

Research Article

# Gompertz Function Approach: Numerical Integration for Microbial Growth Problem

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## Abstract

In this work, we developed a numerical integrator using the Gompertz function model approach with the basic parameters as highlighted by Gompertz in finding and measure the growth in human cells as a basis function involving exponential, logarithmic, and polynomial, hence implemented the numerical integrator to solve problems arising in microbial growth staging. Microbial growth, synonymous to mildew or mold, which is a fungi family commonly found both indoors and outdoors. The indoors occur especially when there is humidity, moisture, oxygen, organic matters and low sunlight. Microbial growth which is the increase in the number of microbial cells which can also be in term of bacterial growth. It can be influenced by various factors to grow including temperature, Water, availability of oxygen, and other nutrient content. The growth staging can be in four phases such as lag, logarithmic, stationary and death phases. A culture of bacterial was taken, the approximate number of strand that was originally present and the growth were calculated using the numerical integration, the results obtained shows a significant, effective and robust improvement on the strand when compared the results with the exact solution. The properties of the integrator were analyzed, considering that Microbial Growth is an increase in the number of bacteria cells in a system when the proper nutrients and environment are provided. Therefore with the approach of Gompertz, the numerical integrator can be applied further to find the growth in each of the phases as they occurs.

## Keywords

Numerical Integrator, Gompertz Function, Microbial Growth and Bacteria Cells

## 1. Introduction

### 1.1. Microbial Growth

A microbial culture is a method of multiply microbial organism by letting them reproduce in predetermined culture medium under controlled conditions. Bacteria Culture's

growth measuring rate can inform Scientists of their Physiological and Metabolic functions, which is also useful in obtaining an accurate cell number of the Bacteria for downstream applications. [7]

Microbial Growth also refer to as Bacteria Growth has been

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studying extensively. It is a common phenomena that Bacteria cells produces asexually. It's engage in a process called binary fission which is single cells splits into two equally sized cells. It has been determined in the laboratory that if provided with the right conditions, can grow very rapidly depending on the situation. Bacteria can grow in a predictable pattern resulting in a growth curve of 4 phases, the lag phase, the exponential phase or log phase, the stationary phase and the death phase.

While growth for multicellular organisms is typically measured in terms of the increase in size of a single organism, Microbial or Bacteria growth is measured by the increase in population, either by measuring the increase in cell number or the increase in overall mass. [9, 14, 15]

### 1.2. Gompertz Function

According to Laird (1960), [4] A Gompertz Function, is a sigmoid function. It is a type of mathematical model for a time series, where growth is slowest at the beginning and end at a time period. The future value asymptote of the function is approach much more gradually by thr curve. In contrast to the simple Logistic function in which both asymptote are approach by the curve symmetrically.

The general formular for a Gompertz function is

$$y(t) = ae^{-be^{-ct}} \tag{1}$$

Where a is an asymptote, since  $ae^{be^{-\infty}} = ae^0 = a$

b, c are positive numbers

b sets the displacement along the x axis

c sets the growth rate (y scalling)

e is Euler's number  $e = 2.71828$

The Growth equation proposed which form the basic equation applied in this research is:

$$\frac{dB_N}{dt} = rB_N \ln\left(\frac{K}{B_N}\right) \tag{2}$$

where

$B_N = B_N(t)$  is the population of Bacteria cells.

r is the constant intrinsic growth of cells, with  $r > 0$

K is the carrying capacity of the growth, that is, the maximum size that it can achieve with the available nutrients. Based on the findings, Gompertz generated the following facts that the carrying capacity, K, of a Tumour like Bacteria be intimately related with quantity of growth cells,  $B_N(t)$  and the value assigned is  $10^{13}$  cells, the rate  $r = 0.0060$ , and  $N(0) = 10^9$  where these values was used for Gompertz equation. [4]

Consider Bacteria which reproduce asexually. Its most commonly engage in a process known as binary fission, where a single cell splits into two equally sized cells. It's however experimented and deduce that the entire process or cycle can take as little as 20 Minutes for an active culture. [12]

### 1.3. Growth of Bacteria Cells

Laird for the first time successfully used the Gompertz curve to fit data of growth, growth in a cellular populations growing in a confined space where the availability of nutrients is limited. Therefore, this growth can be likened with that of Bacteria growth. [3, 10, 11]

Denoting the Bacteria cell size as  $B_N(t)$  it is useful to write the Gompertz Curve as follows:

$$B_N(t) = K \exp\left(\log\left(\frac{B_N(0)}{K}\right) \exp(-at)\right) \tag{3}$$

where:

$B_N(0)$  is the Bacteria cells size at the starting observation time;

K is the carrying capacity, i.e. the maximum size that can be reached with the available nutrients. In fact it is:

$$\lim_{t \rightarrow +\infty} B_N(t) = K \tag{4}$$

independently on  $B_N(0) > 0$ . Note that, in absence of therapies, usually it is  $B_N(0) < K$ , whereas, in presence of therapies, it may be  $B_N(0) > K$ ;

$\alpha$  is a constant related to the proliferative ability of the cells.

Log () refers to the natural log.

It is easy to verify that the dynamics of  $B_N(t)$  is governed by the Gompertz differential equation:

$$B_N'(t) = \alpha \log\left(\frac{K}{B_N(t)}\right) B_N(t) \tag{5}$$

i.e. is of the form:

$$B_N'(t) = F(B_N(t))B_N(t), F'(B_N) \leq 0 \tag{6}$$

where  $F(B_N)$  is the instantaneous proliferation rate of the cellular population, whose decreasing nature is due to the competition for the nutrients due to the increase of the cellular population, similarly to the logistic growth rate. However, there is a fundamental difference: in the logistic case the proliferation rate for small cellular population is finite:

$$F(B_N) = \alpha \left(1 - \left(\frac{B_N}{K}\right)^v\right) \Rightarrow F(0) = \alpha < +\infty \tag{7}$$

where as in the Gompertz case the proliferation rate is unbounded:

## 2. Derivatives of the Equation

### 2.1. Representation of Interpolating Function

Let us assume that the theoretical solution  $y(x)$  to the initial value problem

$$w' = f(x, w), \quad w(x_0) = w_0 \quad (8)$$

can be locally represented in the interval  $[x_n, x_{n+1}]$ ,  $n \geq 0$ . Considering eq (1) and (3) as bases, we developed the non-polynomial interpolating function with transcendental function;

$$\frac{F(x,w)}{K} = \alpha_1 e^{\beta x} + \alpha_2 B^x + \alpha_3 \cos x \quad (9)$$

where  $\alpha_1, \alpha_2, \alpha_3$  are real undetermined coefficients,  $\beta$  and  $B$  are the shape and scale parameters,  $K$  represent the saturation level using Gompertz approach. The intervals defined are  $x \in [0,1]$  and  $k \in (0,1]$ . [8, 13]

Let's shall assume  $w_n$  is a numerical estimate to the theoretical solution  $y(x)$  and  $f_n = f(x_n, w_n)$ , and define mesh points as follows

$$x_n = a + nh, n = 0, 1, 2, \dots \quad (10)$$

then, impose the following constraints on the interpolating function (10) in order to get the undetermined coefficients, [2, 5, 6]

Hence we required that

$$F(x_n, w_n) = K(\alpha_1 e^{\beta x_n} + \alpha_2 B^{x_n} + \alpha_3 \cos x_n)$$

and

$$F(x_{n+1}, w_{n+1}) = K(\alpha_1 e^{\beta x_{n+1}} + \alpha_2 B^{x_{n+1}} + \alpha_3 \cos x_{n+1}) \quad (11)$$

## 2.2. Derivatives of the Ordinary Differential Equation

The derivatives of the interpolating function (9) are required to coincide with the differential equation as well as its first, second, and third derivatives with respect to  $x$  at  $x = x_n$  and  $x = x_{n+1}$

We denote the  $i$ -th total derivatives of  $f(x, w)$  with respect to  $x$  with  $f^{(i)}$  such that

$$F^1(x_n) = f_n, F^2(x_n) = f_n^1, F^3(x_n) = f_n^2, \quad (12)$$

This implies that,

$$f_n = k\alpha_1 \beta e^{\beta x_n} + k\alpha_2 B^{x_n} \log B - k\alpha_3 \sin x_n \quad (13)$$

$$f_n^1 = k\alpha_1 \beta^2 e^{\beta x_n} + k\alpha_2 B^{x_n} (\log B)^2 - k\alpha_3 \cos x_n \quad (14)$$

$$f_n^2 = k\alpha_1 \beta^3 e^{\beta x_n} + k\alpha_2 B^{x_n} (\log B)^3 + k\alpha_3 \sin x_n \quad (15)$$

Solving for  $\alpha_1, \alpha_2$ , and  $\alpha_3$ , from (13) to (15) these form a system of linear equation which can be solved using Cramer's rule and Marple software.

$$\begin{pmatrix} K\beta e^{\beta x_n} & KB^{x_n} \log B & -K \sin x_n \\ K\beta^2 e^{\beta x_n} & KB^{x_n} (\log B)^2 & -K \cos x_n \\ K\beta^3 e^{\beta x_n} & KB^{x_n} (\log B)^3 & K \sin x_n \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} f_n \\ f_n^1 \\ f_n^2 \end{pmatrix} \quad (16)$$

When taking (16) as a system of equations, according to [8],  $AX = B$ , it gives

$$\alpha_1 = \frac{f_n (\log B)^2 \sin x_n + (\log B)^3 \cos x_n - (\log B) (f_n^1 \sin x_n + f_n^2 \cos x_n)}{K\beta e^{\beta x_n} ((\log B)^2 \sin x_n + (\log B)^3 \cos x_n) - K \log B e^{\beta x_n} (\beta^2 \sin x_n + \beta^3 \cos x_n) - K \sin x_n e^{\beta x_n} (\beta^2 (\log B)^3 - \beta^3 (\log B)^2)} \quad (17)$$

$$\alpha_2 = \frac{\beta (f_n^1 \sin x_n + f_n^2 \cos x_n) - f_n (\beta^2 \sin x_n + \beta^3 \cos x_n) - \sin x_n (\beta^2 f_n^2 - \beta^3 f_n^1)}{K\beta B^{x_n} ((\log B)^2 \sin x_n + (\log B)^3 \cos x_n) - KB^{x_n} \log B (\beta^2 \sin x_n + \beta^3 \cos x_n) - KB^{x_n} \sin x_n (\beta^2 (\log B)^3 - \beta^3 (\log B)^2)} \quad (18)$$

$$\alpha_3 = \frac{\beta (\log B^2 f_n^2 - \log B^3 f_n^1) - \log B (\beta^2 f_n^2 - \beta^3 f_n^1) + f_n (\beta^2 \log B^3 - \beta^3 \log B^2)}{K\beta B^{x_n} ((\log B)^2 \sin x_n + (\log B)^3 \cos x_n) - KB^{x_n} \log B (\beta^2 \sin x_n + \beta^3 \cos x_n) - KB^{x_n} \sin x_n (\beta^2 (\log B)^3 - \beta^3 (\log B)^2)} \quad (19)$$

## 3. Formation of Numerical Integration

Since  $F(x_{n+1}) = w(x_{n+1})$  and  $F(x_n) = w(x_n)$

$$\text{Implies that } w(x_{n+1}) = w_{n+1} \text{ and } w(x_n) = w_n \quad (20)$$

$$F(x_{n+1}) - F(x_n) = w_{n+1} - w_n$$

Therefore, from (20)

$$w_{n+1} - w_n = K(\alpha_1 e^{\beta x_{n+1}} + \alpha_2 B^{x_{n+1}} + \alpha_3 \cos x_{n+1}) - K(\alpha_1 e^{\beta x_n} + \alpha_2 B^{x_n} + \alpha_3 \cos x_n) \quad (21)$$

$$= K\alpha_1 [e^{\beta x_{n+1}} - e^{\beta x_n}] - K\alpha_2 [B^{x_{n+1}} - B^{x_n}] + K\alpha_3 [\cos x_{n+1} - \cos x_n] \quad (22)$$

$$\text{Recall [1], that } x_n = a + nh, x_{n+1} = a + (n + 1)h \text{ with } n = 0,1,2 \dots \tag{23}$$

by expansion

$$w_{n+1} - w_n = K\alpha_1 e^{\beta x_n} (e^{\beta h} - 1) - K\alpha_2 B^{x_n} (B^h - 1) + K\alpha_3 \cos(x_n + h) - \cos x_n \tag{24}$$

Substituting for  $\alpha_1, \alpha_2$ , and  $\alpha_3$  in (24), we have

$$w_{n+1} = w_n + P + Q + R \tag{25}$$

where

$$P = \frac{(e^{\beta h} - 1)[(f_n(\log B)^2 - f_n^1 \log B - f_n^1 (\log B)^3 + f_n^2 (\log B)^2) \sin x_n + (f_n(\log B)^3 - f_n^2 \log B) \cos x_n]}{(\beta(\log B)^2 - \beta^2 \log B - \beta^2 (\log B)^3 + \beta^3 (\log B)^2) \sin x_n + (\beta(\log B)^3 - \beta^3 \log B) \cos x_n}$$

$$Q = \frac{(B^h - 1)[(\beta f_n^1 - \beta^2 f_n - \beta^2 f_n^2 + \beta^3 f_n^1) \sin x_n + (\beta f_n^2 - \beta^3 f_n) \cos x_n]}{(\beta(\log B)^2 - \beta^2 \log B - \beta^2 (\log B)^3 + \beta^3 (\log B)^2) \sin x_n + (\beta(\log B)^3 - \beta^3 \log B) \cos x_n}$$

$$R = \frac{[\cos(x_n + h) - \cos x_n][\beta f_n^2 \log B^2 - \beta^2 f_n^2 \log B - \beta^2 f_n \log B^3 - \beta f_n^1 \log B^3 + \beta^3 f_n^1 \log B + \beta^3 f_n \log B^2]}{(\beta(\log B)^2 - \beta^2 \log B - \beta^2 (\log B)^3 + \beta^3 (\log B)^2) \sin x_n + (\beta(\log B)^3 - \beta^3 \log B) \cos x_n}$$

Equation (25) is the new numerical integration for solution of the first order differential equation.

$$\frac{dB_N}{dt} - KB_N = 0 \tag{26}$$

### 4. Implementation of the Integration (25) to Solve Microbial Growth Problem

**Problem 1:** A Bacteria Culture is taken and known to grow at a rate proportional to the amount present. After one hour, 1000 strands of the bacteria are observed in the culture, and after three hours, 2100 strands were observed. Find (i)

An expression for the approximate number of strands of the bacteria present in the culture at any time  $x$

(ii) The approximate number of strands of the bacteria originally in the culture. [1]

**MATHEMATICAL INTERPRETATION OF THE PROBLEM:**

Let  $B_N(x)$  denote the number of bacteria strands in the culture at time ( $x$ ).

Hence, using the general model

which is both linear and separable.

Therefore when solved,  $B_N(x) = Ce^{kx}$

At  $x = 1, B_N = 1000$ , hence  $1000 = Ce^{k \cdot 1}$ .  $B_N(1) = 1000$

At  $x = 3, B_N = 2100$ , hence  $2100 = Ce^{3k}$ .  $B_N(3) = 2100$

Solving the two,

$$K = 0.371 \text{ and } C = 690$$

Hence,

$B_N(x) = 690e^{0.371x}$ , is an expression for the amount of bacteria present at any time  $x$ , this can be seen in the table below:

**Table 1.** The approximated value of strands at time  $x = 1$  (hour),  $B_N = 1000$ .

XN	EXACT VALUE	NUMERICAL VALUE	ABSOLUTE ERROR
[0.00]	[6.9000000000000000e+02]	[6.9000000000000000e+02]	[0.0000000000000000]
[0.10]	[7.160778840233803e+02]	[7.160797887769264e+02]	[0.001904753546114]
[0.20]	[7.431416518597050e+02]	[7.431453099924750e+02]	[0.003658132769942]
[0.30]	[7.712285674083900e+02]	[7.712338211738768e+02]	[0.005253765486827]
[0.40]	[8.003772966540263e+02]	[8.003839880634949e+02]	[0.006691409468658]
[0.50]	[8.306279623334798e+02]	[8.306359378448428e+02]	[0.007975511363043]
[0.60]	[8.620222005488230e+02]	[8.620313143803757e+02]	[0.009113831552668]

XN	EXACT VALUE	NUMERICAL VALUE	ABSOLUTE ERROR
[0.70]	[8.946032192663178e+02]	[8.946133355370953e+02]	[0.010116270777530]
[0.80]	[9.284158587292990e+02]	[9.284268526788766e+02]	[0.010993949577596]
[0.90]	[9.635066538705012e+02]	[9.635184124074153e+02]	[0.011758536914158]
[1.00]	[9.999238988403082e+02]	[9.999363206367847e+02]	[0.012421796476474]

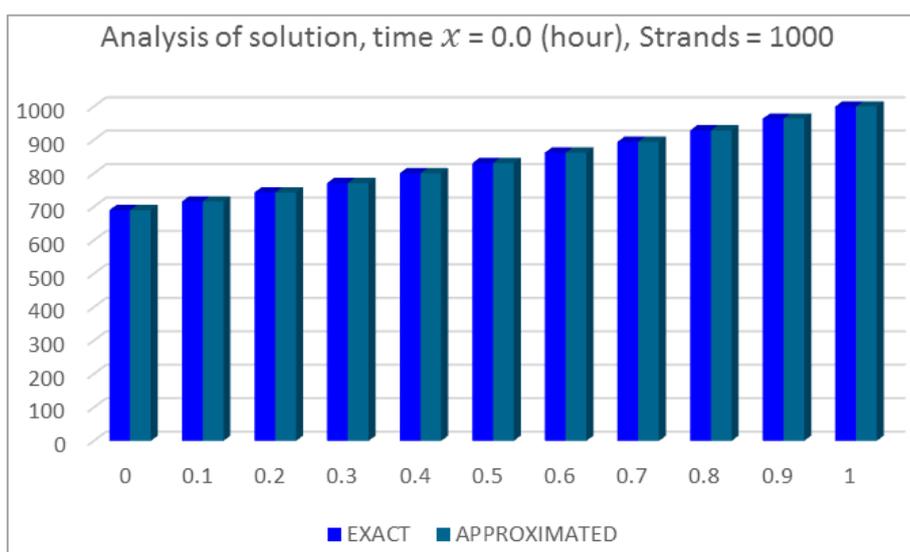


Figure 1. The Bar graph analysis of Exact and Numerical Solutions of time (1hour) and 1000 Strands.

Table 2. The approximated value of strands at time  $x = 3$  (hour),  $B_N = 2100$ .

XN	EXACT VALUE	NUMERICAL VALUE	ABSOLUTE ERROR
[1.00]	[9.999238988403082e+02]	[9.999363206367847e+02]	[0.012421796476474]
[1.10]	[1.037717713779482e+03]	[1.037730709089806e+03]	[0.012995310323049]
[1.20]	[1.076940113965789e+03]	[1.076953604307765e+03]	[0.013490341976649]
[1.30]	[1.117645081458867e+03]	[1.117658999268484e+03]	[0.013917809617396]
[1.40]	[1.159888639359365e+03]	[1.159902927711315e+03]	[0.014288351950654]
[1.50]	[1.203728928786705e+03]	[1.203743541271387e+03]	[0.014612484682402]
[1.60]	[1.249226288661646e+03]	[1.249241189529282e+03]	[0.014900867635333]
[1.70]	[1.296443338345589e+03]	[1.296458503086325e+03]	[0.015164740735827]
[1.80]	[1.345445063113798e+03]	[1.345460479779864e+03]	[0.015416666066358]
[1.90]	[1.396298902254159e+03]	[1.396314574157198e+03]	[0.015671903039674]
[2.00]	[1.449074839057345e+03]	[1.449090790331349e+03]	[0.015951274004237]
[2.10]	[1.503845490193300e+03]	[1.503861778346467e+03]	[0.016288153166215]
[2.20]	[1.560686184267167e+03]	[1.560702934185552e+03]	[0.016749918384903]
[2.30]	[1.619674967396317e+03]	[1.619692503558144e+03]	[0.017536161826911]
[2.40]	[1.680891364439784e+03]	[1.680911689610851e+03]	[0.020325171067043]
[2.50]	[1.744426237563714e+03]	[1.744444764708992e+03]	[0.018527145278085]

XN	EXACT VALUE	NUMERICAL VALUE	ABSOLUTE ERROR
[2.60]	[1.810361320250889e+03]	[1.810379186443232e+03]	[0.017866192343490]
[2.70]	[1.878788232127490e+03]	[1.878805718020890e+03]	[0.017485893399225]
[2.80]	[1.949801305920745e+03]	[1.949818553207653e+03]	[0.017247286907832]
[2.90]	[2.023498339839777e+03]	[2.023515445991693e+03]	[0.017106151916323]
[3.00]	[2.099980799476793e+03]	[2.099997845148666e+03]	[0.017045671873348]

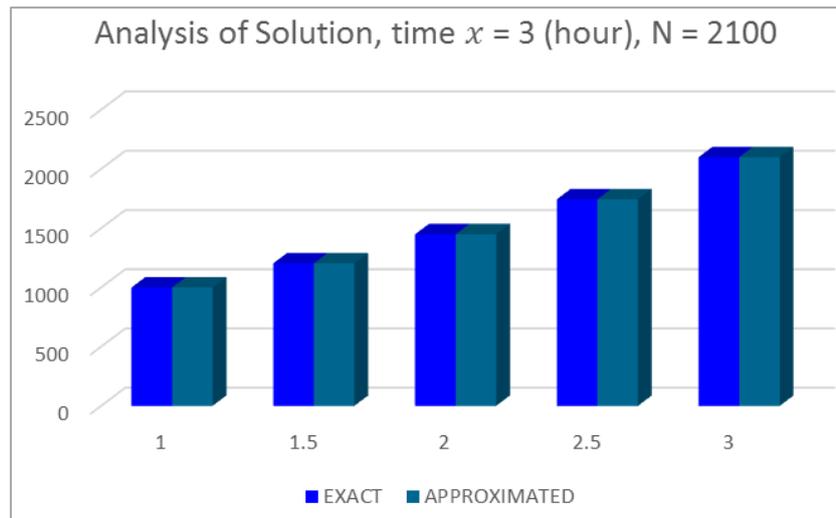


Figure 2. The Bar graph analysis of Exact and Numerical Solutions of time (3hour) and 2100 Strands.

b. We require  $B_N(x)$  at  $x = 0$ .

Substituting  $x = 0$  into  $B_N(x) = 690e^{0.371x} = 690$ .

This can be seen in Table 1 above where  $x = 0, B_N(0) = 690$ .

**Problem 2:** After two and half hour, Bacteria Count is taken and known to grow at a rate proportional to the amount present. 1500 strands of the bacteria are observed in the culture, and after five and half hours, 3500 strands were observed. Find

An expression for the approximate number of strands of the bacteria present in the culture at any time  $x$ .

The approximate number of strands of the bacteria originally in the culture.

**MATHEMATICAL INTERPRETATION:**

Let  $B_N(x)$  denote the number of bacteria strands in the culture at time ( $x$ ).

Hence, consider

$$\frac{dB_N}{dt} - KB_N = 0$$

which is both linear and separable

At  $x = 2.5, B_N = 1500$ , hence  $1500 = Ce^{2.5k}$ .  $B_N(2.5) = 1500$

At  $x = 5.5, B_N = 3500$ , hence  $3500 = Ce^{5.5k}$ .  $B_N(5.5) = 3500$

Solving the two,

$$K = 1/3 \ln(7/3) = 0.2824 \text{ and } C = 740$$

Hence,

(a (i))  $B_N(x) = 740e^{0.2824x}$ , is an expression for the amount of bacteria present at any time t, this can be seen in the table below:

Table 3. The approximated value of strands at time  $x = 2.5$  (hour),  $B_N = 1500$ .

XN	NUMERICAL VALUE	EXACT VALUE	ABSOLUTE ERROR
[0.00]	[7.400000000000000e+02]	[7.400000000000000e+02]	[0.000000000000000]
[0.10]	[7.611940193364643e+02]	[7.611954714643300e+02]	[0.001452127865718]

XN	NUMERICAL VALUE	EXACT VALUE	ABSOLUTE ERROR
[0.20]	[7.829952571355008e+02]	[7.829980348348699e+02]	[0.002777699369176]
[0.30]	[8.054211076484687e+02]	[8.054250787591524e+02]	[0.003971110683665]
[0.40]	[8.284894584619219e+02]	[8.284944899395517e+02]	[0.005031477629814]
[0.50]	[8.522187059356297e+02]	[8.522246673988334e+02]	[0.005961463203676]
[0.60]	[8.766277709962579e+02]	[8.766345371543065e+02]	[0.006766158048549]
[0.70]	[9.017361151942914e+02]	[9.017435673122800e+02]	[0.007452117988578]
[0.80]	[9.275637569999386e+02]	[9.275717835948619e+02]	[0.008026594923308]
[0.90]	[9.541312883582614e+02]	[9.541397853114870e+02]	[0.008496953225631]
[1.00]	[9.814598915475674e+02]	[9.814687617879083e+02]	[0.008870240340912]
[1.10]	[1.009571356394995e+03]	[1.009580509265756e+03]	[0.009152870760431]
[1.20]	[1.038488097906053e+03]	[1.038497448286142e+03]	[0.009350380089018]
[1.30]	[1.068233174366297e+03]	[1.068242641571181e+03]	[0.009467204883777]
[1.40]	[1.098830305978242e+03]	[1.098839812417671e+03]	[0.009506439429060]
[1.50]	[1.130303894110867e+03]	[1.130313363617628e+03]	[0.009469506761207]
[1.60]	[1.162679041272865e+03]	[1.162688396920748e+03]	[0.009355647882330]
[1.70]	[1.195981571996938e+03]	[1.195990733054317e+03]	[0.009161057378833]
[1.80]	[1.230238054998969e+03]	[1.230246932316555e+03]	[0.008877317586212]
[1.90]	[1.265475827418515e+03]	[1.265484315759792e+03]	[0.008488341277143]
[2.00]	[1.301723023199130e+03]	[1.301730986980392e+03]	[0.007963781261424]
[2.10]	[1.339008611873735e+03]	[1.339015854532793e+03]	[0.007242659057738]
[2.20]	[1.377362472291458e+03]	[1.377368654985544e+03]	[0.006182694086192]
[2.30]	[1.416815649015988e+03]	[1.416819976637718e+03]	[0.004327621729999]
[2.40]	[1.457403569206754e+03]	[1.457401283914634e+03]	[0.002285292120177]
[2.50]	[1.499142956754331e+03]	[1.499144942462323e+03]	[0.001985707992390]
[2.60]	[1.542080660279232e+03]	[1.542084244960776e+03]	[0.003584681543998]

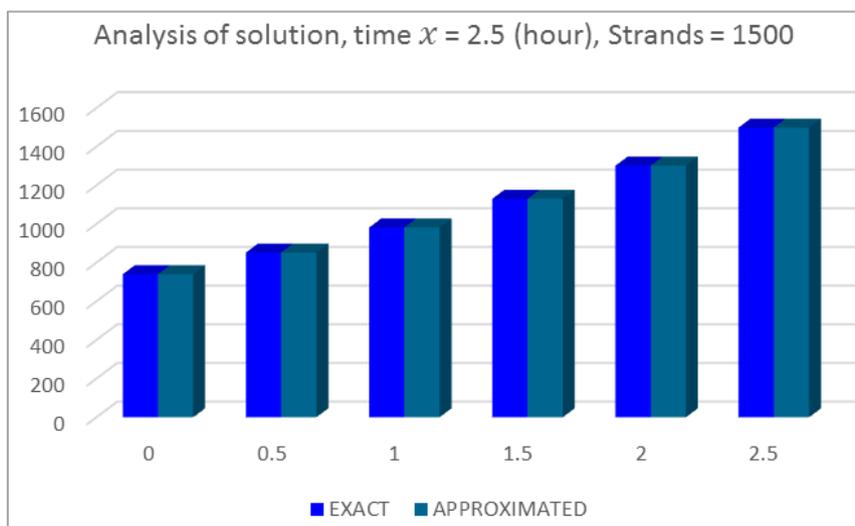


Figure 3. The Bar graph analysis of Exact and Numerical Solutions of time (2.5hour) and 1500 Strands.

a (ii) The approximate number of strands, expression for the amount of bacteria present at time  $x = 5.5$  (hour),  $B_N = 3500$ .

**Table 4.** The approximated value of strands at time  $x = 5.5$  (hour),  $B_N = 3500$ .

XN	NUMERICAL VALUE	EXACT VALUE	ABSOLUTE ERROR
[2.50]	[1.499142956754331e+03]	[1.499144942462323e+03]	[0.001985707992390]
[2.60]	[1.542080660279232e+03]	[1.542084244960776e+03]	[0.003584681543998]
[2.70]	[1.586248878558372e+03]	[1.586253437676532e+03]	[0.004559118159932]
[2.80]	[1.631682492046569e+03]	[1.631687747775814e+03]	[0.005255729244254]
[2.90]	[1.678417614250424e+03]	[1.678423411419974e+03]	[0.005797169550533]
[3.00]	[1.726491460768530e+03]	[1.726497702665671e+03]	[0.006241897141308]
[3.10]	[1.775942340117185e+03]	[1.775948963192809e+03]	[0.006623075623565]
[3.20]	[1.826809670957244e+03]	[1.826816632883971e+03]	[0.006961926727172]
[3.30]	[1.879134007933489e+03]	[1.879141281279721e+03]	[0.007273346232523]
[3.40]	[1.932957071317150e+03]	[1.932964639934869e+03]	[0.007568617719244]
[3.50]	[1.988321778832438e+03]	[1.988329635701488e+03]	[0.007856869050784]
[3.60]	[2.045272279039605e+03]	[2.045280424965263e+03]	[0.008145925657345]
[3.70]	[2.103853986009496e+03]	[2.103862428862431e+03]	[0.008442852934422]
[3.80]	[2.164113615173862e+03]	[2.164122369505444e+03]	[0.008754331581258]
[3.90]	[2.226099220303008e+03]	[2.226108307246215e+03]	[0.009086943206967]
[4.00]	[2.289860231594508e+03]	[2.289869679006682e+03]	[0.009447412173813]
[4.10]	[2.355447494872385e+03]	[2.355457337707251e+03]	[0.009842834866049]
[4.20]	[2.422913311902861e+03]	[2.422923592824576e+03]	[0.010280921714639]
[4.30]	[2.492311481833677e+03]	[2.492322252111015e+03]	[0.010770277337997]
[4.40]	[2.563697343759523e+03]	[2.563708664509035e+03]	[0.011320749511924]
[4.50]	[2.637127820405147e+03]	[2.637139764294787e+03]	[0.011943889640406]
[4.60]	[2.712661462896397e+03]	[2.712674116486085e+03]	[0.012653589688398]
[4.70]	[2.790358496550046e+03]	[2.790371963550961e+03]	[0.013467000914716]
[4.80]	[2.870280867539193e+03]	[2.870295273454084e+03]	[0.014405914890631]
[4.90]	[2.952492290146399e+03]	[2.952507789079351e+03]	[0.015498932951232]
[5.00]	[3.037058294018004e+03]	[3.037075079068058e+03]	[0.016785050053841]
[5.10]	[3.124046270169809e+03]	[3.124064590113213e+03]	[0.018319943404094]
[5.20]	[3.213525512879304e+03]	[3.213545700751685e+03]	[0.020187872380575]
[5.30]	[3.305567250164846e+03]	[3.305589776697094e+03]	[0.022526532247866]
[5.40]	[3.400244641220348e+03]	[3.400270227757587e+03]	[0.025586537239178]
[5.50]	[3.497632660495767e+03]	[3.497662566383866e+03]	[0.029905888099620]
[5.60]	[3.597807433386859e+03]	[3.597844467894197e+03]	[0.037034507338376]
[5.70]	[3.700839388350303e+03]	[3.700895832424399e+03]	[0.056444074095907]
[5.80]	[3.806869796429933e+03]	[3.806898848652250e+03]	[0.029052222316750]
[5.90]	[3.915916951661997e+03]	[3.915938059347115e+03]	[0.021107685117386]

XN	NUMERICAL VALUE	EXACT VALUE	ABSOLUTE ERROR
[6.00]	[4.028083883464806e+03]	[4.028100428797081e+03]	[0.016545332275200]

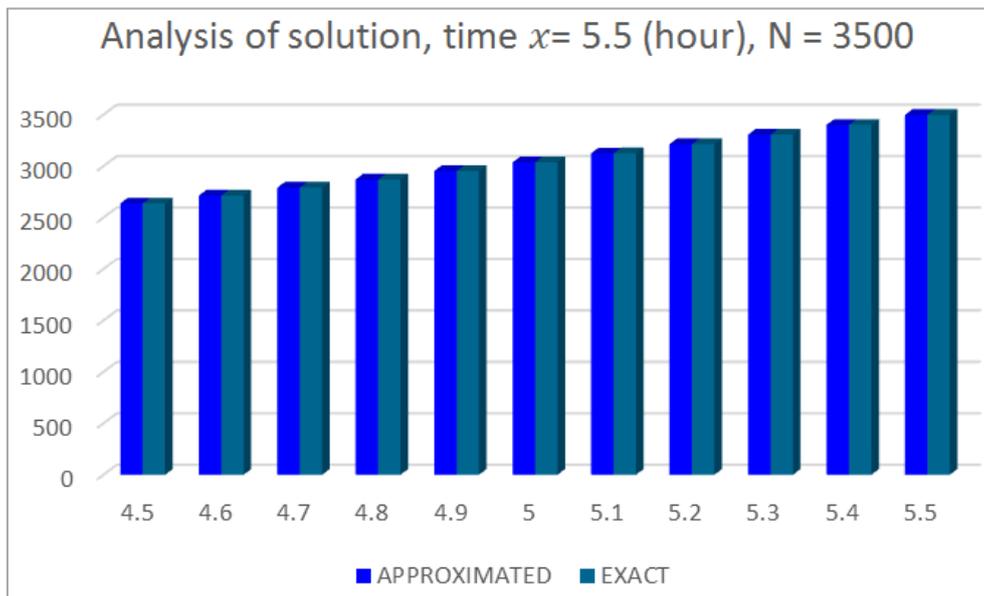


Figure 4. The Bar graph analysis of Exact and Numerical Solutions of time (5.5hour) and 3500 Strands.

b. The approximate number of strands of the bacteria originally in the culture

We require  $B_N(x)$  at  $x = 0$ .

Substituting  $x = 0$  into  $B_N(x) = 740e^{0.2824x}$

where  $x = 0, B_N(0) = 740$ .

Table 5. The approximated value of strands at time  $x = 0$  (hour),  $B_N = 740$ .

XN	NUMERICAL VALUE	EXACT VALUE	ABSOLUTE ERROR
[0.00]	[7.400000000000000e+02]	[7.400000000000000e+02]	[0.000000000000000]
[0.10]	[7.611940193364643e+02]	[7.611954714643300e+02]	[0.001452127865718]
[0.20]	[7.829952571355008e+02]	[7.829980348348699e+02]	[0.002777699369176]
[0.30]	[8.054211076484687e+02]	[8.054250787591524e+02]	[0.003971110683665]
[0.40]	[8.284894584619219e+02]	[8.284944899395517e+02]	[0.005031477629814]
[0.50]	[8.522187059356297e+02]	[8.522246673988334e+02]	[0.005961463203676]
[0.60]	[8.766277709962579e+02]	[8.766345371543065e+02]	[0.006766158048549]
[0.70]	[9.017361151942914e+02]	[9.017435673122800e+02]	[0.007452117988578]
[0.80]	[9.275637569999386e+02]	[9.275717835948619e+02]	[0.008026594923308]
[0.90]	[9.541312883582614e+02]	[9.541397853114870e+02]	[0.008496953225631]
[1.00]	[9.814598915475674e+02]	[9.814687617879083e+02]	[0.008870240340912]

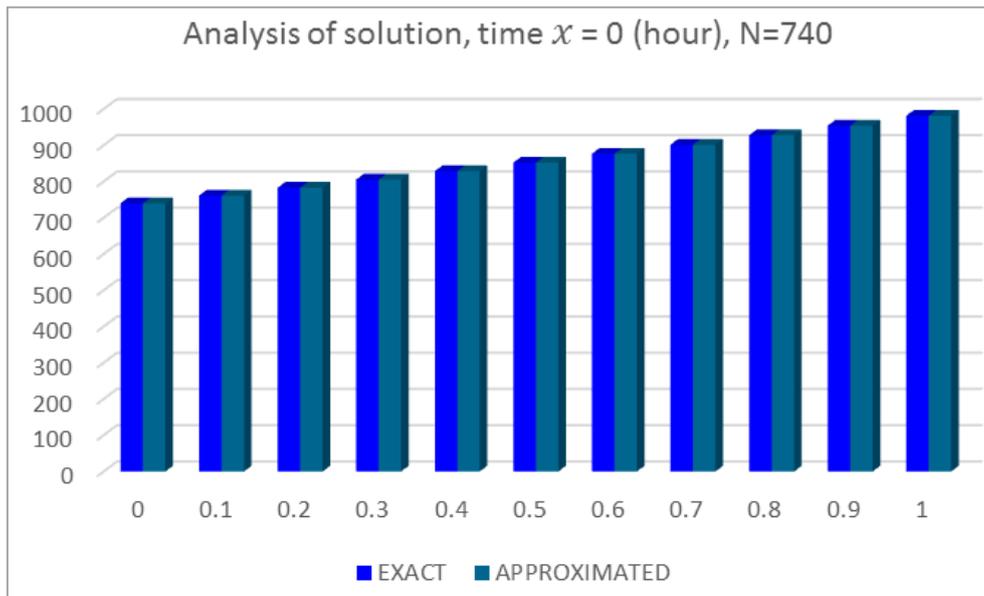


Figure 5. The Bar graph analysis of Exact and Numerical Solutions of time (0.0 hour) and 740 Strands.

## 5. Discussion

It is of note that Numerical methods are to give approximated values, hence the efficiency of the method is shown in the tables such that the approximated values of the bacteria in the colony at the given period is approximated equal to the exact values.

The figures in Table 1, indicates the approximated value of strands  $B_N = 1000$ , at time  $x = 1$  (hour). However, from the table, the approximated value of strands at 1 hour time is  $999.99 \approx 1000$ .

The figures in Table 2 is the approximated value of strands  $B_N = 2100$  at time  $x = 3$  (hour), Which from the table the values gotten is  $2099.981 \approx 2100$ .

The figures in Table 3 is the approximated value of strands  $B_N = 1500$  at time  $x = 2.5$  (hour). The last figure in the table showed  $1499.95 \approx 1500$ .

Similarly, the figures in Table 4 gives the approximated value of strands  $B_N = 3500$  at time  $x = 5.5$  (hour). From the last figure of time  $x = 5.5$  hour, the approximated value obtained is  $3497.63 \approx 3500$ .

## 6. Conclusion

Looking at the figures in the tables, clearly it showed the level of convergency which could be observed from the numerical solution as it tends to the exact solution indicated in the bar graphs at any point in time, hence we have a robust numerical method which could favourably performed effectively. The errors obtained are significantly tends to zero which shows the zero stability of the method. The method can still be applied further to solve bacteria growth problem as

identified by many scholars solving analytically. Therefore, numerical integration for solving microbial growth problem using gompertz function approach was developed and implemented.

## Abbreviations

$B_N = B_N(t)$	The Population of Bacteria Cells
$r$	The Constant Intrinsic Growth of Cells, with $r > 0$
$K$	The Carrying Capacity of the Growth, That is, the Maximum Size That It Can Achieve with the Available Nutrients

## Conflicts of Interest

The authors declare no conflicts of interest.

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