International Journal of Materials Science and Applications
2015; 4(3): 173-188
Published online April 28, 2015 (http://www.sciencepublishinggroup.com/j/ijmsa)
doi: 10.11648/j.ijmsa.20150403.15
ISSN: 2327-2635 (Print); ISSN: 2327-2643 (Online)

The Life Predicting Calculations Based on Conventional Material Constants from Short Crack to Long Crack Growth Process

Yangui Yu

Zhejiang GuangXin New Technology Application Academy of Electromechanical and Chemical Engineering, Hangzhou, China

Email address:
gx_yyg@126.com

To cite this article:
Yangui Yu. The Life Predicting Calculations Based on Conventional Material Constants from Short Crack to Long Crack Growth Process. *International Journal of Materials Science and Applications*. Vol. 4, No. 3, 2015, pp. 173-188. doi: 10.11648/j.ijmsa.20150403.15

Abstract: To use the theoretical approach, to adopt the multiplication-method of two-parameters, by means of the traditional and the modern material constants, thereby to establish some of new calculation models in all crack growth process. In which are the equations of the driving forces, the crack-growth-rate-linking-equation in whole process, and the life predictions; and to propose yet some calculating expressions under different loading conditions. For key material parameters give their new concepts, and provide new functional formulas, define their physical and geometrical meanings. For the transition crack size from micro to macro crack growth process, provide concrete calculation processes and methods. Thereby realize the lifetime predicting calculations in whole process based on conventional materials constants and by the multiplication method of two parameters.

Keywords: Short Crack and Long Crack, Calculating Modeling, Lifetime Prediction, High Cycle Fatigue, Low Cycle Fatigue

1. Introduction

As everyone knows for the traditional material mechanics, that is a calculable subject, and it has made valuable contributions for every industrial engineering designs and calculations. But it cannot accurately calculate the life problems for some structures when it is used for pre-existing flaws and under repeated loading. In that it has no to contain such calculable parameters in its calculating models as the crack variable a or as the damage variable D. But, for the fracture mechanics or the damage mechanics, due to there are these variables, they can all calculate above problems. However, nowadays latter these disciplines are all mainly depended on tests by fatigue, damage and fracture.

Author studies their compositions of traditional mathematical models and modern models for above disciplines; and researches crack growth behaviors for some steels of containing micro and macro flaws, thinks that in the mechanics and the engineering fields, also to exist such scientific principles of genetic and clone technology as life science [1]. Author has done some of works used the theoretical approach as above the similar principles [1-8]. For example, for some strength calculation models from micro to macro crack had been provided by reference [2], for some rate calculation models from micro to macro damage (or crack) growth had been proposed by references [3-8] which were all models in each stage even in whole process, and to be under different loading conditions. Two years ago, in order to do the lifetime calculations in whole process on fatigue-damage-fracture for an engineering structure, author was by means of Google Scholar to search the lifetime prediction models, as had been no found for this kind of calculation equations. After then, author bases on the comprehensive figure 1 of material behaviors has provided some models as in references [2-3]; Recently sequentially complemented for it [1], and continues to research this item, still applies above genetic principles to study and analyze data in references, thereby to provide another some new calculable models for the crack growth driving force and for the lifetime predictions. The purpose is to try to make the fracture mechanics step by step become calculable discipline as the traditional material mechanics. That way, may be having practical significances for decrease experiments, for promoting engineering applying and developing for relevant disciplines.

2. The Life Prediction Calculations for Elastic-Plastic Steels Containing Pre-Flaws

Under repeating load, for some elastic-plastic steels containing pre-cracks, their crack growth rate and life predicting calculations for short crack growth and micro-damage growth process had been calculated by means of some methods in reference [9-10], that are the Q_1-factor method by two-parameters multiplication $\varepsilon \cdot \sigma$ with stress and strain; For the long crack growth process, this paper also adopts similar to above method, that is the Q_2-factor method by the two-parameters multiplication $K_2\delta_{2t}$ with the stress intensity factor K_2 and the crack tip open displacement δ_{2t}; Moreover, here to provide another a new calculating method---the stress (σ)-method. Thereby achieve severally the purpose for life's prediction calculations in different stage.

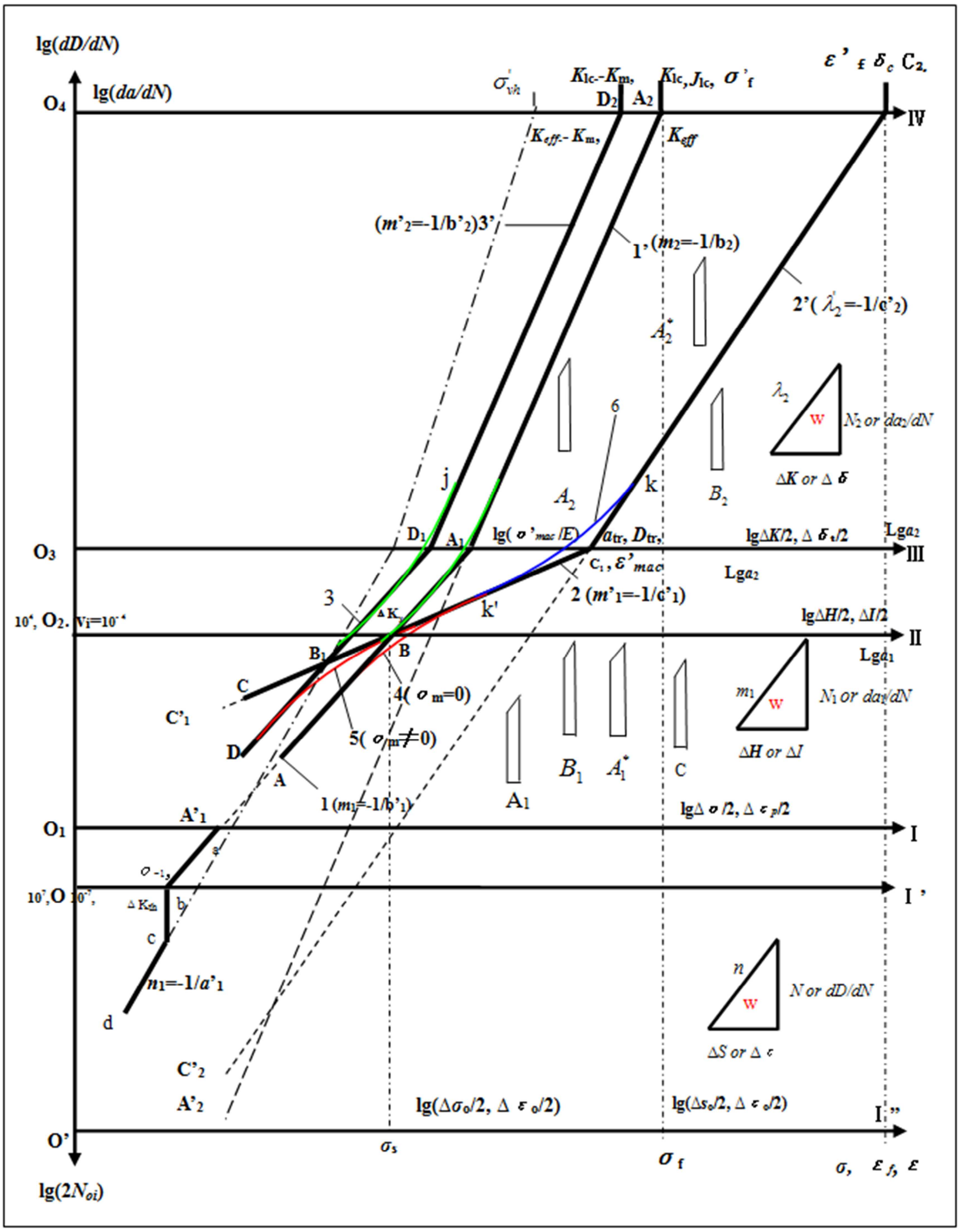

Figure 1. *Comprehensive figure of material behaviors (Bidirectional combined coordinate system and simplified schematic curves in the whole process).*

2.1. The Life Prediction Calculations in Short Crack Growth Process (or Called the First Stage)

The life curves of short crack growth in crack forming stage (the first stage) are just described with curves 1 ($\sigma<\sigma_s, \sigma_m=0$), 2 ($\sigma>\sigma_s$) and 3 ($\sigma<\sigma_s, \sigma_m\neq 0$)) in reversed direction coordinate system as in attached figure1, where About their constituting relationship and meaning of each curve are explained in [1-3], here for them are only described with the Multiplication method of two parameters in short crack growth process.

(1) Under work tress $\sigma<\sigma_s$ condition (high cycle loading)

In attached fig.1, under work tress $\sigma<\sigma_s$ condition, the life prediction equation corresponded to reversed curves 1 and 3 can be calculated in short growth process as following form

$$N_1=\int_{a_1}^{a_{tr}}\frac{da_1}{A_1^*\times(\Delta Q_1)^{\frac{m_1m'_1}{m_1+m'_1}}},\text{(cycle)} \tag{1-1}$$

or

$$N_1=\int_{a_1}^{a_{tr}}\frac{da_1}{A_1^*\times(\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1m'_1}{m_1+m'_1}}a_1},\text{(cycle)} \tag{1-2}$$

Where the Q_1 is defined as two-parameter stress-strain factor under monotonous loading; the ΔQ_1 is defined as two-parameter stress-strain factor range under fatigue loading.

$$Q_1=(\varepsilon\cdot\sigma)\ a_1^{1/\frac{m_1m'_1}{m_1+m'_1}} \tag{2}$$

$$\Delta Q_1=(\Delta\varepsilon\cdot\Delta\sigma)\ a_1^{1/\frac{m_1m'_1}{m_1+m'_1}} \tag{3}$$

$$A_1^*=2[4(\sigma'_f\,\varepsilon'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v_{eff})^{-1},(\text{MPa}^{\frac{m_1m'_1}{m_1+m'_1}}\,\text{mm}/cycle),\quad(\sigma_m=0) \tag{4}$$

$$A_1^*=2[4(\sigma'_f\,\varepsilon'_f)(1\text{-}\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v_{eff})^{-1},(\text{MPa}^{\frac{m_1m'_1}{m_1+m'_1}}\text{mm}/cycle),\ (\sigma_m\neq 0) \tag{5}$$

Here the eqn (2) is driving force of short crack growth under monotonic loading, and the eqn (3) is driving force under fatigue loading. $\Delta\varepsilon=\Delta\sigma/E$. The A_1^* in eqn (4) is defined the comprehensive material constant, that is corresponding reversed curves 1 (A_1A) (attached fig.1), its mean stress $\sigma_m=0$; The A_1^* in eqn (5) is corresponding reversed curves 3(D_1D) (attached Fig.1), its mean stress is $\sigma_m\neq 0$. Here for mean stress $\sigma_m\neq 0$ to adopt as the correctional method in reference [11]. Author research and think, the A_1^* is a calculable parameter of having function relation with other parameters $\sigma'_f, b'_1\ \psi$, its unit of A_1^* is the "$MPa^{m_1m'_1/(m_1+m'_1)}\cdot mm/cycle$", its physical meaning is a concept of power, that is to give out an energy to resist outside force, it just is a maximal increment value to give out energy in one cycle, before the specimen material makes failure. Its geometrical meaning is a maximal micro-trapezium area approximating to beeline (Fig1), that is a projection of corresponding to curve 1 ($\sigma_m=0$) or 3 $\sigma_m\neq 0$ on the y-axis, also is an intercept between O_1-O_3. Its slope of micro-trapezium bevel edge just is corresponding to the exponent $m_1m'_1/(m_1+m'_1)$ of the formula (4-5). Here the parameters m_1 and m'_1 are respectively material constants under high cycle or low cycle fatigue. The $m_1=-1/b'_1$, b'_1 is the fatigue strength exponent under high cycle fatigue; the $m'_1=-1/c'_1$, c'_1 is the fatigue ductility exponent under low cycle fatigue.

Here

$$v_{eff}=\ln(a_{1f}/a_0)/N_{1fc}-N_{01}=[\ln(a_{1f}/a_0)\text{-}\ln a_1/a_{01})]/N_{1f}-N_{01},(mm/\text{cycle}) \tag{6}$$

Or

$$v_{eff}=[a_{1f}\ln(1/1\text{-}\psi)]/N_{1f}-N_{01},\text{(mm/cycle)} \tag{7}$$

The v_{eff} in eqn (6-7) is defined as an effective rate correction factor in first stage, its physical meaning is the effective rate to cause whole failure of specimen material in a cycle, its unit is the $mm/cycle$. ψ is a reduction of area. a_0 is pre-micro size which has no effect for fatigue damage under prior cycle loading [12]. a_{01} is an initial micro size, a_f is a critical crack size before failure, N_{01} is initial life, $N_{01}=0$; N_{1f} is failure life, $N_{1f}=1$. It should yet point the a_{tr} in eqn (1) is a transitional crack size transited from micro to macro damage, $a_{tr} \approx a_{mac}$, a_{mac} is a macro crack value corresponded to forming macro-crack size a_{mac}. a_{oi} is a medial crack value between initial micro crack and transitional crack size corresponding medial life N_{oi}. By the way, here is also to adopt those material constants $\sigma'_f, b_1', \varepsilon'_f, c_1'$ as "genes" inside the fatigue damage subject. So, for the eqn (1), its final expansion equation corresponded reversed to curves 1 (A_1A) is as below form:

$$N_1 = \frac{\ln a_{tr} - \ln a_1}{2(4\sigma'_f \times \varepsilon'_f)^{-\frac{m_1 m'_1}{m_1 + m'_1}} \times (v_{eff})^{-1} (\Delta\sigma \cdot \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1 + m'_1}}}, (Cycle), (\sigma < \sigma_s, \sigma_m = 0) \tag{8}$$

And its final expansion equation corresponded reversed to curves 3 (D_1D) should be:

$$N_{oi} = \frac{\ln a_{oi} - \ln a_1}{2\left[4\sigma'_f \times \varepsilon'_f (1\text{-}\sigma_m / \sigma'_f)\right]^{-\frac{m_1 m'_1}{m_1 + m'_1}} \times (v_{eff})^{-1} (\Delta\sigma \cdot \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1 + m'_1}}} (\sigma_m \neq 0) \tag{9}$$

(2) Under work stress $\sigma > \sigma_s$ condition (or low cycle loading)

When under $\sigma > \sigma_s$ condition, due to plastic strain occurring cyclic hysteresis loop effect, its life equation corresponded to reversed direction curve C_1C is as following

$$N_1 = \int_{a_1}^{a_{tr}} \frac{da_1}{A_1^* \times (0.25\Delta Q_1)^{\frac{m_1 m'_1}{m_1 + m'_1}}}, (Cycle) \tag{10-1}$$

Or

$$N_1 = \int_{a_1}^{a_{tr}} \frac{da_1}{A_1^* \times (0.25\Delta\sigma \times \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1 + m'_1}} \times a_1}, (Cycle) \tag{10-2}$$

Therefore its final expansion equation for eqn (10) is as below form,

$$N_1 = \frac{\ln a_{tr} - \ln a_1}{2(4\sigma'_f \varepsilon'_f)^{-\frac{m_1 m'_1}{m_1 + m'_1}} \times (v_{eff})^{-1} \times (0.25\Delta\sigma \times \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1 + m'_1}}}, (Cycle), (\sigma_m = 0) \tag{11}$$

$$N_{oi} = \frac{\ln a_{oi} - \ln a_{01}}{2[4(\sigma'_f \varepsilon'_f)(1\text{-}\sigma_m / \sigma'_f)]^{-\frac{m_1 m'_1}{m_1 + m'_1}} \times (v_{eff})^{-1} \times (0.25\Delta\sigma \times \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1 + m'_1}}}, (\sigma_m \neq 0) \tag{12}$$

Here influence of mean stress in eqn (12) can be ignored. But it must point that the total strain range $\Delta\varepsilon$ in eqn (11-12) should be calculated by Masing law as following eqn. [13]

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \tag{13}$$

2.2. The Life Predicting Calculations in Long Crack Growth Process (or Called the Second Stage)

In Fig.1, the residual life curves of long crack growth are just described with curves 1' ($\sigma < \sigma_s, \sigma_m = 0$), 2' $(\sigma > \sigma_s)$ and 3' $(\sigma < \sigma_s, \sigma_m \neq 0)$) in reversed direction coordinate system. For the driving force and the life calculating problems in long crack growth process, here to adopt the Q_2 -method of two parameters $K_1\delta_t$ and the stress σ- method are described and calculated for them.

(1) Under work tress $\sigma < \sigma_s$ condition

1) Q_2-factor method

For its life prediction equation corresponded reversed curves A_2A_1 and D_2D_1 in fig.1 should be as below

$$N_2 = \int_{a_{tr}}^{a_{eff}} \frac{da_2}{A^*_2 \times (\Delta Q_2)^{\frac{m_2 m'_2}{m_2 + m'_2}}}, (\text{cycle}) \tag{14}$$

Or

$$N_2 = \int_{a_{tr}}^{a_{eff}} \frac{da_2}{A^*_2 \times \left(y_2(a/b) \times \Delta K_2 \cdot \Delta\delta_2\right)^{\frac{m_2 m'_2}{m_2 + m'_2}}}, (\text{cycle}) \tag{15}$$

Where

$$Q_2 = K_2 \delta_{2t}, (MPa \cdot \sqrt{m} \cdot mm) \tag{16}$$

$$\Delta Q_2 = (\Delta K_2 \cdot \Delta\delta_{2t}), (MPa \cdot \sqrt{m} \cdot mm) \tag{17}$$

Where the Q_2 is defined as two-parameter stress-strain factor under monotonous loading; the ΔQ_2 is defined as two-parameter stress-strain factor range under fatigue loading, they are all drive force for long crack growth in second stage. The $y_2(a/b)$ is correction factor [14] related to long crack form and structure size. When corresponding curve A_2A_1

(fig.1) its calculable comprehensive constant A_2^* is as following

$$A_2^* = 2(4K_{2fc}\delta_{2fc})^{-\frac{m_2 m'_2}{m_2+m'_2}} \times v_{pv}, (MPa\cdot\sqrt{m}\cdot mm)^{\frac{m_2 m'_2}{m_2+m'_2}} \cdot mm/cycle, (\sigma_m = 0) \tag{18}$$

And the A_2^* corresponded to curve D_1D_2 (fig.1), it should be as following form

$$A_2^* = 2[4K_{2fc}\delta_{2fc}(1-K_{2m}/K_{2fc})]^{-\frac{m_2 m'_2}{m_2+m'_2}} \times v_{pv}, (MPa\cdot\sqrt{m}\cdot mm)^{\frac{m_2 m'_2}{m_2+m'_2}} \cdot mm/cycle, (\sigma_m \neq 0) \tag{19}$$

Where the K_{2fc} is a critical stress intensity factor, K_{2m} is mean stress intensity factor; δ_{2fc} is a critical crack tip open displacement. It should be point that their physical and geometrical meanings for the A_2^* are all similar to that concept in short crack growth process mentioned above. But should explain the unit of A_2^* is $(MPa\cdot\sqrt{m}\cdot mm)^{\frac{m_2 m'_2}{m_2+m'_2}} \cdot mm/cycle$.

Where

$$v_{pv} = \frac{(a_{2pv} - a_{02})}{N_{2eff} - N_{02}} \approx 3\times10^{-5} \sim 3\times10^{-4} = v^*(mm/cycle) \tag{20}$$

The v_{pv} is defined to be the virtual rate, its physical meaning is an equivalent crack rate, that is the contributed crack growth rate in a cycle related to the precrack, the dimension can take similar to v^*- dimension in reference [15-16], but the unit is different, because where its unit is $v^* = 3\times10^{-8} \sim 3\times10^{-7}(m/cycle)$, here it is the $mm/cycle$. The crack a_{2pv} is a virtual crack size, a_{02} is an initial size as equivalent to a precrack. N_{02} is an initial life, $N_{02} = 0$. N_{pv} is a virtual life, $N_{2eff} = 1$.

Moreover,

$$K_{2fc} = \sigma'_f\sqrt{\pi a_{2f}}\ , \ (MPa\sqrt{m}) \tag{21}$$

$$K_{2m} = (K_{2\max} + K_{2\min})/2,\ (MPa\sqrt{m}) \tag{22}$$

$$\delta_{2fc} = 2\pi\sigma_s a_f(\sigma'_f/\sigma_s)^2/E, (mm) \tag{23}$$

So the effective life expanded equation corresponding reversed direction curve A_2A_1 is following forming.

$$N_{2eff} = \frac{\frac{4(m_2+m'_2)}{4m_2+4m'_2-6m_2m'_2}\left(a_{2eff}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}} - a_{02}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}}\right)}{2[4(K'_{2fc}\times\delta'_{2fc})]^{-\frac{m_2m'_2}{m_2+m'_2}} \times v_{pv} \times (y_2(a/b)\Delta K_2\cdot\Delta\delta_t]^{\frac{m_2m'_2}{m_2+m'_2}}}, (Cycle), (\sigma_m = 0) \tag{24}$$

In reference [17-18] refer to the effective stress intensity factor, same, here there are also two effective values K_{2eff} and δ_{2eff} corresponding to the critical K_{2fc} and δ_{2fc}, here to propose as follow

$$K_{2eff} \approx \sqrt{K_{th}K_{2fc}} \ \text{ or } \ K_{2eff} \approx (0.25-0.4)K_{2fc} \tag{25-1}$$

$$\delta_{2eff} = (0.25-0.4)\delta_{2fc} \tag{25-2}$$

Where the K_{th} is a threshold stress intensity factor value. The a_{2eff} in (24) is an effective long crack size, it is obtained and calculated from eqns (21), (23) and (25-1), (25-2), and to take less value.

Such, the effective life expanded equation corresponding reversed direction curve D_2D_1 should be

$$N_{2eff} = \frac{\frac{4(m_2+m'_2)}{4m_2+4m'_2-6m_2m'_2}\left(a_{2eff}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}} - a_{02}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}}\right)}{2[4K_{2fc}\delta_{2fc}(1-K_{2m}/K_{2fc})]^{-\frac{m_2m'_2}{m_2+m'_2}} \times v_{pv} \times [y_2(a/b)\Delta K_2\cdot\Delta\delta_t]^{\frac{m_2m'_2}{m_2+m'_2}}}, (Cycle), (\sigma_m \neq 0) \tag{26}$$

Its medial life N_{2oj} in second stage is

$$N_{2oj}=\frac{\frac{4(m_2+m'_2)}{4m_2+4m'_2-6m_2m'_2}\left(a_{2oj}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}}-a_{02}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}}\right)}{2[4K_{2fc}\delta_{2fc}(1-K_{2m}/K_{2fc})]^{-\frac{m_2m'_2}{m_2+m'_2}}\times v_{pv}\times[(y_2a/b)\ \Delta K_2\cdot\Delta\delta_t]^{\frac{m_2m'_2}{m_2+m'_2}}},(Cycle)(\sigma_m\neq 0) \tag{27}$$

2) σ-stress method

Due to word stress is still $\sigma/\sigma_s<<1$($\sigma\le 0.5\sigma_s$), the long crack growth residual life equation of corresponding reversed direction curve A_2A_1 in fig.1 is as following form

$$N_{2eff}=\int_{a_{tr}}^{a_{2eff}}\frac{da_2}{A_2^*\times(\Delta Q_2)^{\frac{m_2m'_2}{m_2+m'_2}}},(Cycle) \tag{28}$$

Here

$$Q_2=\frac{\sigma^3}{E\sigma_s}(\sqrt{\pi a_2})^3,\left(MPa\cdot(\sqrt{mm})^3\right) \tag{29}$$

$$\Delta Q_2=\frac{\Delta\sigma^3}{E2\sigma_s}(\sqrt{\pi a_2})^3,\left(MPa\cdot(\sqrt{mm})^3\right) \tag{30}$$

The Q_2 -factor is a driving force model for long crack growth under monotonous load; the ΔQ_2 - factor is driving force under fatigue loading, which for above Q_2 - factor and ΔQ - factor are all calculated by the stress. And the comprehensive material constants A_2^* in eqn (28) can also expressed by the stress and to be also calculable material constant, that are as following:

$$A_2^{*"}=2\left(\frac{\sigma'^3_f}{E2\sigma_s}(\sqrt{\pi a_{2f}})^3\right)^{-\frac{m_2m'_2}{m_2+m'_2}}\times v_{pv},\ \left(MPa\cdot(\sqrt{mm})^3\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle)\ (\sigma_m=0) \tag{31}$$

$$A_2^*=2\left(\frac{\sigma'^3_f}{E2\sigma_s}(\sqrt{\pi a_{2f}})^3(1-\sigma_m/\sigma'_f)\right)^{-\frac{m_2m'_2}{m_2+m'_2}}\times v_{pv},\left(MPa\cdot(\sqrt{mm})^3\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle)\ (\sigma_m\neq 0) \tag{32}$$

So for $\sigma_m=0$, its final expansion equation corresponded to reversed direction curve A_2A_1 is as below form,

$$N_{2ff}=\frac{\frac{4(m_2+m'_2)}{4m_2+4m'_2-6m_2m'_2}\left(a_{2eff}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}}-a_{02}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}}\right)}{2\left(\frac{\sigma'^3_{2f}}{E2\sigma_s}(\sqrt{\pi a_{2eff}})^3\right)^{-\frac{m_2m'_2}{m_2+m'_2}}\times v'_{pv}\times\left(y_2(a/b)\frac{\Delta\sigma^3}{E2\sigma_s}(\sqrt{\pi a_2})^3\right)^{\frac{m_2m'_2}{m_2+m'_2}}}, \tag{33}$$

For $\sigma_m\neq 0$, the life equation corresponded to reversed direction curve D_2D_1 is following

$$N_{2eff}=\frac{\frac{4(m_2+m'_2)}{4m_2+4m'_2-6m_2m'_2}\left(a_{2eff}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}}-a_{02}^{\frac{2m_2+2m'_2-3m_2m'_2}{2(m_2+m'_2)}}\right)}{2\left(\frac{(\sigma'_f)^3}{E2\sigma_s}(\sqrt{\pi a_{2eff}})^3(1-\sigma_m/\sigma'_f)\right)^{-\frac{m_2m'_2}{m_2+m'_2}}\times v_{pv}\times\left(y_2(a/b)\frac{(\Delta\sigma)^3}{E2\sigma_s}(\sqrt{\pi a_2})^3\right)^{\frac{m_2m'_2}{m_2+m'_2}}} \tag{34}$$

(2) Under work tress $\sigma>\sigma_s$ condition

2) Q_2 -factor method

Under $\sigma>\sigma_s$ condition, due to the materials occur plastic strain, the exponent of its equation also to show change from m_2' to λ_2 ; and due to occur cyclic hysteresis loop effect, its effective life calculable models corresponded to reversed curve C_2C_1 in attached figure 1 is as below form

$$N_{2eff}=\int_{a_{tr}}^{a_{eff}}\frac{da_2}{B_2^*\times(\Delta Q_2/2)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}},(Cycle),(\sigma>\sigma_s) \tag{35}$$

Where B_2^* is also calculable comprehensive material constant, on exponent as compared with above eqn (18) and (19) that is not different,

$$B_2^* = 2[4(K_{2c}\delta_{2c})]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv}, \left(MPa\cdot(\sqrt{mm})^3\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle), (\sigma_m = 0) \tag{36}$$

$$B_2^* = 2[4K_{2fc}\delta_{2fc}(1-K_{2m}/K_{2fc})]^{-\frac{m_2\lambda'_2}{m_2+\lambda'_2}} \times v_{pv}, \left(MPa\cdot(\sqrt{mm})^3\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle), (\sigma_m \neq 0) \tag{37}$$

Where m_2 is an linear elastic exponent in long crack growth process, $m_2 = -1/b_2^{'}$. And λ_2 is a ductility exponent, $\lambda_2 = -1/c_2^{'}$. It should note its unit of the B_2^* is $\left(MPa\cdot(\sqrt{mm})^3\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle)$.

So the effective life expanded equation corresponded to reversed direction curve D_2D_1 (attached fig.1) should be

For $\sigma_m = 0$

$$N_{2eff} = \frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(a_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}} - a_{02}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2[4K_{2c}\delta_{2c}]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv}\times[0.25y_2(a/b)\Delta K_2\cdot\Delta\delta_t]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}}, (Cycle), (\sigma_m = 0) \tag{38}$$

For $\sigma_m \neq 0$

$$N_{2eff} = \frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(a_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}} - a_{02}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2[4K_{2fc}\delta_{2fc}(1-K_{2m}/K_{2fc})]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv}\times[0.25y_2(a/b)\Delta K_2\cdot\Delta\delta_t]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}}, (Cycle), (\sigma_m \neq 0), \tag{39}$$

Here

$$a_{2eff} = \frac{E\times\delta_{2eff}}{\pi\sigma_s(\sigma'_f/\sigma_s+1)}, (mm) \tag{40}$$

$$\delta_{eff} = (0.25\sim0.4)\delta_c, (mm) \tag{41}$$

σ-stress method

Due to work stress $\sigma > \sigma_s$, if adopt stress to express it, the B_2^* in eqn (35) should be

For $\sigma = 0$

$$B_2^* = 2\{[\frac{\sigma'_f\cdot\sigma_s(\sigma'_f/\sigma_s+1)}{E}(\sqrt{\pi a_{2f}})^3]\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv}, \left(MPa\cdot(\sqrt{mm})^3\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle) \tag{42}$$

For $\sigma \neq 0$

$$B_2^* = 2\{[\frac{\sigma'_f\cdot\sigma_s(\sigma'_f/\sigma_s+1)}{E}(\sqrt{\pi a_{2f}})^3](1-\sigma_m/\sigma'_f)\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv}, \left(MPa\cdot(\sqrt{mm})^3\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle) \tag{43}$$

Therefore the residual life equation of corresponded to reversed direction curve D_2D_1 in fig.1, its final expansion equation is as below form,

For $\sigma_m = 0$,

$$N_{2eff}=\frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(a_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}-a_{02}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2\{[\frac{\sigma'_f\cdot\sigma_s(\sigma'_f/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3]\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}}\times\frac{1}{\left([0.5\ \sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}},(cycle) \tag{44}$$

For $\sigma_m \neq 0$,

$$N_{2eff}=\frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(a_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}-a_{02}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2\{[\frac{\sigma'_f\cdot\sigma_s(\sigma'_f/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}}\times\frac{1}{\left([0.5\ \sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}},(cycle) \tag{45}$$

Here, influence to mean stress in eqn (45) usually can ignore. And it must be point that the units in the life equations are all cycle number.

2.3. The Life Prediction Calculations for Whole Crack Growth Process

Due to the behaviors of the short crack and the long crack are different, for availing to life calculation in whole process, author proposes it need to take a crack transitional size a_{tr} at transition point between two stages from short crack to long crack growth process, and the crack size a_{tr} at transition point can be derived to make equal between the crack growth rate equations by two stages, for instance [19],

$$(da_1/dN_1)_{a_{01}->a_{tr}} <= da_{tr}/dN_{tr} =< (da_2/dN_2)_{a_{tr}->a_{eff}} \tag{46}$$

Here the equation (46) is defined as the crack-growth-rate-linking-equation in whole process.

(1) Under work stress $\sigma < \sigma_s$

For $\sigma < \sigma_s, \sigma_m = 0$, its expanded crack-growth-rate-linking-equation for (46) corresponding to positive curve AA_1A_2 (attached fig.1) is as following form

$$\frac{da_1}{dN_1}=\left\{2[4(\sigma'_f\,\varepsilon'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v_{eff})^{-1}\times(\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}a_1\right\}_{a_1->a_{tr}} <= \frac{da_{tr}}{dN_{tr}} =<$$
$$\frac{da_2}{dN_2}=\left\{2\left(\frac{\sigma'^3_{eff}}{E2\sigma_s}(\sqrt{\pi a_{2eff}})^3\right)^{-\frac{m_2m'_2}{m_2+m'_2}}\times v_{pv}\times\left(y_2(a/b)\frac{\Delta\sigma^3}{E2\sigma_s}(\sqrt{\pi a_2})^3\right)^{\frac{m_2m'_2}{m_2+m'_2}}\right\}_{a_{tr}->a_{eff}},(\sigma_m=0)(mm/cycle) \tag{47}$$

For $\sigma < \sigma_s, \sigma_m \neq 0$, its extended crack-growth-rate-linking-equation for (45) corresponding to positive curve DD_1D_2 (attached fig.1)is as following form

$$\frac{da_1}{dN}=\left\{2[4\sigma'_f\,\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v_{eff})^{-1}\times(\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1m'_1}{m_1+m'_1}}a_1\right\}_{a_{01}->a_{tr}} <= \frac{da_{tr}}{dN_{tr}}$$
$$=< \frac{da_2}{dN}=\left\{2\left(\frac{\sigma'^3_{eff}}{E2\sigma_s}(\sqrt{\pi a_{2eff}})^3(1-\sigma_m/\sigma'_f)\right)^{-\frac{m_2m'_2}{m_2+m'_2}}\times v_{pv}\times\left(y_2(a/b)\frac{\Delta\sigma^3}{E2\sigma_s}(\sqrt{\pi a_2})^3\right)^{\frac{m_2m'_2}{m_2+m'_2}}\right\}_{a_{tr}->a_{eff}},dmm/cycle,(\sigma\neq 0) \tag{48}$$

And the life equations in whole process corresponding to reversed direction curves A_2A_1A and D_2D_1D should be as below

$$\sum N = N_1+N_2=\int_{a_{01}}^{a_{tr}}\frac{da}{A_1^*\times(\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}\times a}+\int_{a_{tr}}^{a_{2eff}}\frac{da}{A_2^*[y_2(a/b)\Delta K_2\cdot\Delta\delta_t]^{\frac{m_2m'_2}{m_2+m'_2}}}, \tag{49}$$

Its expanded equation corresponding to reversed direction curves A_2A_1A is as following form

$$\sum N = N_1 + N_2 = \int_{a_{01}}^{a_{tr}} \frac{da}{2[4(\sigma'_f \varepsilon'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1} \times (\Delta\sigma \cdot \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}} \times a}$$
$$+ \int_{a_{tr}}^{a_{2eff}} \frac{da}{2\left(\frac{\sigma'^3_{eff}}{E2\sigma_s}(\sqrt{\pi a_{2eff}})^3\right)^{-\frac{m_2 m'_2}{m_2+m'_2}} \times v_{pv}\left(y_2(a/b)\frac{\Delta\sigma^3}{E2\sigma_s}(\sqrt{\pi a})^3\right)^{\frac{m_2 m'_2}{m_2+m'_2}}}, (\sigma_m = 0) \tag{50}$$

But for $\sigma_m \neq 0$, its expanded equation corresponding to reversed direction curves $D_2 D_1 D$ should be

$$\sum N = N_1 + N_2 = \int_{a_{01}}^{a_{tr}} \frac{da}{2[4\sigma'_f \varepsilon'_f (1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1} \times (\Delta\sigma \cdot \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}} \times a}$$
$$+ \int_{a_{tr}}^{a_{2eff}} \frac{da}{2\left(\frac{\sigma'^3_{eff}}{E2\sigma_s}(\sqrt{\pi a_{2eff}})^3\right)^{-\frac{m_2 m'_2}{m_2+m'_2}} (1-\sigma_m/\sigma'_f) \times v_{pv}\left(y_2(a/b)\frac{\Delta\sigma^3}{E2\sigma_s}(\sqrt{\pi a_2})^3\right)^{\frac{m_2 m'_2}{m_2+m'_2}}}, (\sigma_m \neq 0) \tag{51}$$

(2) Under work stress $\sigma > \sigma_s$

Under work stress $\sigma > \sigma_s$, its expanded link rate equation for eqn (46) corresponded to positive curve CC_1C_2 is as following form

$$\frac{da_{1\text{tr}}}{dN_1} = \left\{ 2[4\sigma'_f \varepsilon'_f (1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1} \times (0.25\Delta\sigma \times \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}} a \right\}_{a_{01} -> a_{tr}} = \frac{da_{\text{tr}}}{dN}$$
$$= \frac{da_{2\text{tr}}}{dN_2} = \left\{ \begin{array}{l} 2\{[\frac{\sigma'_f \cdot \sigma_s (\sigma'_f/\sigma_s + 1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv} \\ \times\left([0.5\, \sigma \cdot \sigma_s (\sqrt{\pi a_2})^3 (\sigma/\sigma_s + 1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \end{array} \right\}_{a_{tr} -> a_{eff}}, (\sigma \neq 0), mm/cycle, \tag{52}$$

And the life equations in whole process corresponding to reversed direction curve C_2C_1C (attached fig.1)should be as following

$$\sum N = N_1 + N_2 = \int_{a_{01}}^{a_{tr}} \frac{da}{B_1 \times (0.25\Delta Q_1)^{m_1} \times D} + \int_{a_{tr}}^{a_{2eff}} \frac{da}{B_2 (0.25\Delta Q_2)^{\lambda_2}}, \tag{53}$$

So the expanded life prediction expression in whole process corresponded to reversed curve C_2C_1C, it should be

$$\sum N = \int_{a_{01}}^{a_{tr}} \frac{da}{2[4\sigma'_f \varepsilon'_f (1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1} \times (0.25\Delta\sigma \times \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}}} +$$
$$+ \int_{a_{tr}}^{a_{2eff}} \frac{da}{2\{[\frac{\sigma'_f \cdot \sigma_s (\sigma'_f/\sigma_s + 1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma'_f)\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv}} \times \frac{1}{\left([0.5\, \sigma \cdot \sigma_s (\sqrt{\pi a})^3 (\sigma/\sigma_s + 1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}} \tag{54}$$

It should point that the calculations for rate and life in whole process should be according to different stress level, to select appropriate calculable equation. Here must explain that its meaning of the eqns (46-48, 52) is to make link between the first stage rate and the second stage rate, Calculation method for which it should be calculated by the short crack growth rate equation before the transition point a_{tr}; it should be calculated by the long crack growth rate equation after the transition point a_{tr}, that is not been added together by the rates for two stages. But for the life calculations in whole process can be added together by two stages life. About calculation tool, it can be calculated by means of computer doing computing by different crack size [21].

3. Computing Example

3.1. Contents of Example Calculations

To suppose a pressure vessel is made with elastic-plastic steel 16MnR, its strength limit of material $\sigma_b = 573MPa$, yield limit $\sigma_s = 361MPa$, fatigue limit $\sigma_{-1} = 267.2MPa$, reduction of area is $\psi = 0.51$, modulus of elasticity $E = 200000MPa$; Cyclic strength coefficient $K' = 1165MPa$, strain-hardening exponent $n' = 0.187$; Fatigue strength coefficient $\sigma'_f = 947.1MPa$, fatigue strength exponent $b'_1 = -0.111$, m_1=9.009; Fatigue ductility coefficient $\varepsilon'_f = 0.464$, fatigue ductility exponent $c'_1 = -0.5395$, $m'_1 = 1.8536$. Threshold value $\Delta K_{th} = 8.6\text{MPa}\sqrt{\text{m}}$, critical stress intensity factor $K_{2c} = K_{1c} = 92.7\text{MPa}\sqrt{\text{m}}$. Its working stress $\sigma_{\max} = 450MPa$, $\sigma_{\min} = 0$ in pressure vessel. Suppose that for long crack shape has been simplified via treatment become an equivalent through-crack, the correction coefficient $y_2(a/b)$ of crack shapes and sizes equal 1. Other computing data are all in table 1.

Table 1. *Computing data.*

$K_{1c}, MPa\sqrt{m}$	$K_{eff}, MPa\sqrt{m}$	$K_{th}, MPa\sqrt{m}$	v_{pv}	m_2	δ_c, mm	λ_2	$y_2(a/b)$
92.7	28.23	8.6	2×10^{-4}	3.91	0.18	2.9	1.0

3.2. Required Calculating Data

Try to calculate respectively as following different data and depicting their life curves：

(1) To calculate the transitional point crack size a_{tr} between two stages;

(2) To calculate the crack growth rate for transitional point a_{tr};

(3) To calculate the life N_1 in first stage from short crack growth $a_1 = 0.02mm$ growth to transitional point a_{tr};

(4) To calculate the effective life N_{2eff} in second stage from transitional point crack size a_{tr} to long crack $a_{2\text{eff}} = 5mm$;

(5) Calculating the whole service life $\sum N$

(6) To depict life curves in whole process.

3.3. Calculating Processes and Methods

The concrete calculation methods and processes are as follows

(1) Calculations for relevant parameters

1) Total strain

To calculate stress-strain data by reference (31):

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}} = \frac{450}{200000} + \left(\frac{450}{1165}\right)^{\frac{1}{0.187}},$$
$$= 2.25\times10^{-3} + 6.178\times10^{-3} = 8.4278\times10^{-3}(m) = 8428\mu\varepsilon$$

$$\varepsilon_{\max} = 8.4278\times10^{-3}(m) = 8428\mu\varepsilon;\ \varepsilon_{\min} = 0$$

$$\varepsilon_m = (\varepsilon_{\max} + \varepsilon_{\max})/2 = (8.4278\times10^{-3} + 0)/2 = 4.2148\times10^{-3}$$

2) Stress range calculation:

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} = 450 - 0 = 450(MPa)$$

3) Mean stress calculation: $\sigma_m = (\sigma_{\max} + \sigma_{\min})/2 = (450 - 0)/2 = 225MPa$

4) Total strain range:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} = \frac{450}{200000} + 2\left(\frac{450}{2\times1165}\right)^{\frac{1}{0.187}} = 2.25\times10^{-3} + 3.034\times10^{-4} = 2.5534\times10^{-3}(m) = 2.553mm$$

5) Elastic strain range:

$$\Delta\varepsilon_e = 2.25\times10^{-3}(m) = 2.25mm;$$

6) Plastic strain range:

$$\Delta\varepsilon_p = 3.034\times10^{-4}(m) = 0.3034mm$$

7) According to formulas (7), calculation for correction coefficient v_{eff} in first stage

$$v_{eff} = a_{eff}\ln[1/(1-\psi)] = 2\times\ln[1/(1-0.51)] = 1.43,(\text{mm/cycle})$$

8) Take virtual rate v_{pv} in second stage by eqn (20)

$$v_{pv} = \frac{a_{2eff} - a_{02}}{N_{2f} - N_{02}} \approx 2.0\times10^{-4}(mm/Cycle),\quad N_{2f} = 1,$$
$$N_{02} = 0$$

9) According to formulas (38), Calculating effective size

a_{eff}

$$a_{eff} = \frac{E\times\delta_{eff}}{\pi\sigma_s(\sigma_f/\sigma_s+1)} = \frac{200000\times 0.25\times 0.18}{\pi 361(947.1/361+1)} = 2.1(mm),$$

take $a_{eff} = 2.0mm$.

Here take effective crack size $a_{1eff} = a_{2eff} = 2mm$ in first and the second stage.

(2) To calculate the transitional point crack size a_{tr} between two stages:

1) Calculation for comprehensive material constant A_1^* in first stage by eqn (5)

$$A_1^* = 2[4\sigma'_f\,\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}}\times(a_{eff}\times v_f)^{-1}$$

$$= 2[4(947.1\times 0.464)(1-225/947.1)]^{-\frac{9.009\times 1.8536}{9.009+1.8536}}\times(2\times 0.713)^{-1}$$

$$= 2.216\times 10^{-5}\,(\mathrm{MPa}^{\frac{m_1 m'_1}{m_1+m'_1}}\,\mathrm{mm}/cycle)$$

2) Calculation for comprehensive material constant B_2^* in second stage by eqn (43)

$$B_{2eff}^* = 2\{[\frac{\sigma'_f\cdot\sigma_s(\sigma'_f/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv}$$

$$= 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv} =$$

$$2\{[\frac{947.1\times 361(947.1/361+1)}{200000}(\sqrt{\pi\times a_{2eff}})^3](1-225/947.1)\}^{-\frac{3.91\times 2.9}{3.91+2.9}}\times 2\times 10^{-4}$$

$$= 2\{[6.1945\times\pi^{1.5}2^{1.5}]0.7624\}^{-1.665}\times 2\times 10^{-4} = 2\{74.381\}^{-1.665}\times 2\times 10^{-4} = 3.0625\times 10^{-7}$$

$$[MPa\cdot(\sqrt{mm})^3]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle) .$$

3) Calculation for transitional point crack size a_{tr}

According to the equations (46) and (52),

$$(da_1/dN_1)_{a_{01}->a_{tr}} <= da_{tr}/dN_{tr} =< (da_2/dN_2)_{a_{tr}->a_{eff}}$$

Then, for the transitional point crack size a_{tr} between two stages it can make link between the rate expansion at left side and the rate one at right side as following

$$A_1^*\times(0.25\times\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1 m'_1}{m_1+m'_1}}\times a_{tr} = A_2^*[(\Delta\sigma/2)\cdot 0.5\sigma_s(\sqrt{\pi a_{tr}})^3(\Delta\sigma/2\sigma_s+1)/E]^{\frac{m_2\lambda_2}{m_2+\lambda_2}};$$

$$2[4\sigma'_f\,\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}}\times(a_{1eff}\times v_f)^{-1}\times(0.25\times\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1 m'_1}{m_1+m'_1}}\times a_{tr}$$

$$= 2\{[\frac{\sigma'_f\cdot\sigma_s(\sigma'_f/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv}\times[0.5(\Delta\sigma/2)\cdot\sigma_s(\sqrt{\pi a_{tr}})^3(\Delta\sigma/2\sigma_s+1)/E]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}$$

$$2[4(947.1\times 0.464)(1-225/947.1)]^{-\frac{9.009\times 1.8536}{9.009+1.8536}}\times(2\ \times 0.7133)^{-1}\times(0.25\times 2.553\times 10^{-3}\times 450)^{\frac{9.009\times 1.8536}{9.009+1.8536}}\times a_{tr} =$$

$$= \{[\frac{947.1\times 361(947.1/361+1)}{200000}(\sqrt{\pi\times 2})^3](1-225/947.1)\}^{-\frac{3.91\times 2.9}{3.91+2.9}}$$

$$\times 2\times 10^{-4}\times[0.5(450/2)\cdot 361(\sqrt{\pi a_{tr}})^3(450/2\times 361+1)/E]^{\frac{3.91\times 2.9}{3.91+2.9}}$$

Then make simplified calculation:

$$3.22\times 10^{-6}a_{tr} = 2.6695\times 10^{-6}\times a_{tr}^{2.4975};$$

$$a_{tr} = 1.2062^{0.6678} = 1.133(mm);$$

So obtain the transitional point crack size $a_{tr} = 1.133(mm)$ between two stages.

The crack growth rate calculations for transitional point a_{tr}

$$da_1/dN_1 = da_{tr}/dN_{tr} = 3.22\times 10^{-6}\times a_{tr} = 3.22\times 10^{-6}\times 1.133 = 3.648\times 10^{-6}(mm/cycle)$$

$$da_2/dN_2 = da_{tr}/dN_{tr} = 2.6695\times 10^{-6}a_{tr}^{2.4975} = 2.6695\times 10^{-6}\times 1.133^{2.4975} = 3.646\times 10^{-6}(mm/cycle)$$

Here it can be seen, the crack growth rate at the transition point crack size $a_{tr}=1.113(mm)$ is same, it is 3.646×10^{-6} $(mm/cycle)$.

(3) Life prediction calculations in whole process

1) Select life predicting calculation equation (12), the calculable life N_1 in first stage from short crack $a_{01}=0.02mm$ to transitional point $a_{tr}=1.113(mm)$ is as follow,

$$N_1=\frac{\ln a_{tr}-\ln a_{01}}{2[4\sigma'_f\,\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(a_{1eff}\times v_f)^{-1}\times(0.25\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1m'_1}{m_1+m'_1}}}$$

$$=\frac{\ln1.133-\ln0.02}{2[4(947.1\times0.464)(1-225/947.1)]^{-\frac{9.009\times1.8536}{9.009+1.8536}}\times(2\times0.7133)^{-1}}\times\frac{\ln1.133-\ln0.02}{3.22\times10^{-6}}=\frac{4.0369}{3.22\times10^{-6}}=1253693(Cycle)$$

So first stage life $N_1=1253693(Cycle)$

And from above we can derive simplified life equation in first stage corresponded to different damage value as follow form $N_1=\dfrac{1}{3.22\times10^{-6}a_1}$

Select life predicting calculation equation (45), the calculable life N_2 in second stage from transitional point crack size $a_{tr}=1.113(mm)$ to $a_{eff}=5mm$ is as follow,

From above formula we can also derive simplified life equation corresponding different damage value as follow form → $N_2=\dfrac{1}{2.6695\times10^{-6}a^{2.9}}$

Therefore, whole process life is $\sum N=N_1+N_2=1253693+185014=1438707(Cycle)$

If use integral equation (53) and (54) to calculate the service life in whole process, it is

$$\sum N=\int_{a_{01}}^{a_{tr}}\frac{da}{A_1^*\times Q^{\frac{m_1m'_1}{m_1+m'_1}}}+\int_{a_{tr}}^{a_{2eff}}\frac{da}{A_2^*\times Q^{\frac{m_2\lambda_2}{m_1+\lambda_2}}}=\int_{a_{01}}^{a_{tr}}\frac{da}{2[4\sigma'_f\,\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(a_{1eff}\times v_f)^{-1}\times(0.25\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1m'_1}{m_1+m'_1}}\times a}$$

$$+\int_{a_{tr}}^{a_{2eff}}\frac{da_2}{2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv}}\times\frac{1}{[0.5(\Delta\sigma/2)\cdot\sigma_s(\sqrt{\pi a_2})^3(\Delta\sigma/2\sigma_s+1)/E]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}}=$$

$$=\int_{0.02}^{1.133}\frac{da}{2[4(947.1\times0.464)(1-225/947.1)]^{-\frac{9.009\times1.8536}{9.009+1.8536}}\times(2\times0.713)^{-1}}\times\frac{1}{(0.25\times2.553\times10^{-3}\times450)^{1.5373}\times a_1}$$

$$+\int_{1.133}^{5}\frac{da_2}{2\{[\frac{947.1\times361(947.1/361+1)}{200000}(\sqrt{\pi\times2})^3](1-225/947.1)\}^{-\frac{3.91\times2.9}{3.91+2.9}}\times2\times10^{-4}}$$

$$\times\frac{1}{[0.5(450/2)\cdot\sigma_s(\sqrt{\pi a_2})^3(450/2\sigma_s+1)/200000]^{\frac{3.91\times2.9}{3.91+2.9}}}$$

$$=\int_{0.02}^{1.133}\frac{da}{2.216\times10^{-5}\times0.1469\times a_1}+\int_{1.133}^{5}\frac{da}{3.0625\times10^{-7}\times\{3.670933\}^{1.665}a_2^{2.4975}}$$

$$=\int_{0.02}^{1.133}\frac{da}{3.22\times10^{-6}a_1}+\int_{1.133}^{5}\frac{da}{2.6695\times10^{-6}\times a_{tr}^{2.4975}}=1300000+185020=1485020(Cycle)$$

3.4. Calculating Results

3.4.1. Life Data of Each Stage and Whole Process

Calculating results are that the first stage life is $N_1=1300000$ cycle from micro-crack size 0.02 mm to transition point size 1.113mm; the second stage life is 185020 cycle from transition point-crack size 1.113 mm growth to long crack length 5 mm; and the service life in whole process is 1485020 cycle. This result is consistent with expansion equations calculation results data $\sum N=N_1+N_2=1253693+185014=1438707(Cycle)$.

The life data corresponded to different crack length are all included in table 2, 3 and 4. Author finds these data are calculated by two-parameter-multiplication-method that are basically close as compared with another result data: which are calculated by the single-calculated method, it has been published recently in reference [21]. It should be point that the data calculated by two-parameter-multiplication-method are nicer and steady in whole process.

Table 2. *Crack growth life data in whole process.*

Data point of number	**1**	**2**	**3**	**4**	**5**
Crack size (mm)	0.02	0.04	0.1	0.2	0.4
Data of the first stage	15527950	7763975	3105590	1552795	776398
Data of the second stage	Useless section				

Table 3. *Crack growth life data in whole process.*

Data point of number	**6**	**7**	**8 transition point**	**9**
Crack size (mm)	0.6	0.7	1.113	1.5
Data of the first stage	517598	44365	274103	207039
Data of the second stage	1341644	912931	274240	136076

Table 4. *Crack growth life data in whole process.*

Data point of number	**9**	**10**	**11**	**12**	**13**
Crack size (mm)	1.5	2.0	3.0	4	5
Data of the first stage	207039	155280	Useless section		
Data of the second stage	136076	66336	24097	11747	6728

3.4.2. To Depict the Life Curves in Each Stage and Whole Process

According to crack growth data for deferent crack size depicting the curves in whole process, which are showed in figures 2 and 3.

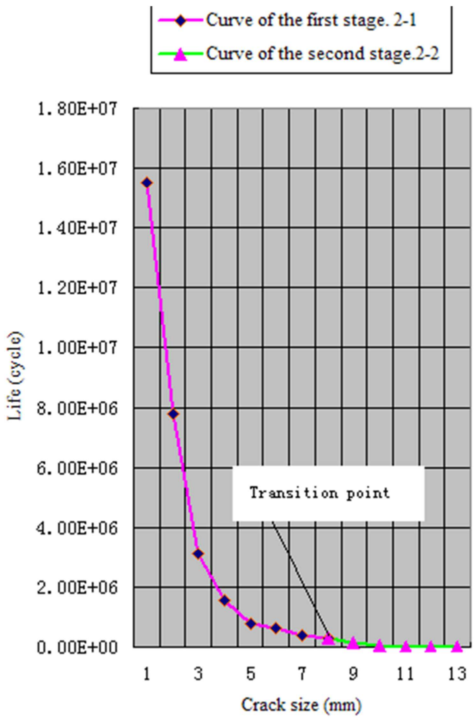

Figure 2. *Life curve in whole course (in decimal coordinate system).*

(A) 2-1-data curve in first stage obtained by two-parameters calculating method;
(B) 2-2-data curve in second stage obtained by two-parameters calculating method;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5 mm is just at eighth point (crack size 1.113*mm*).

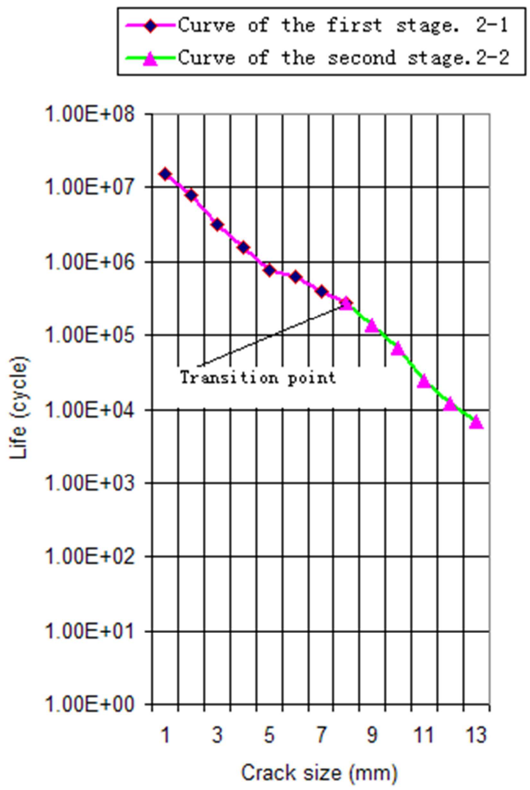

Figure 3. *Life curve in whole course (in logarithmic coordinate system).*

(A) 2-1-data curve in first stage obtained by two-parameters calculating method;
(B) 2-2-data curve in second stage obtained by two-parameters calculating method;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5mm is just at eighth point (crack size1.113*mm*).

4. Discussions

Author puts forward such a view point: in the mechanics, the aviation, the machinery and the civil engineering etc fields, there are a scientific law of similar to gene principle and cloning technique. Because where there are common scientific laws as life science:

1) Each unit cell combined in a genetic structure has all its own genetic (or inheritable) character;

2) To have the clonable (or can copy) and the transferable characters, and can be recombined together by inherent relationship;

3) The new combined structures there are also with new characters and functions.

Please note: Those parameters inside from equations (1) to (54) in this paper, which there are all as "genetic character of unit cells", e.g. as the stress parameter σ , the strain ε and their material constants σ_s , ψ, E etc subjected traditional materials mechanics; and these "genetic elements" all to maintain their original calculable properties and functions. But once they are transferred into new areas as the fracture mechanics or the damage mechanics, they have been formed by new structure-equations. Then the new equations have also been shown with new calculable functions, as those equations in the traditional material mechanics. Author just is according to such thinking logics and methodologies, and by means of various relatedness among the material constants, e.g. $m_1 = -1/b_1^{'}$, $m_1^{'} = -1/c_1^{'}, n^{'} = b_1^{'}/c_1^{'}, m_1 = -1/c_1^{'}n^{'}$; $m_2 = -1/b_2^{'}$, $\lambda_2 = -1/c_2^{'}$ etc, and according to the cognition for their physical and geometrical significance for key parameters, thereby derives the above mentioned a lot of calculable models.

5. Conclusions

In above text, author adopts the σ -method and the Q_1 -factor method to calculate various parameters and a mass of data, here can obtain following common conclusions:

(1) About comparison of calculating results: As compared with the single-parameter method to find, that above mentioned calculated data by the multiplication-method of two-parameters are nicer and steady in whole process.

(2) About new cognitions for key parameters: True material constants must show the inherent characters of materials, such as the σ_s and E, δ, ψ etc in the material mechanics; for instance the σ'_f , ε'_f , $b_1^{'}$, c'_1 and so on in the fatigue- damage mechanics; for example the $K_{2c}(=K_{1c})$, A_2 and m_2 and so on in the fracture mechanics; which could all be checked and obtained from general handbooks; But some new material constants in the fracture mechanics that are essentially to have functional relations with other materials constants, for which they can be calculated by means of the relational conventional materials constants under the condition of combining experiments to verify, e.g. eqns (4-5), (18-19), (31-32), (36-37), (42-43), (46-48), (52), etc. Therefore for this kind of material constants should be defined as the comprehensive materials constants.

(3) About cognitions of the physical and the mathematical meanings for key parameters: The parameters A_1^* in the first stage and A_2^*, $A_2^{*''}, B_2^*$ in the second stage, Their physical meanings are all a concept of the power, just are a maximal increment value paying energy in one cycle before to cause failure. Their geometrical meanings are all a maximal micro-trapezium area approximating to beeline.

(4) About the methods for crack propagation rate and lifetime calculations: Calculation for crack transition size a_{tr} it can be calculated form the crack-growth-rate-linking-equations (46-48) and (52) in whole process; before the transition point it should be calculated by short crack growth rate equations (da_1/dN_1); after transition point a_{tr} it should be calculated via long crack growth rate (da_2/dN_2) equations. But for the lifetime calculations in whole process can be added together by life cycle number of two stages.

(5) Based on the traditional material mechanics is a calculable subject; in consideration of the conventional material constants there are "the hereditary characters"; In view of the relatedness and the transferability between related parameters among each disciplines; And based on above viewpoints and cognitions of the (1)~(4); then for the fatigue and the fracture disciplines, if make them become calculable subjects, that will be to exist possibility.

Acknowledgments

At first author sincerely thanks scientists David Broek, Miner, P. C. Paris, Coffin, Manson, Basquin, Y. Murakami, S. Ya. Yaliema, Morrow J D, Chuntu Liu, Shaobian Zhao, Jiazhen Fan, etc, they have be included or no included in this paper reference, for they have all made out valuable contributions for the fatigue-damage-fracture subjects. Due to they hard research, make to discover the fatigue damage and crack behavioral law for materials, to form the modern fatigue-damage-fracture mechanics; due to they work like a horse, make to develop the fatigue-damage-fracture mechanics subjects, gain huge benefits for accident analysis, safety design and operation for which are mechanical equipments in engineering fields. Particularly should explain that author cannot have so many of discovery and provide above the calculable mathematical models and the combined figure 1, if have no their research results.

Author thanks sincerity the Zhejiang Guangxin New Technology Application Academy of Electromechanical and Chemical Engineering gives to support and provides research funds.

Nomenclature

a_{o1} and a_{oi}	initial crack and medial short crack in the first stage
a_{tr} and a_{mac}	crack transition size from short crack to long crack and macro crack size, $a_{tr} \approx a_{mac}$
N_{oi}	medial life in first stage,
N_1	life in first stage
$m_1, b_1^{'}$	fatigue strength exponent under high cycle fatigue in first stage, $m_1 = -1/b_1^{'}$
$m'_1, c_1^{'}$	fatigue ductility exponent under low cycle fatigue in first stage, $m'_1 = -1/c_1^{'}$
A_1^*	comprehensive material constant in first stage
ϕ	reduction of area
$\sigma_f^{'}$	fatigue strength coefficient under fatigue loading
$\varepsilon_f^{'}$	fatigue ductility coefficient under fatigue loading
Q_1 and ΔQ_1	two-parameter stress-strain factor and stress-strain factor range of short crack in first stage
a_{02}	initial crack size in the second stage
$a_{fc} = a_c$	critical crack size to make fracture in one cycle
$m_2, b_2^{'}$	fatigue strength exponent under high cycle fatigue in second stage, $m_2 = -1/b_2^{'}$
$\lambda_2, c_2^{'}$	fatigue ductility exponent under low cycle fatigue in second stage, $\lambda_2 = -1/c_2^{'}$
A_2^*	comprehensive material constant in the second stage
A_{2eff}^*	effective comprehensive material constant in the second stage
$\delta_{2t}, \Delta\delta_{2t}$	crack tip open displacement and crack tip open displacement range in second stage
K_{2eff}	effective stress intensity factor in second stage
δ_{eff}	effective crack tip open displacement in second stage
K_m	mean stress intensity factor
δ_c	critical crack tip open displacement
$K_{1c} = K_{2c}$	fracture toughness.
$Q_2, \Delta Q_2$	two-parameter stress-strain factor and stress-strain factor range of long crack in second stage
v_{pv}	virtual rate in the second stage
$a_{o2}, a_{oj}, a_{eff}, a_{2c}$	initial, medial , effective and critical size of crack in the second stage, $a_{o2} \le a_{oj} \le a_{eff} < a_{2c}$
$N_{o2}, N_{oj}, N_{eff}, N_{2c}$	initial life, medial life , effective and critical life in second stage, $N_{o2} \le N_{oj} \le N_{eff} < N_{2c}$
N_2	life in second stage
$\sum N$	lifetime in whole process

References

[1] Yangui Yu. Life Predictions Based on Calculable Materials Constants from Micro to Macro Fatigue Damage Processes. American Journal of Materials Research. Vol. 1, No. 4, 2014, pp. 59-73.

[2] Yu Yangui, Sun Yiming, MaYanghui and XuFeng. The Computing of intersecting relations for its Strength Problem on Damage and Fracture to Materials with short and long crack. 2011; In: International Scholarly Research Network ISRN. Mechanical Engineering Volume, Article ID 876396 (2011). http://www.hindawi.com/isrn/me/.

[3] Yangui Yu. The Calculations of Evolving Rates Realized with Two of Type Variables in Whole Process for Elastic-Plastic Materials Behaviors under Unsymmetrical Cycle. Mechanical Engineering Research. Canadian Center of Science and Education 2012; 2. (2):77-87; ISSN 1927-0607(print) E-ISSN 1927-0615 (Online).

[4] Yu Yangui, Xu Feng. Studies and Application of Calculation Methods on Small Crack Growth Behaviors for Elastic-plastic Materials. Chinese Journal of Mechanical Engineering. Vol. 43, 2007; 12: 240-245. (In Chinese).

[5] YU Yangui, LIU Xiang, ZHANG Chang sheng and TAN Yanhua. Fatigue damage calculated by Ratio-Method Metallic Material with small crack under un-symmetric Cyclic Loading. Chinese Journal of Mechanical Engineering. Vol. 19, 2006; 2: 312-316.

[6] YU Yangui. Fatigue Damage Calculated by the Ratio-Method to Materials and Its Machine Parts. Chinese Journal of Aeronautics. Vol. 16, 2003; 3: 157-161.

[7] Yu Yangui and LIU Xiang. Studies and Applications of three Kinds of Calculation Methods by Describing Damage Evolving Behaviors for Elastic-Plastic Materials. Chinese Journal of Aeronautics. Vol, 19, 2006; 1: 52-58.

[8] Yangui Yu, Several kinds of Calculation Methods on the Crack growth Rates for Elastic-Plastic Steels. In: 13th International Conference on fracture (ICF13), (Beijing, 2013) In CD, ID S17-045.

[9] Smith K N, Watson P and Topper T H. A stress-strain function of the fatigue of metals. Journal of Materials. Vol. 5, 1970; 4: 767-778.

[10] Yu Yangui, Jiang Xiaoluo, Chen Jianyu and Wu Zhiyuan. The Fatigue Damage Calculated with Method of the Multiplication $\Delta\varepsilon_e\Delta\varepsilon_p$. In: Proceeding of the Eighth International Fatigue Congress, Vol. 5, Stockholm Sweden; June 3-7, 2002. p. 2815-2822.

[11] Morrow J D. Fatigue Design Handbook, Section 3.2, SAE Advances in Engineering; Vol. 4: Society for Automotive Engineers, Warrenddale, PA. 1968. pp. 21-29.

[12] Y. Murakami, S. Sarada, T. Endo. H. Tani-ishi, Correlations among Growth Law of Small Crack, Low-Cycle Fatigue Law and ,Applicability of Miner's Rule, Engineering Fracture Mechanics, Vol.18, 5. 1983: 909-924,

[13] Masing, G. Eigerspannungen and Verfestigung bein Messing, Proceeding of the 2nd International Congress of Applied Mechanics, Zurich, 1976, pp. 332-335.

[14] S. V. Doronin, et al., Ed. RAN U. E. Soken, Models on the fracture and the strength on technology systems for carry structures, (Novosirsk Science, 2005), PP. 160-165. (in Russian)

[15] S. Ya. Yaliema. Correction about Paris's equation and cyclic intensity character of crack. J. Strength Problem. Vol, 147, 1981; 9:20-28. (In Russian).

[16] Yu Yangui, MaYanghuia, The Calculation in whole Process Rate Realized with Two of Type Variable under Symmetrical Cycle for Elastic-Plastic Materials Behavior, in: 19th European Conference on Fracture, (Kazan, Russia, 26-31 August, 2012), In CD, ID 510.

[17] Xian-Kui Zhu, James A. Joyce, Review of fracture toughness (G, K, J, CTOD, CTOA) testing and standardization, Engineering Fracture Mechanics, 85, 2012: 1-46.

[18] U. Zerbst, S. Beretta, G. Kohler, A. Lawton, M. Vormwald, H.Th. Beier, C. Klinger,I. C erny´, J. Rudlin, T. Heckel a, D. Klingbeil, Safe life and damage tolerance aspects of railway axles – A review. Engineering Fracture Mechanics. 98, 214–271 (2013).

[19] Yangui Yu. Calculations for Crack Growth Rate in Whole Process Realized with the Single Stress-Strain-Parameter Method for Elastic-Plastic Materials Contained Crack. Journal of Materials Sciences and Applications. Vol. 1, No. 3, 2015, pp. 98-106.

[20] Yu Yangui, Bi Baoxiang, MaYanghau, Xu Feng. Damage Calculations in Whole Evolving Process Actualized for the Materials Behaviors of Structure with Cracks to Use Software Technique. In: 12th International Conference on Fracture Proceeding. Ottawa, Canada. July 12-19, 2009. CD, Author Index Y, Yangui.

[21] Yangui Yu. The Life Predicting Calculations in Whole Process Realized by Calculable Materials Constants from short Crack to Long Crack Growth Process. International Journal of Materials Science and Applications. Vol. 4, No. 2, 2015, pp. 83-95.doi: 10.11648/j.ijmsa.20150402.13.