



On a Couette Flow of Conducting Fluid

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Abstract: We discuss the thermal radiation effect on unsteady free-convective Couette flow of conducting fluid in the presence of transverse magnetic field. The mathematical model is highly nonlinear due to the effect of the thermal radiation. Both numerical solution by the finite difference method and the analytical solution of the steady state by the perturbation method are presented. The numerical solution at large time agrees with the analytic solution of the steady state.

Keywords: Transverse Magnetic Field, Thermal Radiation, Couette Flow

1. Introduction

Amount of efforts have been instilled in the study of unsteady laminar free convection phenomenon in a vertical channels owing to its importance to chemical, biomedical, and environmental engineering and sciences. The interest in this field relates to its great practical importance to variety of applications. For example, nuclear reactors, solid matrix heat exchangers, thermal insulation, surface catalysis of chemical reactions, oil recovery, dispersion of chemical contaminants in various processes, storage of nuclear waste materials, grain storage and drying and many others Jha *et al.* [13].

A number of studies have been performed to latch on to the transport mechanism of momentum and heat transfer in vertical channels. Indeed, several authors have surveyed thermal radiation effect on Magneto hydrodynamic free-convection fluids in conjunction with other flow parameters [1, 4, 5, 7, 17, 19, 27]. The literatures on unsteady free convective fluid associated with Couette flow are numerous. Singh [26] considered the motion of fluid induced by the impulsive motion of one of the plates in the presence of convection currents due to asymmetric heating condition on the boundaries. Jha [12] further extended it for MHD case. Fang [6] investigated the Couette flow of a viscous, incompressible fluid with porous walls. Muhuri [18] researched on the Couette flow where one of the porous plates moves with impulsive and accelerated motion.

Makinde and Maserumule [16] illustrated the thermal criticality and entropy analysis for a variable viscosity Couette flow. Jha [13, 12] examined the unsteady flow behavior of a natural convection Couette flow of viscous reactive fluid in a vertical channel.

Also steady and unsteady Couette flows associated with magneto hydrodynamics have been subject of interest due to its applications ranging from desalination and viscos metric analysis. It also has application in many devices such as power generators, pumps accelerators, heating electrostatics, precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and droplets-sprays Hazem [10]. Dash and Biswall [3] reported Couette flow of an incompressible viscoelastic fluid with a sudden motion on the lower wall with varying time where n is positive. Hayat *et al.* [9] studied the motion of a conducting Oldroyd-B fluid between two parallel plates where the lower plate is at rest while the upper oscillating in its own plane. Govindarajulu [8] have presented Couette-type flows in hydromagnetic with time-dependent suction and the fluid assumed to be viscous, incompressible and electrically conducting where the walls are taken to be non-conducting. Salama [21] studied flow formation in Couette motion in magneto hydrodynamics with time varying suction and taking into account the effects of heat and mass transfer. Khem *et al.* [14] analyzed Couette motion of an electrically conducting, viscous in compressible fluid through saturated

porous medium bounded by two insulated vertical porous plates.

Sheikholeslami *et al.* [23] studied magnetic field effect on nanofluid flow and heat transfer is studied numerically using Control Volume based Finite Element Method. Sheikholeslami and Gorji-Bandpy [22] investigated the effect of external magnetic source on ferrofluid flow and heat transfer in an enclosure heated from below. Magneto hydrodynamic effect on natural convection heat transfer of Cu-water nanofluid in an enclosure with hot elliptic cylinder was also investigated in the work of Sheikholeslami *et al.* [24].

In this paper, we discuss the behavior of the unsteady free-convective Couette flow under the influence of the transverse magnetic field and the thermal radiation for a simple system consisting of two infinite vertical plates held at different temperature. We use the Roseland approximation to describe the radiative heat flux in the energy equation. We also present the numerical solution by the implicit finite difference method and the analytical solution of the steady state by the perturbation method for the governing time-dependent partial differential equations. We have found an excellent agreement between the numerical solution at the large time and the analytical solution of the steady state. Several graphs illustrating the effect of the various controlling parameters involved in the flow formation are presented.

2. Governing Equations

We consider a time dependent unsteady free convection Couette flow of a viscous, incompressible, electrically conducting and radiating fluid separated between two infinite vertical parallel plates of a distance H . At time $t' \leq 0$, both the fluid and plates are assumed to be at rest at temperature T_0 . At some time $t' > 0$, the temperature of the plate at $y' = 0$ rise to T_w and the plate starts moving on its own plane with impulsive motion with velocity U while the plate at a distance H from it is fixed. A strong homogeneous magnetic field of strength B_0 is imposed normal to the plates in the presence of an incident radiative heat flux of intensity q_r , which is absorbed by the plate and transferred to the fluid. The Cartesian (x', y') co-ordinate systems are taken with x' -axis along the moving plate in the upward direction and the y' -axis normal to it as shown in Figure 1. The plates are infinite in length, the velocity and temperature are functions of y' and t' alone. Using Boussinesq's

approximation, the governing equations for the present physical situation in dimensional form as:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_0) - \frac{\sigma_1 B_0^2 u'}{\rho} \quad (1)$$

$$\frac{\partial u'}{\partial t'} = \alpha \left[\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{K} \frac{\partial q_r}{\partial y'} \right] \quad (2)$$

The quantity q_r appearing on the right hand side of equation (2) represents the radiative heat flux in the x' -direction. The radiative heat flux in the x' -direction is considered insignificant in comparison with that in the y' -direction. The radiative heat flux term in the problem is simplified by using the Rosseland diffusion approximation for an optically thick fluid according to Rashad [20].

$$q_r = -\frac{4\sigma \partial T'^4}{3k^* \partial y'} \quad (3)$$

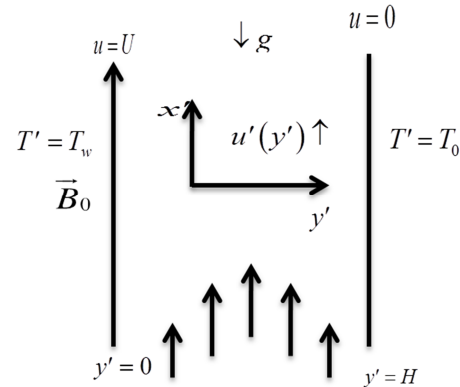


Figure 1. Schematic diagram of the problem.

This approximation is valid for intensive absorption, that is, for an optically thick boundary layer. Hence, the Rosseland approximation has been used with positive result in a variety of problems ranging from the transport of radiation through gases at low density to the study of the effects of radiation on blast waves by nuclear explosion Ali *et al.* [2].

The relevant initial and boundary conditions are:

$$\begin{aligned} t' \leq 0: & \{ u' = 0, \quad T' = T_0 \quad 0 \leq y' \leq H \\ t' > 0: & \begin{cases} u' = U, \quad T' = T_w & \text{at } y' = 0 \\ u' = 0, \quad T' = T_0 & \text{at } y' = H \end{cases} \end{aligned} \quad (4)$$

Table 1. Nomenclature.

Symbol	Description	Symbol	Description
H	Distance between two parallel plates	t'	Dimensional time
C_T	Temperature difference parameter	t	Dimensionless time
g	Acceleration due to gravity	T'	Dimensional temperature of the fluid
B_0	Applied magnetic field	T_0	Initial temperature of fluid and plate ($t' = 0$)
Nu_0	Nusselt number at $y = 0$	T_w	Temperature of the plat $y' = 0$ at $t' > 0$

Symbol	Description	Symbol	Description
Nu_1	Nusselt number at $y = 1$	u'	Dimensional velocities
M	Magnetic number	u	Dimensionless velocities
Gr	Grashof number	U	Velocity of the plate at $y' = 0$
Pr	Prandtl number	x'	Vertical co-ordinate, direction of the fluid
R	Thermal radiation parameter	y'	Dimensional co-ordinate perpendicular to the plate
Greek Symbols			
Symbol	Description	Symbol	Description
β	Co-efficient of thermal expansion	α	Thermal diffusivity
κ^*	Mean n abs	σ	Stefan-Boltzmann constant
K	Thermal conductivity	θ	Dimensionless temperature
ν	Kinematic viscosity	ρ	Density of the fluid
τ_0	Dimensionless skin friction at $y = 0$	τ_1	Dimensionless skin friction at $y = 1$
σ_1	Fluid electrical conductivity		

To obtain the solution of equation (1) and (2) subject to the initial and boundary condition (4) in dimensionless form, the following appropriate dimensionless quantities are introduced.

$$\begin{cases} t = \nu t' / H^2, y = y' / H, u = u' / U, Pr = \nu / \alpha, R = 4\sigma(T_w - T_0) / \kappa^* K, \\ C_T = T_0 / (T_w - T_0), M^2 = \sigma_1 B_0^2 H^2 / \nu \rho, Gr = g \beta H^2 (T_w - T_0) / \nu U, \\ \theta = (T' - T_0) / (T_w - T_0) \end{cases} \quad (5)$$

Using the dimensionless quantities introduced in equation (5), the dimensionless form of equations (1) and (2) are: conditions for this special case can be written as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - M^2 u \quad (6)$$

$$Pr \frac{\partial \theta}{\partial t} = \left[1 + \frac{4R}{3} (C_T + \theta)^3 \right] \frac{\partial^2 \theta}{\partial y^2} + 4R [C_T + \theta]^2 \left(\frac{\partial \theta}{\partial y} \right)^2 \quad (7)$$

while the dimensionless initial and boundary conditions are

$$\begin{aligned} t \leq 0: & \begin{cases} u = 0, \theta = 1 & \text{at } 0 \leq y \end{cases} \\ t > 0: & \begin{cases} u = 1, \theta = 1, & \text{at } y = 0 \\ u = 0, \theta = 0, & \text{at } y = 1 \end{cases} \end{aligned} \quad (8)$$

3. Analytical Solution

The governing equations presented in the previous section are highly nonlinear and exhibited no analytical solutions due to thermal radiation effect. In general analytical solution are very useful in validating computer routines of complicated time dependent two or three-dimensional free convective and radiating conducting fluid and comparison with experimental data. It is therefore of interest to reduce the governing equations of the present problem to the form that can be solved analytically. A special case of the present problem that exhibit analytical solution is the problem of steady state MHD natural convection Couette flow trapped between two infinite vertical plates in the presence of thermal radiation effects. The resulting steady state equations and boundary

In order to obtain approximate analytical solutions of equation (9) and (10) subject to the boundary condition (3. 11), we employ a regular perturbation method by taking a power series expansion in the radiation parameter R such as:

$$\begin{aligned} \theta(y) &= \theta_0(y) + R\theta_1(y) + R^2\theta_2(y) + \dots = \sum_{j=0}^{\infty} R^j \theta_j(y) \\ u(y) &= u_0(y) + Ru_1(y) + R^2u_2(y) + \dots = \sum_{j=0}^{\infty} R^j u_j(y) \end{aligned} \quad (12)$$

where R is the radiation parameter (1). The second and high order terms of R give correction to θ_0 and u_0 that account for thermal radiation effect. Substituting equation (12) in (9) and (10) and equating the like power of R , one can obtain the required solution of the governing momentum and energy equations as:

$$u(y) = \frac{Gr(1-y)}{M^2} + \frac{[M^2 - Gr]}{M^2 \sinh(M)} \sinh(M - My) + R \left\{ \frac{\sinh(My)}{\sinh(M)} [K_1] + \frac{1}{M^2} [K_2 + K_3 y + K_4 y^2 + K_5 y^3 + K_6 y] \right\} \quad (13)$$

$$\theta(y) = 1 - y + R \left[2B^2 (y - y^2) - (4B/3)(y - y^3) + (1/3)(y - y^4) \right] \quad (14)$$

From (13), the steady-state skin frictions on the boundaries are:

$$\tau_0 = \left. \frac{du}{dy} \right|_{y=0} = \frac{[Gr - M^2] \cosh(M)}{M \sinh(M)} - \frac{Gr}{M^2} + R \left[-\frac{MK_1}{\sinh(M)} + \frac{1}{M^2} [K_3] \right] \quad (15)$$

$$\tau_1 = \left. \frac{du}{dy} \right|_{y=1} = \frac{[Gr - M^2]}{M \sinh(M)} - \frac{Gr}{M^2} + R \left\{ -\frac{MK_1 \cosh(M)}{\sinh(M)} + \frac{1}{M^2} [K_3 + 2K_4 + 3K_5 + 4K_6] \right\} \quad (16)$$

Equally from (14) the steady state heat transfer on the boundaries are:

$$Nu_0 = - \left. \frac{d\theta}{dy} \right|_{y=0} = \left[1 - R(2B^2 - 4B/3 + 1/3) \right] \quad (17)$$

$$Nu_1 = - \left. \frac{d\theta}{dy} \right|_{y=1} = \left[1 - R(2B^2 - 4B/3 + 1) \right] \quad (18)$$

while the first and second order space derivatives are approximated by the central difference formula.

$$\frac{\partial \theta}{\partial y}(y_i, t_j) \approx \frac{\theta(y_{i+1}, t_j) - \theta(y_{i-1}, t_j)}{2(\Delta y)} + o((\Delta y)^2) \quad (21)$$

$$\frac{\partial^2 u}{\partial y^2}(y_i, t_j) \approx \frac{u(y_{i+1}, t_j) - 2u(y_i, t_j) + u(y_{i-1}, t_j))}{(\Delta y)^2} + o((\Delta y)^2) \quad (22)$$

$$\frac{\partial^2 \theta}{\partial y^2}(y_i, t_j) \approx \frac{\theta(y_{i+1}, t_j) - 2\theta(y_i, t_j) + \theta(y_{i-1}, t_j))}{(\Delta y)^2} + o((\Delta y)^2) \quad (23)$$

4. Numerical Solution Procedure

The momentum and energy equations given in equations (6) and (7) are solved numerically using implicit finite difference method. The time derivatives in both equations are approximated using backward difference formula as:

$$\frac{\partial u}{\partial t}(y_i, t_j) \approx \frac{u(y_i, t_j) - u(y_i, t_{j-1}))}{\Delta t} + o((\Delta t)^2) \quad (19)$$

$$\frac{\partial \theta}{\partial t}(y_i, t_j) \approx \frac{\theta(y_i, t_j) - \theta(y_i, t_{j-1}))}{\Delta t} + o((\Delta t)^2) \quad (20)$$

Replacing j by $(j+1)$ in (19), (20), (21), (22) and (23) gives an iterative system, which does not restrict the time step. Thus the transport equations (6) and (7) at the grid point (i, j) are linearized. The momentum Equation reads:

$$\frac{u_i^{j+1} - u_i^j}{\Delta t} = \frac{u_i^{j+1} - 2u_i^{j+1} + u_{i+1}^{j+1}}{(\Delta y)^2} - M^2 u_i^j + Gr \theta_i^j \quad (24)$$

and the energy equation becomes:

$$\left(\frac{\theta_i^{j+1} - \theta_i^j}{\Delta t} \right) = \left[1 + \frac{4}{3} R (C_T + \theta_i^j)^3 \right] \left(\frac{\theta_i^{j+1} - 2\theta_i^{j+1} + \theta_{i+1}^{j+1}}{(\Delta y)^2} \right) + 4R [C_T + \theta_i^j]^2 \left(\frac{\theta_{i+1}^j - \theta_{i-1}^j}{2(\Delta y)} \right)^2 \quad (25)$$

with the following new boundary conditions:

$$\left. \begin{aligned} u_{i,0} &= 0, \quad \theta_{i,0} = 0 \text{ for all } i=0 \\ u_{0,j} &= 0, \quad \theta_{0,j} = 1 \\ u_{M,j} &= 0, \quad \theta_{M,j} = 0 \end{aligned} \right\} \quad (26)$$

$$abs|(u, \theta)_{exact} - (u, \theta)_{num}| < 10^{-3} \quad (27)$$

5. Result Validation

In order to confirm the validity of this numerical solution, the numerical result are compared with analytical solution derived for steady state problem using perturbation technique. A series of numerical experiments shows that steady state velocity and temperature occurs when the dimensionless time approaches to the values of Prandtl number of the fluid. During the course of numerical computation, an excellent agreement is found between unsteady and steady state solutions at large values of time see Figures 2 and 3 respectively.

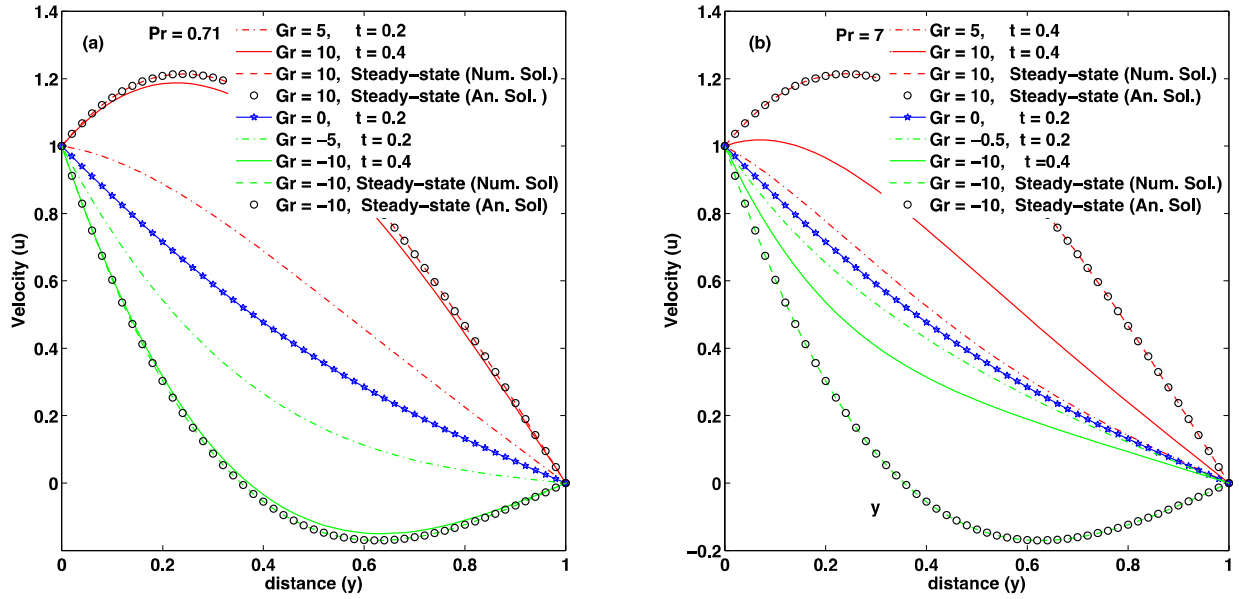


Figure 2. Velocity profile ($R = 0.1, C_T = 0.01, M = 1$)

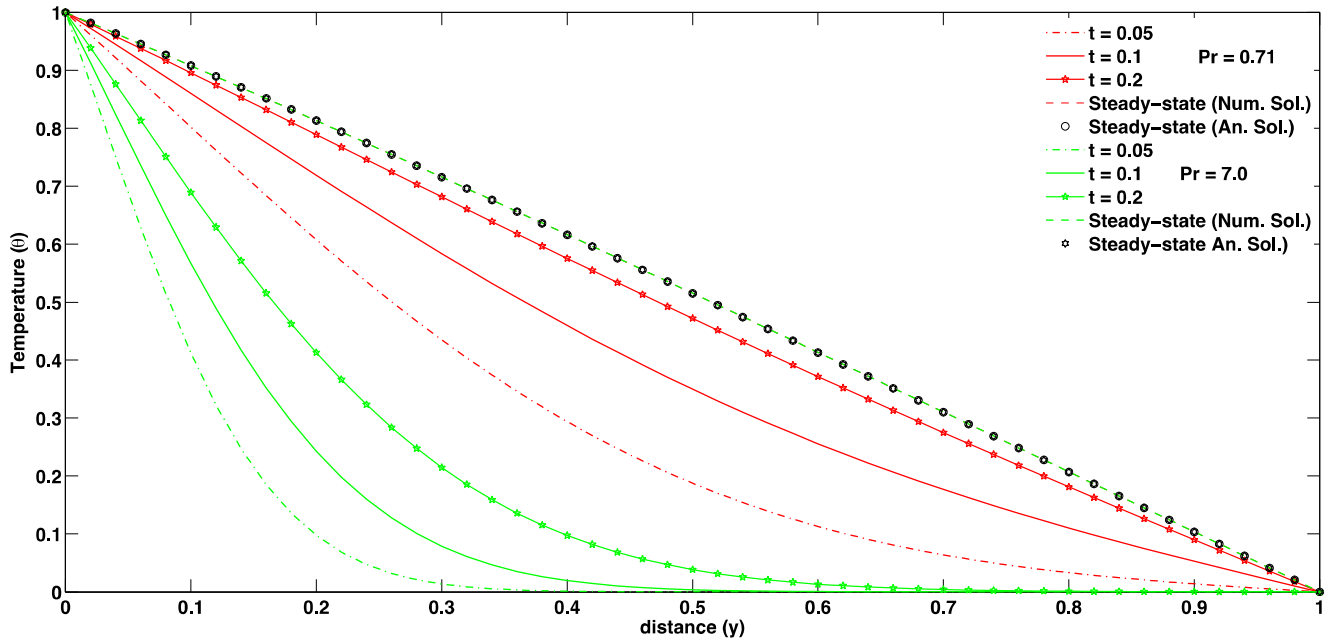


Figure 3. Temperature profile ($R = 0.1, C_T = 0.01$).

6. Result and Discussion

Numerical solutions for time dependent momentum and energy equations using implicit finite difference method are performed to see the impact of various dimensionless parameters controlling the present physical situation. The effect of essential governing parameters, such as magnetic parameter ($R = 0.1, C_T = 0.01, M = 1$), the radiation parameter (R), the temperature difference C_T , the Grashof number and the Prandtl number (Pr) are reported. Fixed values selected for numerical computation are $C_T = 0.01$, $M = 1$, $R = 0.1$ and Pr is chosen as 0.71 and 7.0 that

physically represent air and water, respectively. The value of Gr is taken to indicate the state of the plates. It can be negative, zero or positive depending on the temperature of the plate. Therefore $Gr < 0$ relate to an external heating of the moving plate, while $Gr > 0$ corresponds to external cooling of the moving plate. In the present treatise, time is chosen between $0.05 \leq t \leq 8.5$ so as to capture the unsteady behavior of both velocity and temperature. In the numerical computation the numerical values of R are in the range of $0 \leq R \leq 0.8$. Because large value of R , lead to finite time temperature blow up. Since the terms associated with R in energy equation are strong heat sources Makinde and Chiyoka [21].

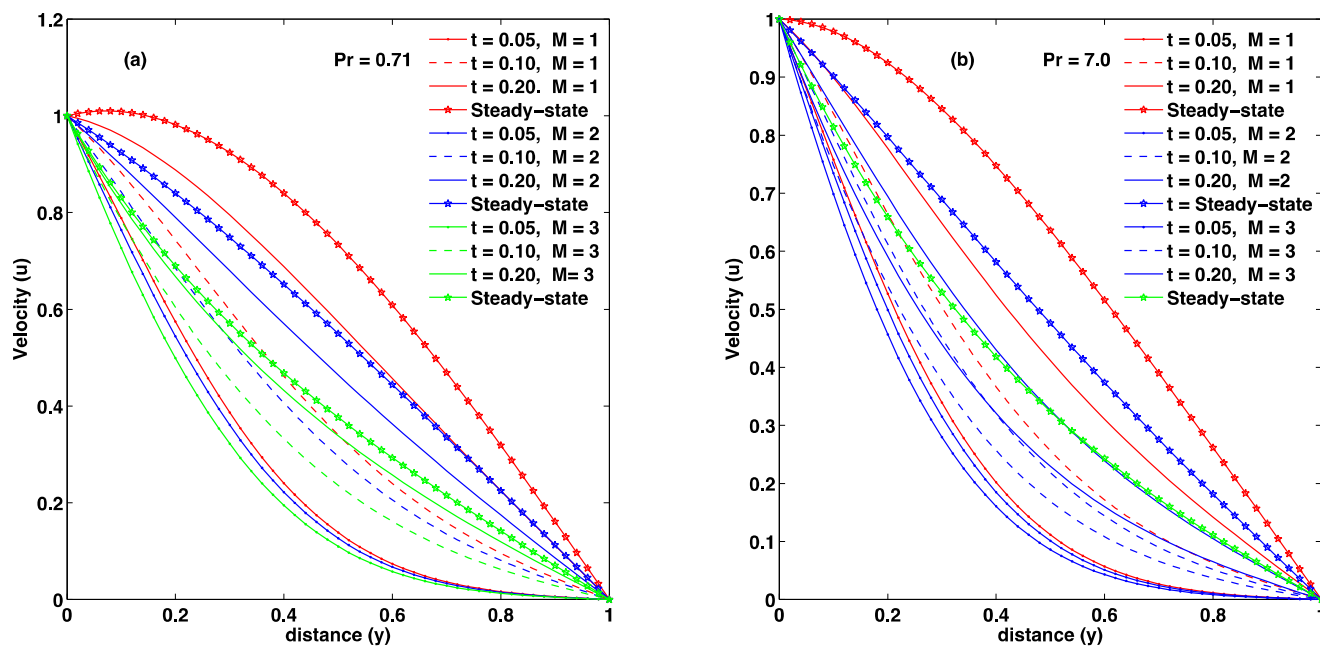


Figure 4. Velocity profile ($R = 0.1, C_T = 0.01, Gr = 5$)

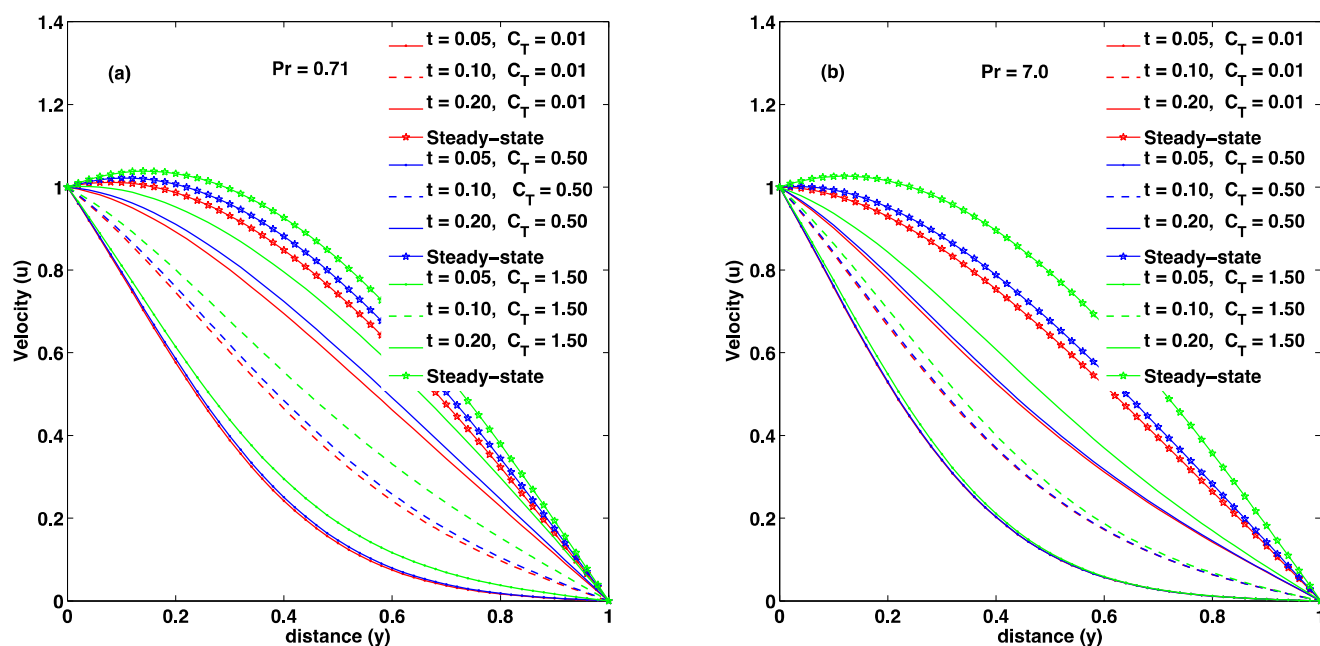


Figure 5. Velocity profile ($R = 0.2, M = 1, Gr = 5$)

Figure 2 illustrates the velocity profile due to change in Gr for fixed values of other parameter. From this Figure, velocity significantly enhanced with the increase in Gr . It is clearly observed from Figure 2a and b, that, the velocity of air and water takes reverse flow and attains steady state as time increases ($Gr < 0$). It also reveals from Figure 2a and b

that for $Gr > 0$, the velocity of air and water increases with time and reached its steady state value. It is important to mention here that, $Gr = 0$ represent a physical situation when flow formation is only due to the movement of one of the boundary (Couette flow).

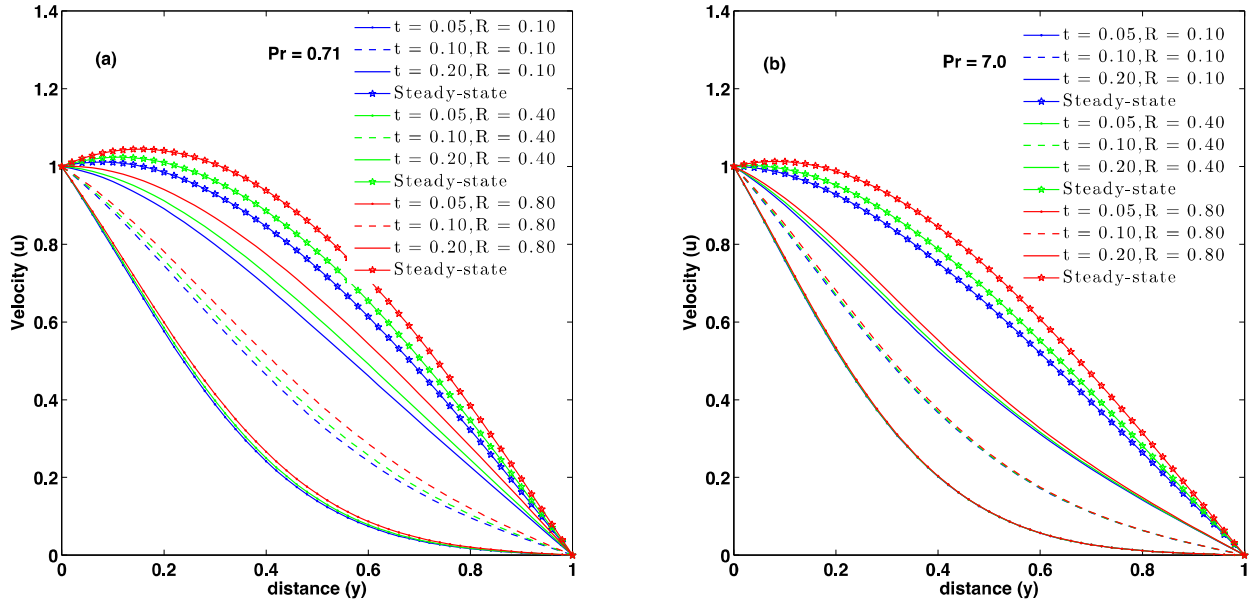


Figure 6. Velocity profile ($C_T = 0.2, M = 1, Gr = 5$)

Figure 3 reports that the increase in time strengthens the temperature until it reached its steady state value. Also, temperature decreases with increase in Prandtl number. This is due to the inverse relation between Prandtl number and thermal diffusivity of fluid.

Figure 4a and b depicts the effect of magnetic parameter on velocity for fixed values of the remaining parameters. It is observed that as the magnetic parameter increases the velocity decreases. This is true since the imposition of a magnetic field to an electrically conducting fluid creates a drag-force. This is an important controlling mechanism in nuclear energy systems heat transfer, where enhancing the magnetic field can reduce momentum development [24].

difference parameter (C_T) on unsteady and steady state velocity when other parameters are fixed. The temperature difference parameter is found to enhance velocity with little effect. However, it is clear that increase in time increases the velocity significantly. The velocity is also found to attain a steady state as the dimensionless time approaches to the values of Prandtl number of the fluid.

From Figure 6a and b it is clear that radiation parameter (R) enhanced both unsteady and steady state velocity. These Figures further reflects that the time required to reach steady state is higher in case of water ($Pr = 7.0$) than air ($Pr = 0.71$). In both cases the time needed to attain steady

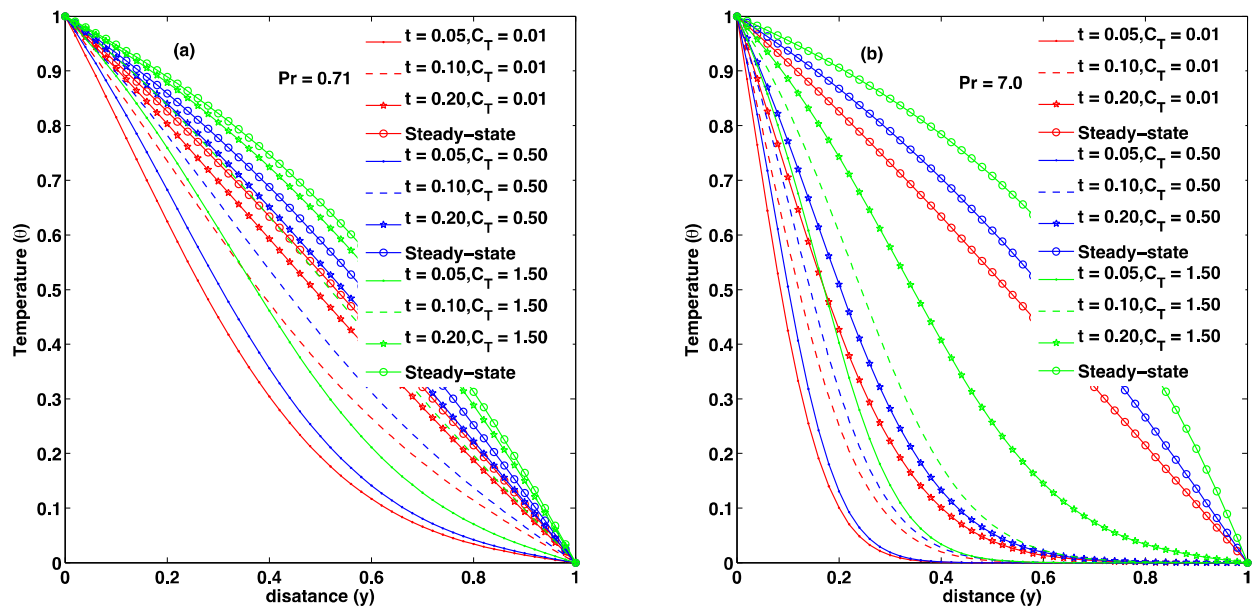


Figure 7. Temperature profile ($R = 0.2$)

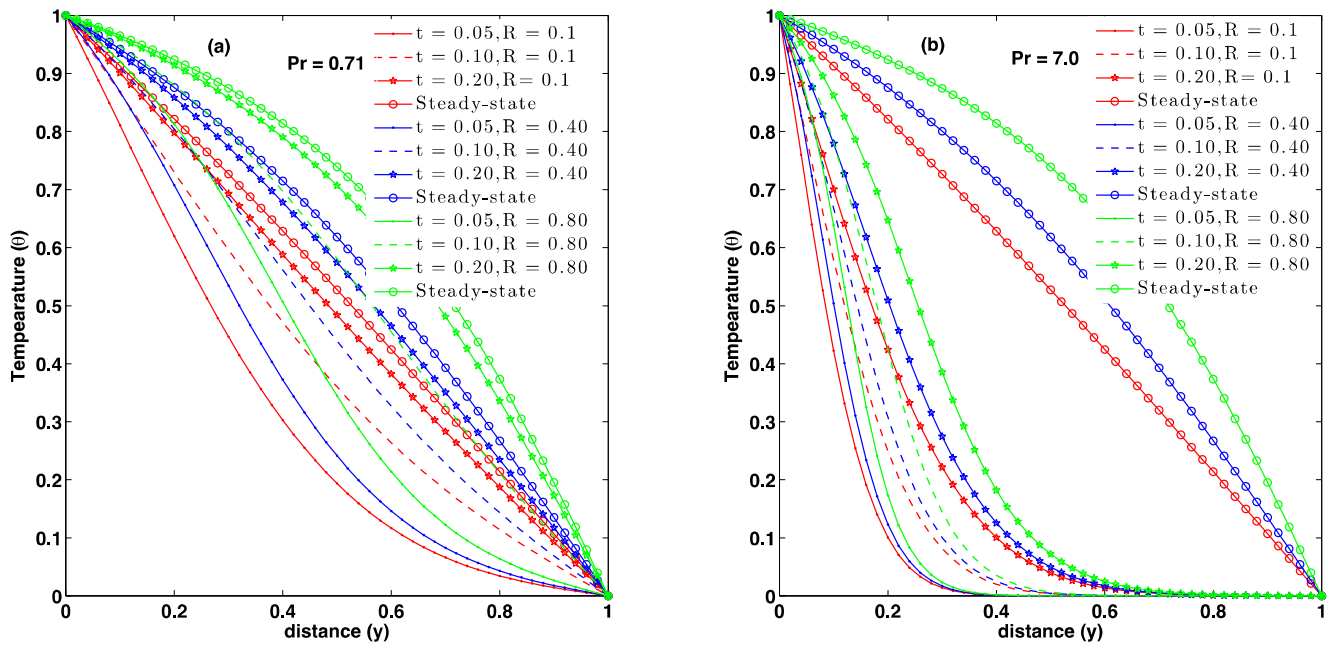
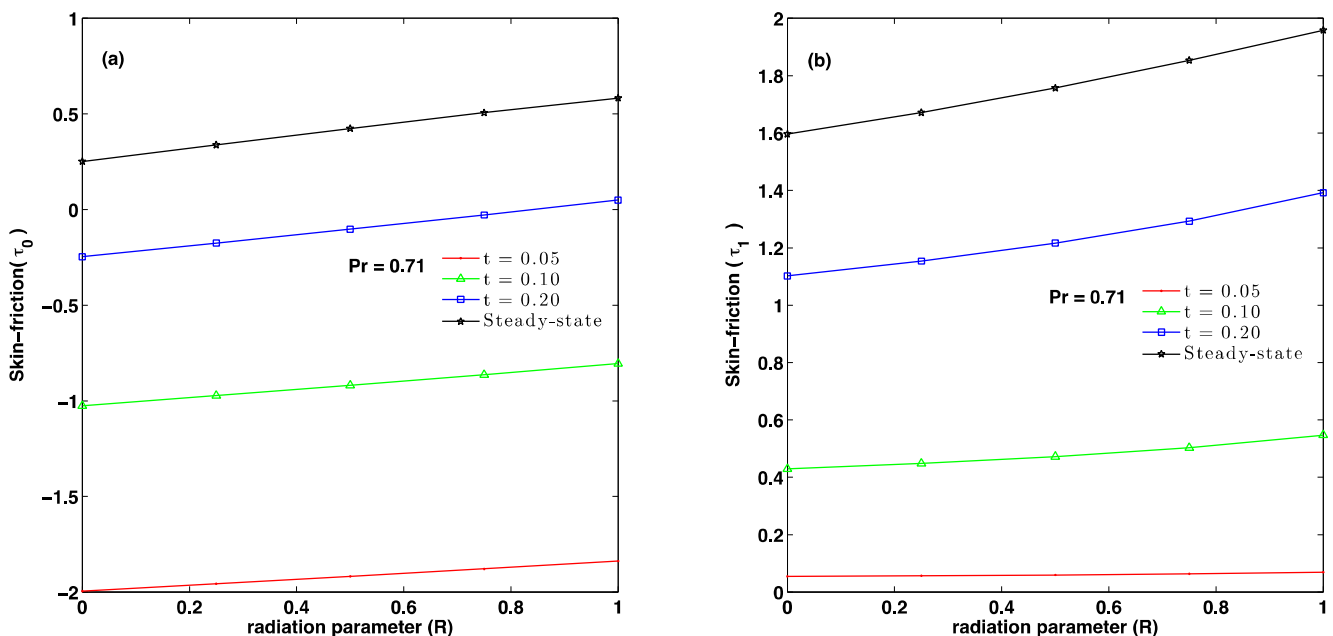


Figure 8. temperature profile ($C_T = 0.2$)

Figures 7 and 8 shows the variations in the temperature profile due to temperature difference (C_T) and radiation (R) parameters when other parameters are fixed. These Figures explain that an increase in C_T and R increases the temperature profiles until the steady state value is reached. Also it is obvious from these Figures that as the value of C_T

temperature increases. It is interesting to report that the time require to reach steady state is higher in Figures 7b and 8b in comparison to 7a and 8a. This is true since thermal diffusivity reduces as Prandtl number increases. Furthermore as time increases, the temperature increases until the steady state value is attained see figure 7 and 8.



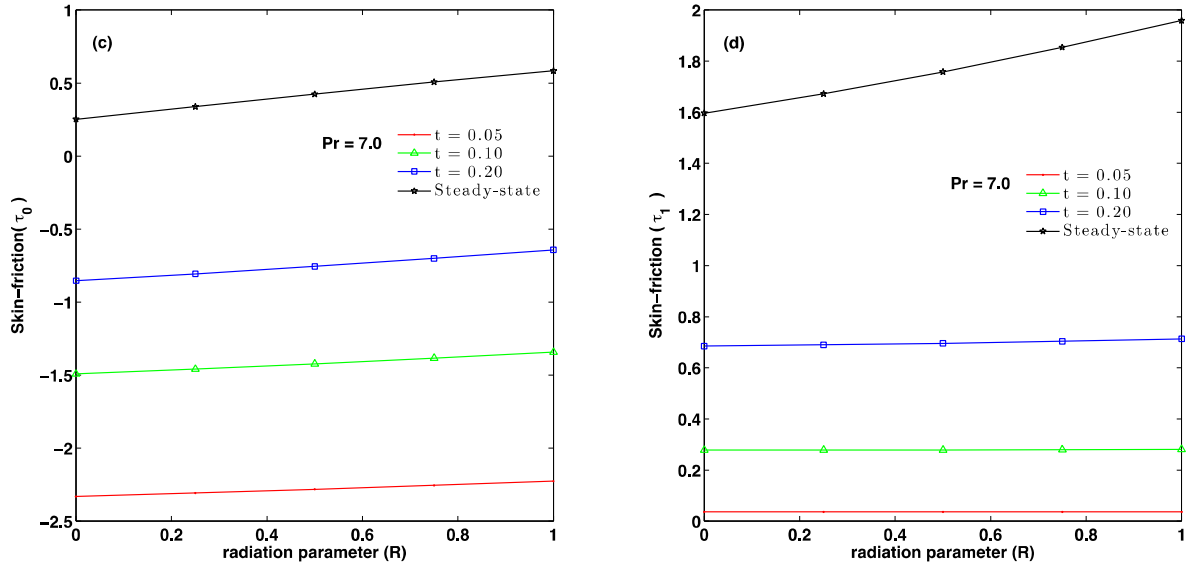


Figure 9. Skin friction against κ : (a & c) at $y=0$, (b & d) at $y=1$.

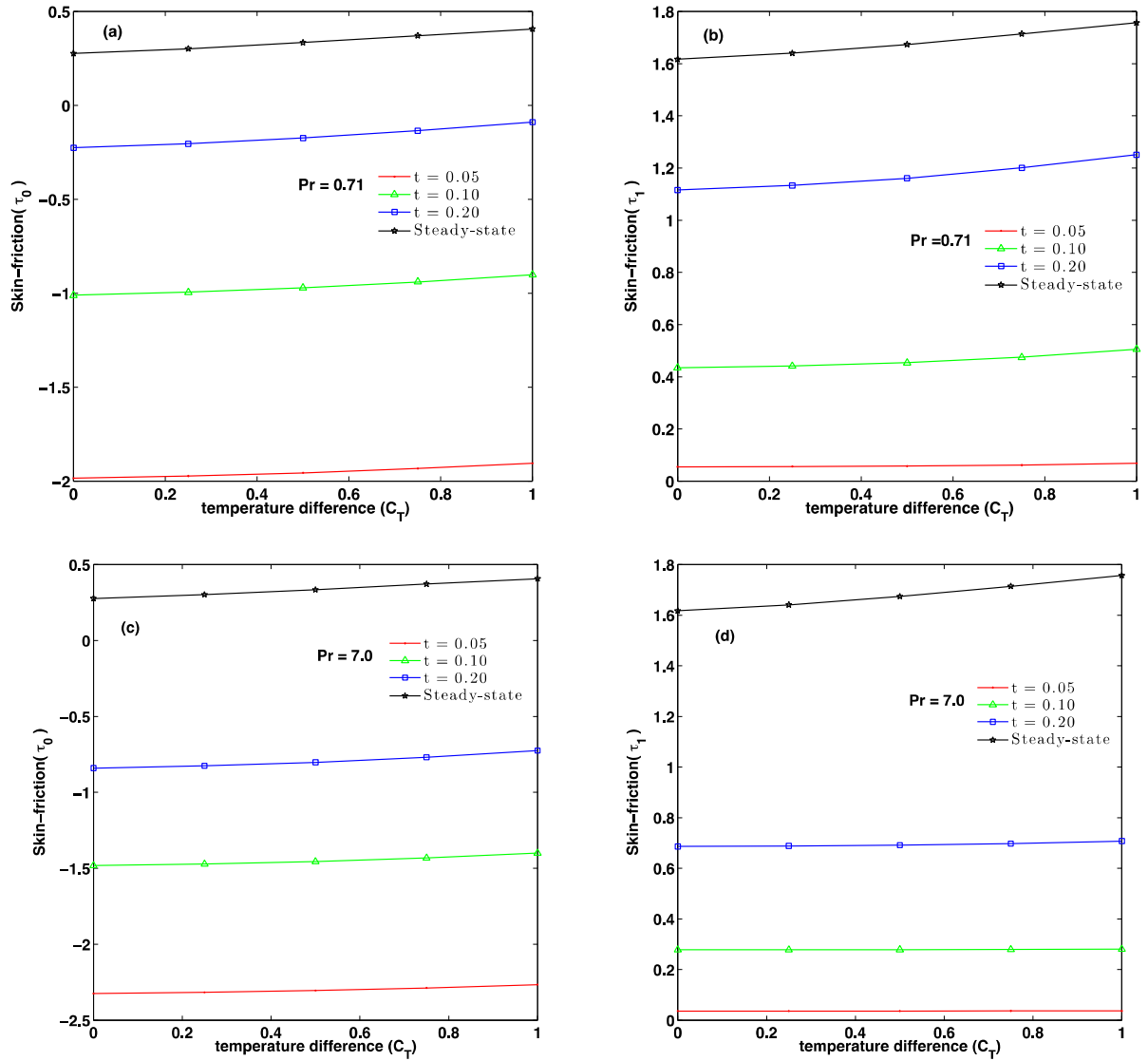


Figure 10. Skin friction against C_T : (a & c) at $y=0$, (b & d) at $y=1$.

In Figures 9 through 12, the variation of skin friction for air ($Pr = 0.71$) and water ($Pr = 7.0$) at the plate $y = 0$ and $y = 1$ are narrated for fixed values of the other parameters. Figures 9a b, c, and d depict the variation of skin friction at $y = 0$ and $y = 1$ due to different values of radiation parameter (R) and time (t). From these Figures is noted that skin friction increases with increase in radiation and dimensionless time (t) on both plates. The physical fact is that, an increase in radiation parameter creates more heat to the fluid leading to an increase in temperature, which consequently increases the convection current.

Figure 10a, b, c, and d illustrate the variation effect of temperature difference (C_T) and dimensionless time (t) on skin friction at $y = 0$ and $y = 1$ respectively when other

that an increase in C_T or t enhances the skin-friction on both plates and at length procure its steady state value.

Figures 11a b, c, and d show the contribution effect of Grashof number (Gr) on skin friction in both walls for fixed values of other controlling parameters. It is observed that the skin friction is elevated with the increase of Gr . Also, the values of skin friction are found to be higher in Figures 11c and d at $y = 1$ in comparison with Figures 11a and b ($y = 0$).

Figure 12a, b, c, and d, presents a variation of skin friction at $y = 0$ and $y = 1$ for different values of magnetic number (M) and fixed values of other parameters. It is interesting to mention that skin friction decreases as magnetic parameter (M) increases. In contrast, as the dimensionless time (t) increases the skin friction increases.

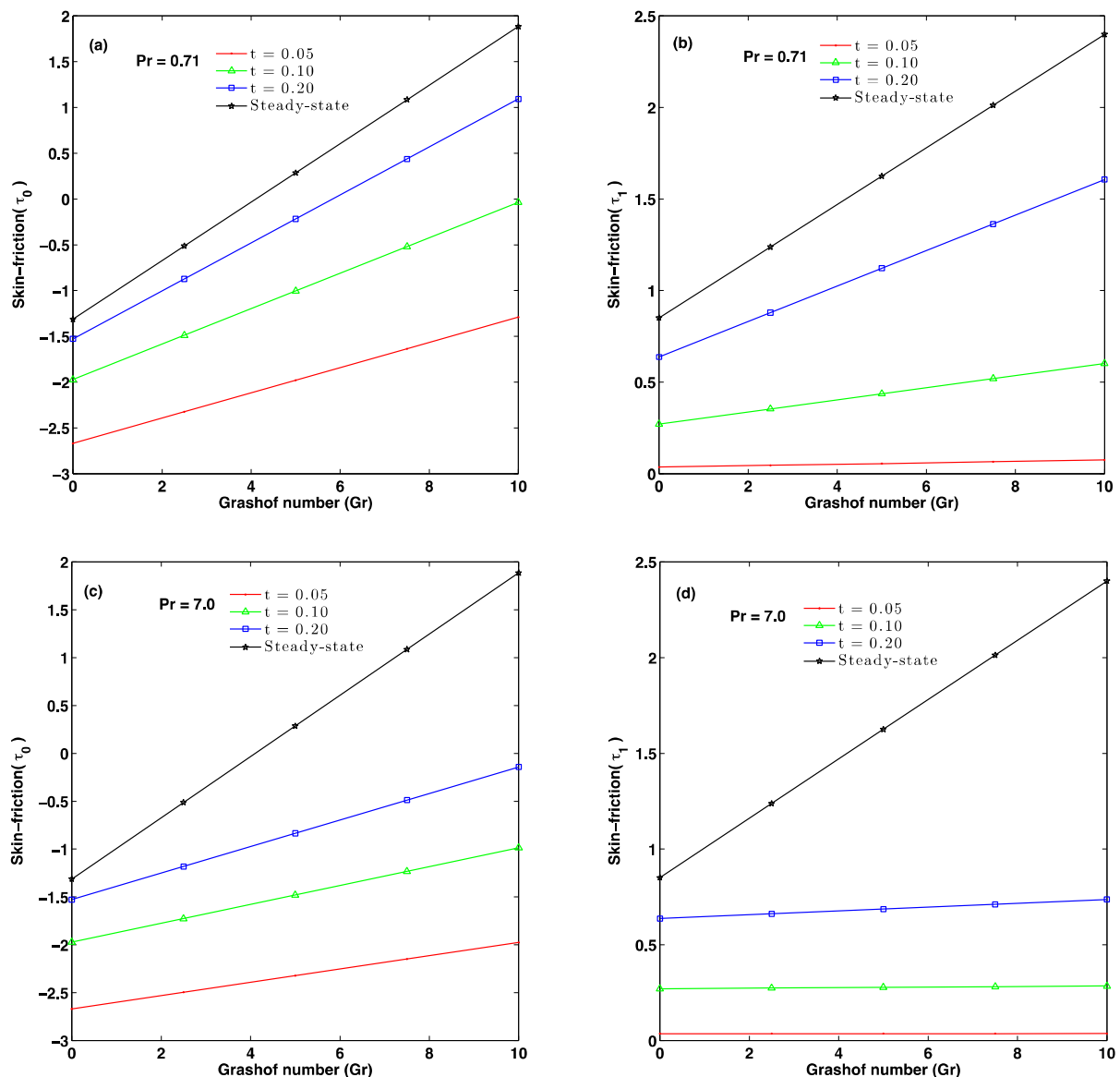


Figure 11. Skin friction against Gr (a & c) at $y = 0$, (b & d) at $y = 1$.

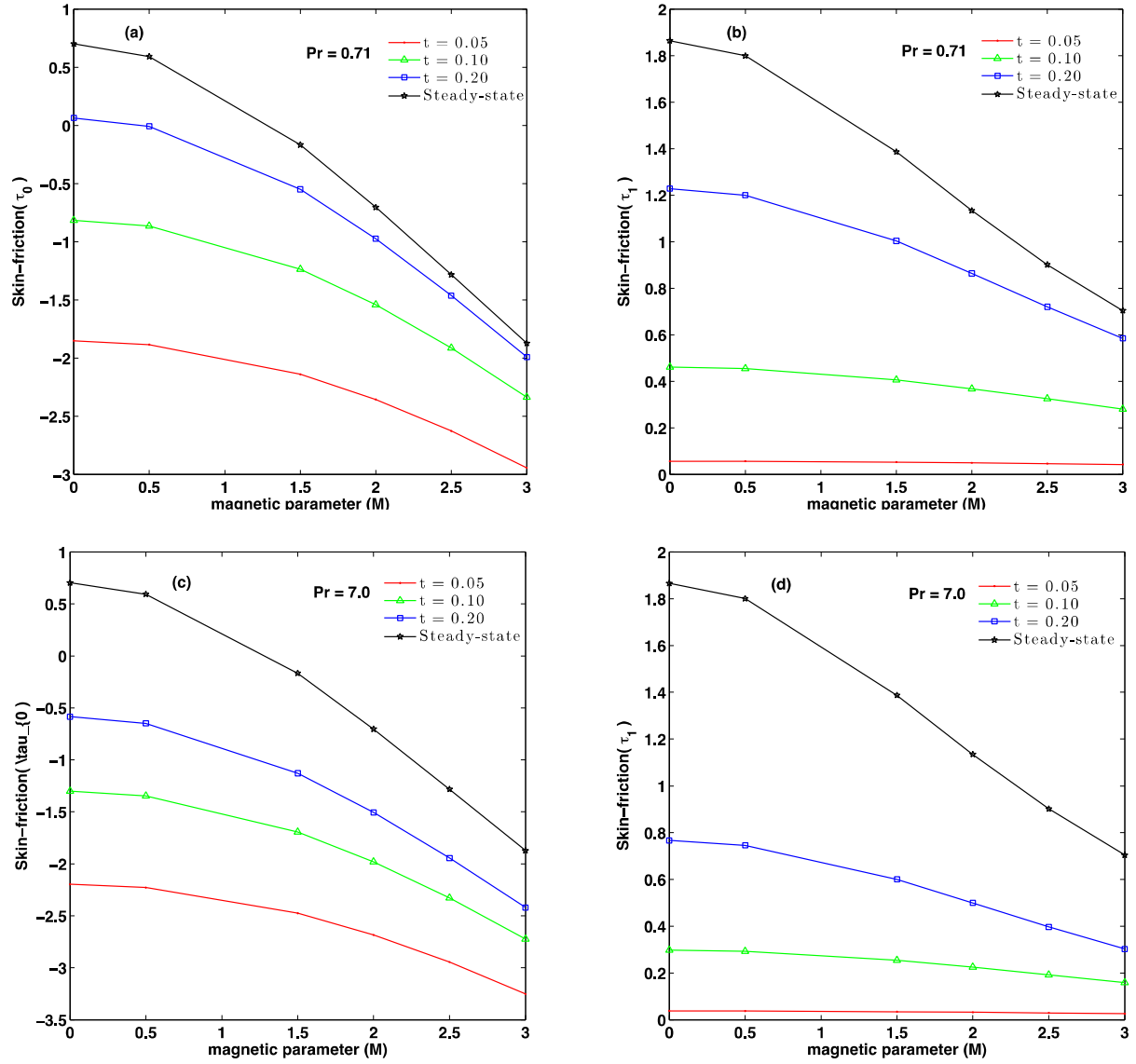
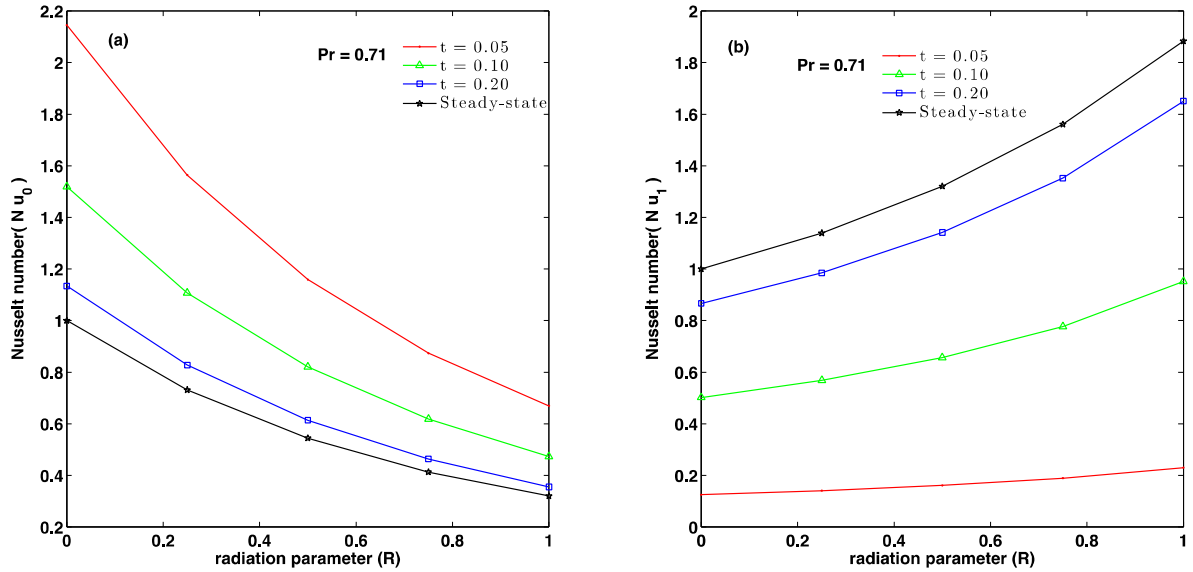


Figure 12. Skin friction against M : (a & c) at $y = 0$, (b & d) at $y = 1$.



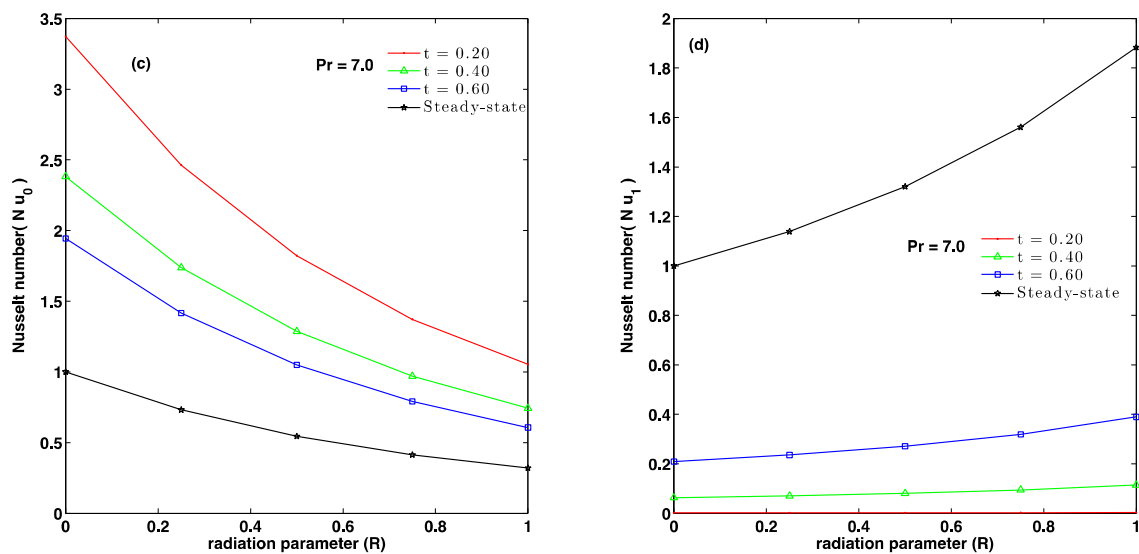


Figure 15. Nusselt number against R : (a & c) at $y=0$, (b & d) at $y=1$.

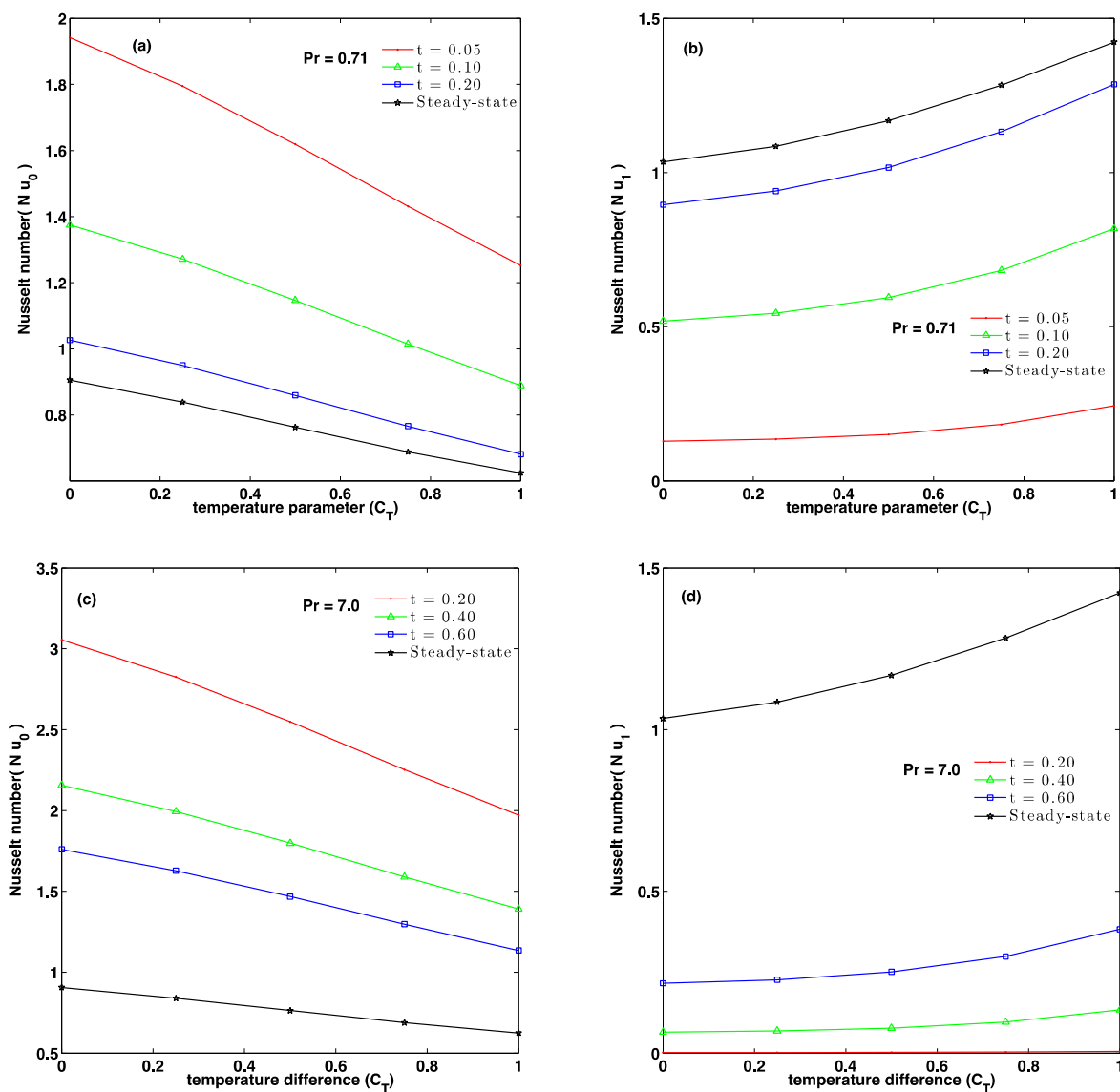


Figure 14. Nusselt number against C_T : (a & c) at $y=0$, (b & d) at $y=1$.

Figures 13 and 14 show influence of radiation parameter and temperature difference parameter on Nusselt number at $y=0$ and $y=1$ respectively. It is observed that as R and C_T increases the values of Nusselt number decreases at $y=0$ and increases with increase in R and C_T at $y=1$.

7. Conclusion

In order to visualize the flow formation of free-convective Couette flow of conducting fluid in presence of thermal radiation and transverse magnetic field, numerical as well as analytical solutions are obtained. The influence of the dimensionless controlling parameters on velocity temperature, skin friction and Nusselt number are demonstrated in figures and discussed. In the absence of thermal radiation and magnetic field the results agree with available result of Singh [9]. Furthermore, in the absence of radiation, the results are in excellent agreement with Jha [10]. From the results obtained, the findings are.

- Velocity and temperature increases with increase in time.
- Skin-friction increases with increase in time, while Nusselt number decreases with increase in time at $y=0$ and increases with time at $y=1$.
- The time required for the velocity, temperature, skin-friction and Nusselt number to attain steady-state strongly rest on the dimensionless parameters M , R , C_T , Gr and Pr .
- The results are found in good agreement between steady state and unsteady state after some sufficiently large time (t)

Appendix

$$B = C_T + 1,$$

$$K_1 = 4B^2Gr/M^4 + 4Gr/M^4 - 8BGr/M^4 + 8Gr/M^6,$$

$$K_2 = 4B^2Gr + 8Gr/M^4,$$

$$K_3 = 2B^2Gr + Gr/3 - 4BGr/3 + 8BGr/M^2,$$

$$K_4 = 2B^2Gr + 4Gr/M^2, K_5 = 4BGr/3, K_6 = Gr/3$$

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