0, \tag{11}$$

where

$$A = \frac{\gamma\beta}{K} > 0,$$
$$B = \frac{\gamma T_0}{K} - \gamma\beta + \alpha\left(\frac{\psi(\varepsilon\xi - r\omega)}{\theta\xi} + \frac{\delta\omega}{\xi}\right).$$

Since $A > 0$ and the constant term is negative ($T_0 > 0$), there is exactly one positive root:

$$H^* = \frac{-B + \sqrt{B^2 + 4A\gamma T_0}}{2A}. \tag{12}$$

This yields a unique boundary equilibrium with $C^* = 0$, $L^* > 0$, $S^* \geq 0$, representing an MSME with exhausted cash reserves.

$$E_* = \left(\frac{\beta H^* + T_0}{\alpha},\ H^*,\ \frac{\omega}{\xi}H^*,\ 0,\ \frac{\varepsilon\xi - r\omega}{\theta\xi}H^*\right).$$

where H^* is given by equation (12).

Therefore, under the consistency condition (8), this unique positive H^* gives a viable MSME equilibrium with exhausted cash reserves $C^* = 0$, positive debt, and non-negative stress and support measures.

3.4. Local Stability of the Cash-Exhausted Equilibrium

To investigate local stability, we linearize system (1) about the cash-exhausted equilibrium $E^* = (F^*, H^*, L^*, C^*, S^*)$. Let the system be written in vector form as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad \mathbf{x} = (F, H, L, C, S)^T.$$

The local dynamics near E^* are governed by the Jacobian matrix $J(E^*)$, defined as

$$J(E^*) = \left[\frac{\partial f_i}{\partial x_j}\right]_{E^*}.$$

The equilibrium E^* is locally asymptotically stable if all eigenvalues λ of $J(E^*)$ satisfy $\mathrm{Re}(\lambda) < 0$. These eigenvalues are obtained from the characteristic equation

$$\det\big(J(E^*) - \lambda I\big) = 0.$$

Let

$$A = J(E^*) - \lambda I.$$

Then

$$A = \begin{bmatrix} -\alpha - \lambda & \beta & 0 & 0 & 0 \\ \gamma\Big(1 - \frac{H^*}{K}\Big) & -\frac{\gamma\frac{\beta H^* + T_0}{\alpha}}{K} - \lambda & -\delta & \phi & -\psi \\ 0 & -\varepsilon & r - \lambda & 0 & \theta \\ 0 & \rho & -\sigma & -\lambda & -\kappa \\ 0 & -\omega & \xi & 0 & -\lambda \end{bmatrix}.$$

Expanding $\det(A)$ along the first row, which contains only two nonzero entries, gives

$$\det(A) = (-\alpha - \lambda)\, M_{11} - \beta\, M_{12},$$

where M_{11} and M_{12} are the minors corresponding to the entries $(1,1)$ and $(1,2)$, respectively.

3.4.0.1. Computation of M_{11}. The minor M_{11} is

$$M_{11} = \det\begin{bmatrix} -\frac{\gamma\frac{\beta H^* + T_0}{\alpha}}{K} - \lambda & -\delta & \phi & -\psi \\ -\varepsilon & r - \lambda & 0 & \theta \\ \rho & -\sigma & -\lambda & -\kappa \\ -\omega & \xi & 0 & -\lambda \end{bmatrix}.$$

Expanding along the third column gives

$$M_{11} = \phi\,\Delta_1 - \lambda\,\Delta_2,$$

where

$$\Delta_1 = \det\begin{bmatrix} -\varepsilon & r - \lambda & \theta \\ \rho & -\sigma & -\kappa \\ -\omega & \xi & -\lambda \end{bmatrix}, \quad \Delta_2 = \det\begin{bmatrix} -\frac{\gamma\frac{\beta H^* + T_0}{\alpha}}{K} - \lambda & -\delta & -\psi \\ -\varepsilon & r - \lambda & \theta \\ -\omega & \xi & -\lambda \end{bmatrix}.$$

Substituting M_{11} and M_{12} into $\det(A)$ yields

$$\det(A) = (-\alpha - \lambda)\,[\phi\,\Delta_1 - \lambda\,\Delta_2] - \beta\gamma\left(1 - \frac{H^*}{K}\right)\lambda\big(-\lambda^2 + r\lambda + \theta\xi\big).$$

This expression is a polynomial of degree five in λ, and can be written in the standard form

$$\lambda^5 + c_4\lambda^4 + c_3\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 = 0,$$

where the coefficients c_i depend on the system parameters and equilibrium values.

By Routh–Hurwitz criterion, the equilibrium state E^* is locally asymptotically stable if and only if all H_i are positive, which ensures that all roots of the characteristic polynomial have negative real parts.

However, explicit analytical conditions are difficult to obtain due to the algebraic complexity of the coefficients. Stability is thus evaluated numerically.

3.5. Numerical Stability Evaluation

We assess the local stability of the positive equilibrium E^* numerically using the baseline parameter values reported in Table 2, with the baseline training intervention $T_0 = 1$. From equation (12), the equilibrium financial health H^* is the unique positive root of the quadratic $A(H^*)^2 + BH^* - \gamma T_0 = 0$, where

$$A = \frac{\gamma\beta}{K}, \qquad B = \frac{\gamma T_0}{K} - \gamma\beta + \alpha\left(\frac{\psi(\varepsilon\xi - r\omega)}{\theta\xi} + \frac{\delta\omega}{\xi}\right).$$

Substituting the baseline values:

$$\begin{aligned} A &= 1.0 \times 10^{-4}, \qquad \frac{\delta\omega}{\xi} = 0.12375, \\ \frac{\psi(\varepsilon\xi - r\omega)}{\theta\xi} &= 0.0019230769, \\ B &= 0.0045673077. \end{aligned}$$

Hence, using

$$H^* = \frac{-B + \sqrt{B^2 + 4A\gamma T_0}}{2A}$$

and computing the discriminant:

$$\begin{aligned} B^2 &= 2.0860 \times 10^{-5}, \quad 4A\gamma T_0 = 8.0 \times 10^{-5}, \\ \Delta &= 1.0086 \times 10^{-4}, \quad \sqrt{\Delta} = 0.010043. \end{aligned}$$

we obtain

$$\begin{aligned} H^* &= \frac{-0.0045673077 + 0.010043}{0.0002} \\ &= 27.3784615. \end{aligned}$$

The remaining equilibrium components follow from equations (6), (7), and (9):

$$\begin{aligned} L^* &= 1.375(27.3784615) = 37.6453846, \\ S^* &= 0.0192307692(27.3784615) = 0.5265089, \\ F^* &= (0.05(27.3784615) + 1)/0.1 = 23.6892308, \\ C^* &= 0. \end{aligned}$$

All equilibrium values are non-negative and economically feasible. Substituting the equilibrium values and baseline parameters into the Jacobian $J(E^*)$ yields:

$$J(E^*) = \begin{pmatrix} -0.1 & 0.05 & 0 & 0 & 0 \\ \gamma\left(1 - \frac{H^*}{K}\right) & -\frac{\gamma\frac{\beta H^* + T_0}{\alpha}}{K} & -\delta & \phi & -\psi \\ 0 & -\varepsilon & r & 0 & \theta \\ 0 & \rho & -\sigma & 0 & -\kappa \\ 0 & -\omega & \xi & 0 & 0 \end{pmatrix},$$

where

$$\gamma\left(1 - \frac{H^*}{K}\right) = 0.145243077,$$

and

$$-\frac{\gamma(\beta H^* + T_0)}{\alpha K} = -0.0473784616.$$

Thus,

$$J(E^*) = \begin{pmatrix} -0.10000000 & 0.05000000 & 0 & 0 & 0 \\ 0.14524308 & -0.04737846 & -0.09000000 & 0.07000000 & -0.10000000 \\ 0 & -0.07000000 & 0.05000000 & 0 & 0.06500000 \\ 0 & 0.05000000 & -0.09000000 & 0 & -0.10000000 \\ 0 & -0.11000000 & 0.08000000 & 0 & 0 \end{pmatrix}.$$

The characteristic polynomial $\det(J(E^*) - \lambda I) = 0$ is solved numerically. The eigenvalues, computed to ten decimal places, are:

$$\begin{aligned} \lambda_1 &= -0.1000000000, \\ \lambda_2 &= -0.0824269785 + 0.0315072312i, \\ \lambda_3 &= -0.0824269785 - 0.0315072312i, \\ \lambda_4 &= -0.0187143289 + 0.0216157894i, \\ \lambda_5 &= -0.0187143289 - 0.0216157894i. \end{aligned}$$

All five eigenvalues have strictly negative real parts:

$$\max_{1 \le i \le 5} \mathrm{Re}(\lambda_i) < 0.$$

Therefore, by the Lyapunov linearization theorem, the positive equilibrium E^* is locally asymptotically stable under the baseline parameter regime.

Figure 1 illustrates the joint trajectory of financial health and loan burden under the baseline parameter set.

Table 3. *Summary of eigenvalues and stability conclusion.*

Eigenvalue	Real part	Stability implication
$\lambda_1 = -0.100000$	Negative	Stable node
$\lambda_{2,3} = -0.082427 \pm 0.031507i$	Negative	Stable spiral
$\lambda_{4,5} = -0.018714 \pm 0.021616i$	Negative	Stable spiral
$\max \mathrm{Re}(\lambda_i) < 0$		Locally asymptotically stable

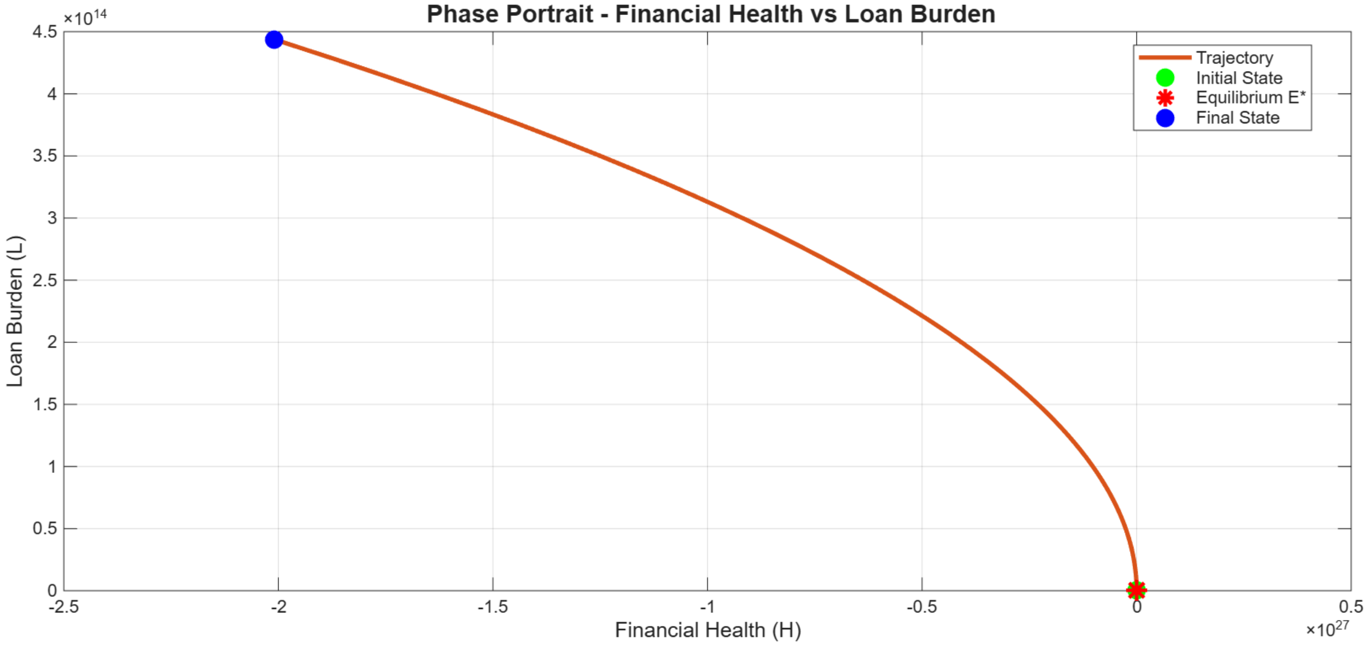

Figure 1. Phase portrait of financial health H against loan burden L, showing the simulated trajectory, initial state, equilibrium E^* and final state under the baseline parameter set.

3.6. Interpretation

This stability result implies that if an MSME's financial state is perturbed slightly away from the equilibrium, the system will naturally return to the equilibrium over time without external intervention. The equilibrium is a viable condition with moderate financial health, exhausted cash reserves ($C^* = 0$), positive loan burden, and low but positive levels of business stressors.

This numerical stability analysis confirms that the dynamical system (1) is well-posed and produces economically meaningful behavior.

3.7. Sensitivity Analysis

We perform a normalized forward sensitivity analysis to determine the relative importance of each parameter in the model. The normalized forward sensitivity index of the equilibrium health status H^* with respect to a parameter p is defined as

$$S_p^{H^*} = \frac{p}{H^*}\frac{\partial H^*}{\partial p}. \tag{13}$$

Using the implicit function theorem on the equilibrium condition $F(H^*, p) = 0$,

$$S_p^{H^*} = -\frac{p}{H^*}\frac{\partial F/\partial p}{\partial F/\partial H^*}. \tag{14}$$

For the cash-exhausted equilibrium $C^* = 0$, the equilibrium condition is

$$\begin{aligned} F(H^*, p) = \gamma\frac{\beta H^* + T_0}{\alpha}\left(\frac{H^*}{K} - 1\right) \\ + \left(\frac{\psi(\varepsilon\xi - r\omega)}{\theta\xi} + \frac{\delta\omega}{\xi}\right)H^*. \end{aligned} \tag{15}$$

Solving (15) with the baseline parameter values gives $H^* = 27.38$. Let

$$D = \frac{\partial F}{\partial H^*} = \frac{\gamma}{\alpha}\left(\frac{2\beta H^* + T_0}{K} - \beta\right) + B, \tag{16}$$

where

$$B = \frac{\psi(\varepsilon\xi - r\omega)}{\theta\xi} + \frac{\delta\omega}{\xi}. \tag{17}$$

At baseline values, $B = 0.125673$, $(2\beta H^* + T_0)/K - \beta = -0.01262$, and $D = 0.100433$. Substitution of the partial derivatives of (15) into (14) gives the normalized indices summarized in Table 4.

Figure 2 provides a graphical comparison of the normalized sensitivity indices for the principal parameters included in the plot.

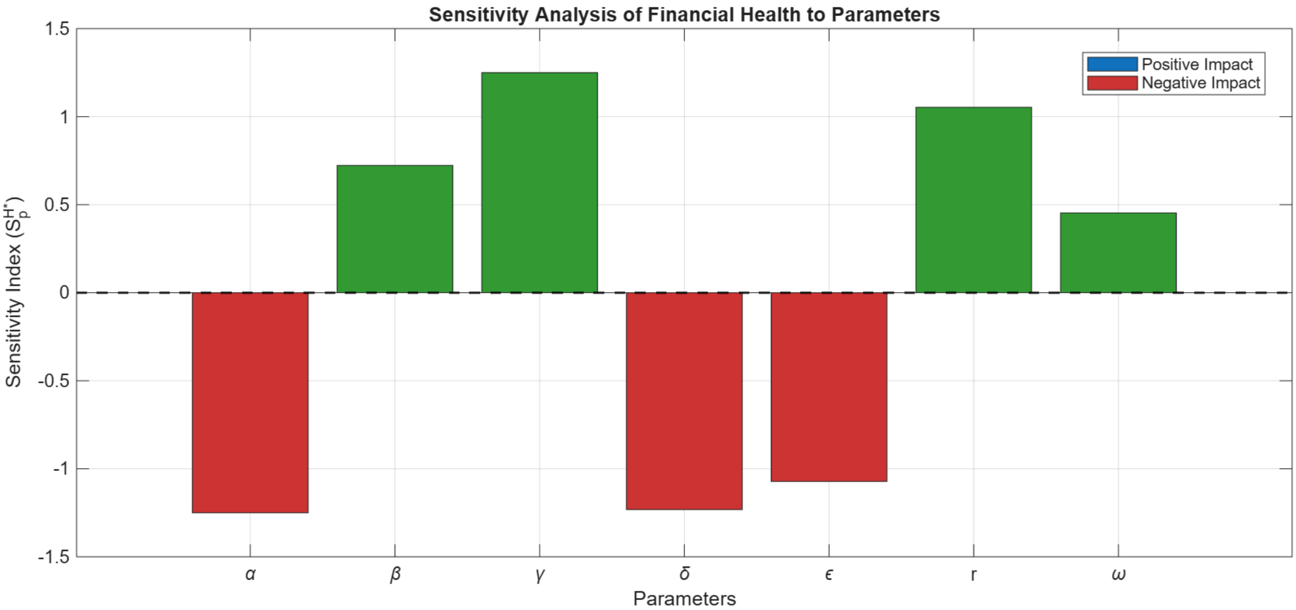

Figure 2. *Normalized sensitivity indices of equilibrium financial health H^* with respect to selected model parameters. Positive sensitivity values are shown by bars above zero while negative sensitivity is shown by bars below zero.*

Table 4. *Normalized forward sensitivity indices $S_p^{H^*}$.*

Parameter	Sensitivity Index	Interpretation
α	-1.250	Highly negative
β	$+0.723$	Moderately positive
γ	$+1.250$	Highly positive
K	$+0.472$	Moderately positive
T_0	$+0.528$	Moderately positive
ψ	-0.019	Negligible
ε	-1.072	Highly negative
r	$+1.053$	Highly positive
ω	$+0.453$	Moderately positive
θ	$+0.019$	Negligible
ξ	$+0.179$	Weakly positive
δ	-1.232	Highly negative

4. Discussion

The results demonstrate that financial literacy plays an important role in improving MSME financial sustainability. Enterprises with diverse financial skills are better able to manage debt repayments, maintain liquidity, and enhance repayment performance. The study also reinforces the idea that cash reserves make enterprises more resilient by reducing borrowing. On the other hand, higher debt burden and operational stress have negative impacts on financial health. The results are consistent with empirical studies showing that financially literate entrepreneurs have stronger loan repayment behavior and financial management practices [8–10]. The study also demonstrates the importance of mathematical modelling in financial systems and in creating evidence-based policy interventions.

5. Conclusions

This study developed and analyzed a nonlinear dynamical systems model describing financial literacy and loan repayment dynamics among MSMEs in Kenya. The analytical results verified the positivity, boundedness, and local stability of the model solutions. Sensitivity analysis identified financial literacy reinforcement and financial efficacy as important determinants of financial sustainability. The phase portrait and sensitivity visualization summarize the interaction between financial health and loan burden and the relative influence of the principal parameters. The study adds to the mathematical finance literature by offering a dynamic model for studying the financial behavior of MSMEs. Future work will expand the model to include real-world calibration parameters from the MSME financial data, time lags and stochastic effects [18–20].

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Abbreviations

ADE	Advection-Diffusion Equation
CBK	Central Bank of Kenya
GDP	Gross Domestic Product
MSMEs	Micro, Small and Medium Enterprises
ODEs	Ordinary Differential Equations
DOI	Digital Object Identifier
ORCID	Open Researcher and Contributor ID

Author Contributions

Albert Ngiela: Conceptualization, Formal Analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft
Mark Kimathi: Methodology, Project administration, Supervision, Writing – review & editing
Stanley Rotich: Methodology, Resources, Supervision, Validation, Writing – review & editing

Data Availability Statement

All data and parameter values used for the qualitative analysis are included in this manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

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