International Journal of High Energy Physics
2017; 4(4): 36-45
http://www.sciencepublishinggroup.com/j/ijhep
doi: 10.11648/j.ijhep.20170404.11
ISSN: 2376-7405 (Print); ISSN: 2376-7448 (Online)

Methodology Article

Interaction of the Electromagnetic Radiation Quantum and Material Particle in a Vector - Potential Space

Andrey Nikolaevich Volobuev[1, *], Eugene Sergeevich Petrov[1], Paul Konstantinovich Kuznetsov[2]

[1]Department of Medical and Biological Physics, Samara State Medical University, Samara, Russia

[2]Department of Electric - Drive, Samara State Technical University, Samara, Russia

Email address:

volobuev47@yandex.ru (A. N. Volobuev)

[*]Corresponding author

To cite this article:

Andrey Nikolaevich Volobuev, Eugene Sergeevich Petrov, Paul Konstantinovich Kuznetsov. Interaction of the Electromagnetic Radiation Quantum and Material Particle in a Vector - Potential Space. *International Journal of High Energy Physics*. Vol. 4, No. 4, 2017, pp. 36-45. doi: 10.11648/j.ijhep.20170404.11

Received: September 2, 2017; **Accepted**: September 15, 2017; **Published**: October 9, 2017

Abstract: Interaction of a photon and material particle (electron and atom) is considered. It is shown that this process needs to be described in the uniform reference system moving with light velocity in space of a vector - potential. On the basis of the Noether's theorem it is shown that in a vector - potential space the volumetric density of photon energy, its velocity and a ring current density of a material particle are conserved. On the basis of the Schrodinger's equation solving for a photon and electron cooperating in vector - potential space it is shown the electron during interaction should represent the quantum oscillator with a discrete set energies. Electron fluctuations or Dirac's electron "jitter" are realized with a light velocity. The problem of an electron magnetic moment (spin) occurrence in a vector - potential space is considered. Conditions of the atom currents quantization in vector - potential space, and also the Heisenberg's uncertainty principle in this space are submitted. The Lamb's frequency shift in vector - potential space is found. The multiphoton system in a vector - potential space is investigated.

Keywords: A Photon, An Electron, Dirac's Electron "Jitter", Nonlinear Schrodinger's Equation, Quantum Oscillator, Magnetic Moment, Spin, Electronic and Atomic Ring Currents

1. Introduction

At calculation of a photon and material particle interaction there is a line of problems. In particular, interaction of radiation and free electrons can take place in the various ways. It can be coherent direct reradiation of a photon on a free electron or reradiation of a photon with change of a direction of its movement (Thomson's scattering), not coherent scattering (Compton's effect) that is typical for high energies quantums: X-ray and γ radiation. The basic paradigm of quantum electrodynamics consists that a photon or quantum an indivisible particle. Consequence of it is two-stage character of a photon and electron interaction. First the photon is completely absorbed by an electron, and then the electron radiates a photon.

In spite of the fact that in reality absorption of a photon by an electron, obviously, takes place, a calculation of this process is inconvenient. The law of an impulse preservation at absorption of a photon by motionless electron looks like $\frac{\hbar\omega}{c} = mV$ where V there is electron velocity after photon absorption, m - its relativistic mass, ω - photon frequency, $\hbar$ - Planck's reduced constant, c - light velocity in vacuum. The law of energy conservation in relativistic form can be written down as:

$$\hbar\omega = mc^2 - m_0c^2 = mc^2\left(1-\frac{m_0}{m}\right) = mc^2\left(1-\sqrt{1-\frac{V^2}{c^2}}\right)$$

where m_0 there is electron rested mass.

Having divided the law of energy conservation on the law

of an impulse preservation we receive $\frac{V}{c}=\left(1-\sqrt{1-\frac{V^2}{c^2}}\right)$, т.е. $V=c$, that is impossible, at least, in Euclidian space.

The received result is not casual. The matter is that the photon exists in the reference system moving with a light velocity. To describe interaction of a photon and electron it is necessary that the electron also it was examined in the reference system moving with a light velocity. In it the physical sense of the received result will consist.

If the electron is not examined in the reference system moving with a light velocity that, for example, in process of the Dirac's equation solving by the method of perturbation theory at interaction of a photon and electron there arise poorly proved with the physical point of view so-called virtual electron states [1].

According to the special theory of a relativity all particles at the movement with velocity V have reduction of the size under the law [2] $l=l_0\sqrt{1-\frac{V^2}{c^2}}$ where l there is the size of a moving particle, l_0 - the size of a particle rested in the given reference system. For a photon if it to consider as a particle [3] moving with a light velocity the length in Euclidian space is equal to zero $l=0$. But if to examine a photon in the inertial reference system moving with a light velocity in which it actually is rested it is possible to receive spatial display of a photon. It has been made in work [4] where a vector - potential space which is propagated with a light velocity was used.

In the reference system moving with a light velocity in a vector - potential space it is possible to investigate various processes. In particular, in the given work processes of a photon and material particles (electron, atom) interaction, occurrence of the electron magnetic moment are examined. However, this reference system has the restrictions. First of all at it there can not be a mass, and a size in Euclidian coordinates. The mass in this reference system aspire to infinity $m=\frac{m_0}{\sqrt{1-\frac{V^2}{c^2}}}$, and the size of any material particle to zero. However, fields, currents, energy, frequency, etc. in it can be present, and it is frequently enough for the analysis of some processes.

2. Conservative Parameters in a Vector - Potential Space

First of all we shall find out what parameters in a vector - potential space are preserved, i.e. are invariant. For this purpose we use Noether's theorem in this space [5].

The volumetric density of action in a vector – potential space is equal $s(\mathbf{q},\dot{\mathbf{q}},t)=\int ldt$ where $\mathbf{q}=-\frac{\mathbf{A}}{c}$ there is generalized (for Lagrange's equation Лагранжа $\frac{d}{dt}\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)=\frac{\partial l}{\partial\mathbf{q}}$ [2]) the independent coordinate connected to vector - potential $\boldsymbol{A}$, l - Lagrangian of systems a photon - electron [4], t - time. The variation of action is size $\delta s(\mathbf{q},\dot{\mathbf{q}},t)=\delta\int ldt$, and according to a principle of the least action (in the nature this principle is always realized) $\delta l=0$ [6].

Let's find a variation of the Lagrangian:

$$\delta l=\frac{\partial l}{\partial\mathbf{q}}\delta\mathbf{q}+\frac{\partial l}{\partial\dot{\mathbf{q}}}\delta\dot{\mathbf{q}}+\frac{\partial l}{\partial t}\delta t\,. \tag{1}$$

We carry out infinitesimal displacement initially time, and then the generalized coordinate. Using Lagrange's equation we shall replace the first term in (1):

$$\begin{aligned}\delta l&=\frac{d}{dt}\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)\delta\mathbf{q}+\frac{\partial l}{\partial\dot{\mathbf{q}}}\delta\dot{\mathbf{q}}+\frac{\partial l}{\partial t}\delta t=\frac{d}{dt}\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)\delta\mathbf{q}+\frac{\partial l}{\partial\dot{\mathbf{q}}}\left(\frac{d(\delta\mathbf{q})}{dt}\right)+\frac{\partial l}{\partial t}\delta t=\\&=\frac{d}{dt}\left(\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)\delta\mathbf{q}\right)+\frac{\partial l}{\partial t}\delta t=\frac{d}{dt}\left(\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)\delta\mathbf{q}\right)+\frac{dl}{dt}\delta t'\end{aligned} \tag{2}$$

where $\delta t\to\delta t'$ there is some infinitesimal a time displacement.

Taking into account $\frac{d(\delta t)}{dt}=\delta 1=0$ we have $\frac{dl}{dt}\delta t=\frac{dl}{dt}\delta t+l\frac{d(\delta t)}{dt}=\frac{d(l\delta t)}{dt}$, and hence $\frac{dl}{dt}\delta t'=\frac{d\left(l\delta t'\right)}{dt}$. Thus:

$$\delta l=\frac{d}{dt}\left(\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)\delta\mathbf{q}\right)+\frac{d\left(l\delta t'\right)}{dt}=\frac{d}{dt}\left(\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)\delta\mathbf{q}+l\delta t'\right) \tag{3}$$

We carry out infinitesimal displacement of the generalized coordinate:

$$\delta\mathbf{q}\to\delta\mathbf{q}'=\delta\mathbf{q}+\frac{\partial\mathbf{q}}{\partial t}\delta t'=\delta\mathbf{q}+\dot{\mathbf{q}}\delta t' \tag{4}$$

Using (4) the formula (3) can be written down as:

$$\delta l=\frac{d}{dt}\left(\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)\left(\delta\mathbf{q}'-\dot{\mathbf{q}}\delta t'\right)+l\delta t'\right)=\frac{dQ}{dt} \tag{5}$$

Let's assume that changes of variations $\delta t\to\delta t'$ and $\delta\mathbf{q}\to\delta\mathbf{q}'$ results a variation of action (or Lagrangian) in zero $\delta l=0$. Hence we shall receive that the size (it sometimes name a Noether's charge) is equal:

$$Q=\left(\frac{\partial l}{\partial\dot{\mathbf{q}}}\right)\left(\delta\mathbf{q}'-\dot{\mathbf{q}}\delta t'\right)+l\delta t'=\left(l-\frac{\partial l}{\partial\dot{\mathbf{q}}}\dot{\mathbf{q}}\right)\delta t'+\frac{\partial l}{\partial\dot{\mathbf{q}}}\delta\mathbf{q}'=const \tag{6}$$

According to Noether's theorem the factors at variations of time $\delta t'$ and the generalized coordinate $\delta\mathbf{q}'$ are conserved.

According to [6] the size $l-\frac{\partial l}{\partial \dot{\mathbf{q}}}\dot{\mathbf{q}}=-w$ i.e. is equal to the volumetric density of the photon energy with the opposite sign. The size $\frac{\partial l}{\partial \dot{\mathbf{q}}}=\frac{\dot{\mathbf{q}}}{4\pi}=\frac{\mathbf{c}}{4\pi}$ i.e. is proportional to the light velocity in space of the generalized coordinates [4].

Thus, the formula (6) can be copied as:

$$Q=-w\delta t'+\frac{\mathbf{c}}{4\pi}\delta\mathbf{q}'=const \tag{7}$$

Hence, according to the Neother's theorem the volumetric density of a photon energy *w* and its velocity ***c*** in space of the generalized coordinates (or a vector - potential) are conserved.

Not stopping on a deduction we shall note that Neother's theorem allows to find and other conservative sizes in a vector - potential space. In particular the density of a ring current *j* is conserved [7].

3. Schrodinger's Equations for a Photon and Electron in Space of Vector - Potential

Let's consider process of a photon and electron interaction in the reference system moving with light velocity in a vector - potential space.

Application of Dirac's equation to electron movement has led, in particular, to representation about so-called electron "jitter" [8, 9, 10]. Dirac in this occasion spoke "There is the electron which is imagined to us slowly moving actually should make oscillatory movement of very big frequency and small amplitude which is summed to uniform movement observable by us. As a result of this oscillatory movement the electron velocity is always equaled the light velocity." [10].

Let's show that presence of electron "jitter" with light velocity is a necessary condition of the electron and photon interaction opportunity. Under the electron "jitter" we mean not its mechanical oscillations which cannot be described in a vector - potential space, and oscillations, first of all, its wave function.

In [4] nonlinear Schrodinger's equation for wave function of the photon Ψ propagating in space of vector - potential has been found:

$$i\hbar\frac{\partial\Psi}{\partial t}+2\pi\hbar^2\frac{\partial^2\Psi}{\partial q^2}+\ln\left|\Psi\right|\Psi=0 \tag{8}$$

All sizes are written down in a vector - potential space. As against Schrodinger's equation in Euclidian space the equation (8) is relativistic. Therefore, in a vector - potential space is not necessity to write down the separate relativistic equation such as Dirac's equation [2].

In spite of the fact that the equation (8) is nonlinear this nonlinearity takes place only in the generalized coordinate space, i.e. vector – potential space. Nonlinearity of Schrodinger's equation (8) is consequence of the generalized coordinate from parameters nonlinear dependence, in particular, from a magnetic field [4]. In Euclidian coordinates the process of electromagnetic radiation quantum propagation has linear character. Therefore, for example, in quantum-mechanical systems the linear principle of superposition is correct.

Let's consider a photon and electron interaction in space of vector - potential. Let wave function of a photon before interaction is equal Ψ_0.

The solution of the equation (8) for wave function Ψ_0 looks like [4]:

$$\Psi_0=\exp\left(\frac{c^2}{8\pi}-\hbar\delta_0+\frac{1}{2}\right)\exp\left[-\frac{(\mathbf{q}-\mathbf{c}t)^2}{8\pi\hbar^2}\right]\exp\left[i\left(\frac{\mathbf{cq}}{4\pi\hbar}-\delta_0 t\right)\right] \tag{9}$$

where δ_0 there is initial photon frequency.

After interaction with electron a wave function of a photon to become equal Ψ. It can be presented as the sum of wave function Ψ_1 of not coherent scattering photon and some small perturbation determined by interaction of a photon and electron. We shall assume this interaction is characterized by the wave function Ψ_2, so $\Psi_2 << \Psi_1$:

$$\Psi=\Psi_1+\Psi_2 \tag{10}$$

We do not postulate any preliminary properties of electron. All its properties will be defined by an opportunity of its interaction with a photon.

Having substituted (10) in (8), we shall receive:

$$i\hbar\frac{\partial(\Psi_1+\Psi_2)}{\partial t}+2\pi\hbar^2\frac{\partial^2(\Psi_1+\Psi_2)}{\partial q^2}+\ln\left|\Psi_1+\Psi_2\right|(\Psi_1+\Psi_2)=0 \tag{11}$$

Let's allocate in (11) the Schrodinger's equation for a secondary photon:

$$\begin{aligned} &i\hbar\frac{\partial\Psi_1}{\partial t}+i\hbar\frac{\partial\Psi_2}{\partial t}+2\pi\hbar^2\frac{\partial^2\Psi_1}{\partial q^2}+2\pi\hbar^2\frac{\partial^2\Psi_2}{\partial q^2}+\\ &+\ln\left|\Psi_1\right|(\Psi_1+\Psi_2)+\ln\left|1+\frac{\Psi_2}{\Psi_1}\right|(\Psi_1+\Psi_2)=0 \end{aligned} \tag{12}$$

In connection with that the secondary photon (as well as primary before interaction) after interaction exists separately it is possible to write down:

$$i\hbar\frac{\partial\Psi_1}{\partial t}+2\pi\hbar^2\frac{\partial^2\Psi_1}{\partial q^2}+\ln\left|\Psi_1\right|\Psi_1=0 \tag{13}$$

$$i\hbar\frac{\partial\Psi_2}{\partial t}+2\pi\hbar^2\frac{\partial^2\Psi_2}{\partial q^2}+\ln\left|\left(1+\frac{\Psi_2}{\Psi_1}\right)\right|(\Psi_1+\Psi_2)+\ln\left|\Psi_1\right|\Psi_2=0 \tag{14}$$

The equation (13) for a separate photon has the solution [4]:

$$\Psi_1=\exp\left(\frac{c^2}{8\pi}-\hbar\delta+\frac{1}{2}\right)\exp\left[-\frac{(\mathbf{q}-\mathbf{c}t)^2}{8\pi\hbar^2}\right]\exp\left[i\left(\frac{\mathbf{cq}}{4\pi\hbar}-\delta t\right)\right] \tag{15}$$

Where δ there is a frequency of a scattering photon in vector – potential space. This frequency can differ from frequency of a primary photon δ_0 according to Compton's effect [1].

The equation (14) reflects interaction of the photon and electron in a vector – potential space. Taking into account $\Psi_2 << \Psi_1$ we shall transform the Schrodinger's equation (14) to a kind:

$$i\hbar\frac{\partial\Psi_2}{\partial t}+2\pi\hbar^2\frac{\partial^2\Psi_2}{\partial q^2}+\ln\left|\Psi_1\right|\Psi_2=0 \qquad (16)$$

Let's note the Schrodinger's equation (16) as against Schrodinger's equation for a free photon (13) is linear. Wave function Ψ_2 characterizes the electron during its interaction with a photon.

4. "Jitter" of Electron in a Vector - Potential Space

The amplitude of wave function for a secondary photon according to (15) is equal:

$$\left|\Psi_1\right|=\exp\left(\frac{c^2}{8\pi}-\hbar\delta+\frac{1}{2}\right)\exp\left[-\frac{\left(\mathbf{q}-\mathbf{c}t\right)^2}{8\pi\hbar^2}\right] \qquad (17)$$

Having substituted (17) in (16) we shall find:

$$i\hbar\frac{\partial\Psi_2}{\partial t}+2\pi\hbar^2\frac{\partial^2\Psi_2}{\partial q^2}+\left(\frac{c^2}{8\pi}-\hbar\delta+\frac{1}{2}-\frac{\left(\mathbf{q}-\mathbf{c}t\right)^2}{8\pi\hbar^2}\right)\Psi_2=0 \qquad (18)$$

Using a standard method, we shall exclude in (18) derivative on time having presented [2]:

$$i\hbar\frac{\partial\Psi_2}{\partial t}=E\Psi_2 \qquad (19)$$

Where *E* there is full electron energy.

The equation (18) will be transformed to a kind:

$$2\pi\hbar^2\frac{\partial^2\Psi_2}{\partial q^2}+\left(\frac{c^2}{8\pi}-\hbar\delta+\frac{1}{2}-\frac{\left(\mathbf{q}-\mathbf{c}t\right)^2}{8\pi\hbar^2}+E\right)\Psi_2=0 \qquad (20)$$

Using an independent variable as $\xi=\frac{\left(\mathbf{q}-\mathbf{c}t\right)}{2\sqrt{\pi}\hbar}$ we shall transform the equation (20) to a kind:

$$\frac{1}{2}\frac{\partial^2\Psi_2}{\partial\xi^2}+\left(\frac{c^2}{8\pi}-\hbar\delta+\frac{1}{2}-\frac{\xi^2}{2}+E\right)\Psi_2=0 \qquad (21)$$

All used physical parameters dependent on a variable $\xi=\frac{\left(\mathbf{q}-\mathbf{c}t\right)}{2\sqrt{\pi}\hbar}$ move with a light velocity *c* in space of vector - potential. Thus the reference system also moving with a light velocity *c* is used.

The equation (21) allows draw a conclusion, electron cooperating with a photon, should move with a light velocity. It confirms Dirac's conclusion [10]. Solving the equation (21), we shall find character of this movement. Let's notice in (21) the wave function $\Psi_2=\Psi_2(\xi)$ depends on time.

Let's enter the following designation:

$$\lambda=2\left(\frac{c^2}{8\pi}-\hbar\delta+\frac{1}{2}+E\right) \qquad (22)$$

The equation (21) will be written down as similar [11]:

$$\frac{\partial^2\Psi_2}{\partial\xi^2}+\left(\lambda-\xi^2\right)\Psi_2=0 \qquad (23)$$

The solution of the equation (23) is well-known [11]. It assumes that electron cooperating with a photon it is quantum oscillator oscillating with a light velocity. The solution exists only at the whole positive values of number *n*:

$$\Psi_{2n}=A_n\exp\left(-\frac{\xi^2}{2}\right)H_n(\xi) \qquad (24)$$

where is $n=0,1,2,\ldots$, and the size A_n - the factor determined by a condition of normalization,

$$H_n(\xi)=(-1)^n\exp\left(\xi^2\right)\frac{d^n\exp\left(-\xi^2\right)}{d\xi^n} \qquad (25)$$

There are Hermit's polynomials.

The condition of normalization has a standard kind:

$$\int_{-\infty}^{+\infty}\Psi_{2n}^2d\xi=1 \qquad (26)$$

Substituting the formula (24) in condition (26) we shall find [11]:

$$A_n\int_{-\infty}^{+\infty}\exp\left(-\xi^2\right)H_n^2(\xi)d\xi=A_n2^n n!\sqrt{\pi}=1 \qquad (27)$$

Hence the factor $A_n=\frac{1}{2^n n!\sqrt{\pi}}$, and the solution (24) looks a kind:

$$\Psi_{2n}=\frac{1}{2^n n!\sqrt{\pi}}\exp\left(-\frac{\xi^2}{2}\right)H_n(\xi) \qquad (28)$$

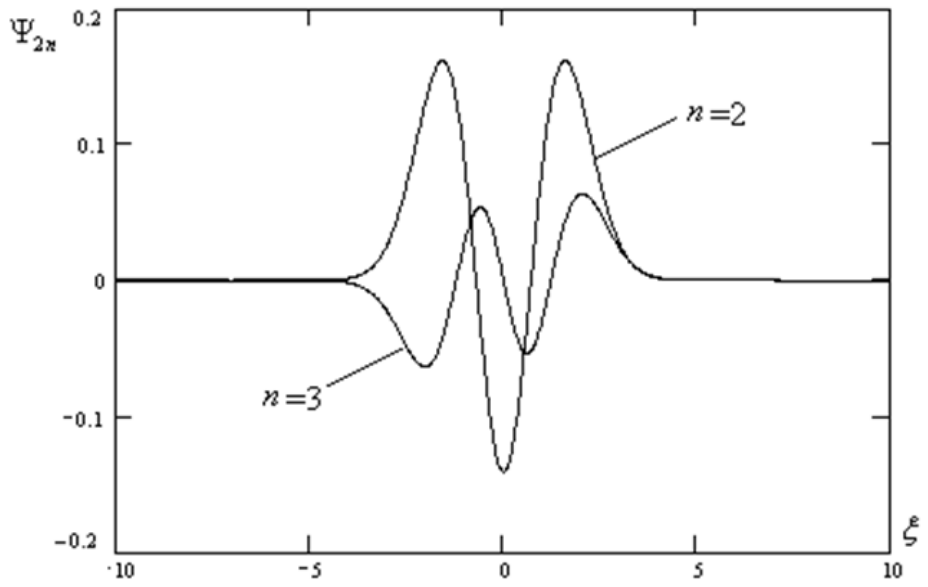

Figure 1. *Wave function* Ψ_{2n} *for* $n=2$ *and* $n=3$.

On Figure 1 the graph of electron wave function is shown for $n=2$ and $n=3$. As can see from the graph the electron wave function cooperating with a photon has oscillating character. The amplitude of oscillations of wave function with increase *n* quickly falls.

Let's find values of energies which the electron - oscillator can have. For this purpose we shall find an own values of parameter λ_n.

Substituting the formula (28) in the differential equation (23) we have:

$$\frac{d^2H_n(\xi)}{d\xi^2}-2\xi\frac{dH_n(\xi)}{d\xi}+(\lambda_n-1)H_n(\xi)=0 \qquad (29)$$

On the other hand Hermit's polynomials (25) are solutions of the equation [11]:

$$\frac{d^2H_n(\xi)}{d\xi^2}-2\xi\frac{dH_n(\xi)}{d\xi}+2nH_n(\xi)=0 \qquad (30)$$

Comparing the equations (29) and (30) we receive:

$$\lambda_n=2n+1 \qquad (31)$$

Taking into account a designation (22) we find that full energy of electron - oscillator can have only discrete values:

$$E_n=n+\hbar\delta-\frac{c^2}{8\pi} \qquad (32)$$

At the lowermost power level at $n=0$ the energy of electron is equal:

$$E_0=\hbar\delta-\frac{c^2}{8\pi} \qquad (33)$$

Using the formula (33) it is possible to write down (32) as:

$$E_n-E_0=n \qquad (34)$$

The formula (34) represents a condition for change of energy at transition electron - oscillator, i.e. at its Dirac's "jitter", from one state in another in space of vector - potential. Energy of electronic "jitter" is quantized.

According to the energy conservation law it is possible to write down:

$$E_n-E_0=n=\hbar\delta_0-\hbar\delta=\hbar\Delta\delta \qquad (35)$$

The formula (35) allows calculate a difference of frequencies of primary and scattered photons and hence is analogue of the formula for change of frequency in Compton's effect in the vector - potential space [1].

Due to the fact that Dirac's "jitter" of electron is supposed very big frequency and small amplitude in the equation (23) for numerical estimations it is possible to accept $\lambda >> \xi^2$. In this case the equation (23) describes classical oscillatory process with the frequency proportional $\sqrt{\lambda_n}$, and according to (28) the amplitude of wave function of electron "jitter" is equal $\Psi_{2n}=\frac{1}{2^n n!\sqrt{\pi}}\exp\left(-\frac{\xi^2}{2}\right)$.

Let's consider the ratio $\left|\frac{\Psi_{2n}}{\Psi_1}\right|$ i.e. the ratio of the wave functions amplitudes of electron "jitter" and a photon. This ratio also characterizes a validity of the assumption $\Psi_2 << \Psi_1$.

Proceeding from formulas (17) and (28), with the account (33), it is possible to find:

$$\left|\frac{\Psi_{2n}}{\Psi_1}\right|\sim\frac{1}{2^n n!\sqrt{\pi}}\exp\left(E_0-\frac{1}{2}\right) \qquad (36)$$

In [4] it is shown the energy of zero oscillations for a photon according to the equation (13) it is equal $\frac{1}{2}$. Assuming energies of zero oscillations for a photon and electron during their interaction are equalized we have $\left|\frac{\Psi_{2n}}{\Psi_1}\right| \sim \frac{1}{2^n n!\sqrt{\pi}}$. Already for the second electron – oscillator power level the ratio is $\left|\frac{\Psi_{2n}}{\Psi_1}\right| \sim 0.07$. At increase n this ratio very quickly to tend to zero.

Proceeding from the lead analysis it is possible to assume also that Dirac's "jitter" of electron is connected to quantum waves (28) arising on an electron surface at its interaction with a photon and propagating with a light velocity c. Fronts of these waves are conditionally shown on Figure 2, curves 1.

5. The Magnetic Moment of Electron in a Vector – Potential Space

Let's consider the physical reasons of occurrence of the electron magnetic moment. Spin of electron i.e. it mechanical characteristic - the moment of movement quantity cannot be described in space of vector - potential.

It is known energy of a magnetic field and a current interaction in classical electrodynamics is expressed by the formula [6]:

$$U = -\frac{1}{c}\mathbf{jA} = \mathbf{jq} \tag{37}$$

where j there is a current density with which the electromagnetic field cooperates.

This formula is similar to the formula $W = e\phi$ for energy of electron with a charge e in an electrostatic field with potential ϕ. The role of a charge in space of vector - potential plays a current density j.

The additional information about an electron we shall introduce assuming existence in it of constant ring currents. By these currents it is determined the electron magnetic moment $\boldsymbol{\mu}$. As in space of vector - potential there is no concept of mass in this space it is possible to identify electron only with density of a quantum ring current j, and the formula (37) represents electron energy in a vector - potential space.

Let's transform the equation (16) using representation (19):

$$\frac{\partial^2 \Psi_2}{\partial q^2} + \frac{1}{2\pi\hbar^2}\left(E + \ln\left|\Psi_1\right|\right)\Psi_2 = 0 \tag{38}$$

The equation (38) is stationary, i.e. not dependent on time.

Let's introduce into the equation (38) the energy electron magnetic moment (or ring currents) supposing:

$$\ln\left|\Psi_1\right| = \mathbf{jq} = jq\cos\theta = \pm jq \tag{39}$$

where θ there is an angle between a direction of an electron current j and the generalized coordinate $\mathbf{q} = -\frac{\mathbf{A}}{c}$. We assume that the electron magnetic moment has two directions that meets $\theta = 0$ and $\theta = 180^0$ (j against vector – potential A of a field, and j on a vector – potential of a field A, Figure 2).

On Figure 2 the spherical electron form is given conditionally since in space of a vector - potential A the Euclidian coordinates are absent. Therefore to estimate the Euclidian velocity of an electron surface rotation it is incorrect. It is accepted an angle $\theta = 180^0$ (the substantiation is lower) i.e. the generalized coordinate q it is directed against of a ring electronic density current j (q and A by determination are directed opposite each other).

The electron magnetic moment $\boldsymbol{\mu}$ is always directed against its spin S and hence on a direction of an electron magnetic field strength $\mathbf{H} = \mathrm{rot}\mathbf{A}$. We shall note that in space of a vector - potential the strength of magnetic field H (as a rotor on Euclidian coordinates) and electron spin S can be shown only conditionally.

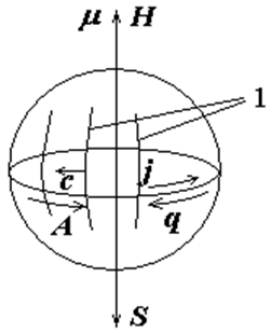

Figure 2. *The scheme of the electron vector parameters.*

Thus the equation (38) will be transformed to a kind:

$$\frac{\partial^2 \Psi_2}{\partial q^2} + \frac{1}{2\pi\hbar^2}\left(E \pm jq\right)\Psi_2 = 0 \tag{40}$$

For the solving of the equation (40) we shall introduce a new variable:

$$\eta = \left(E \pm jq\right)\left(\frac{1}{2\pi\hbar^2 j^2}\right)^{\frac{1}{3}} \tag{41}$$

Passing to a variable η we shall find:

$$\frac{\partial^2 \Psi_2}{\partial \eta^2} + \eta\Psi_2 = 0 \tag{42}$$

The solution of the equation (42) looks like [11]:

$$\Psi_2(\eta) = A\Phi(-\eta) \tag{43}$$

where $\Phi(\eta) = \frac{1}{\sqrt{\pi}}\int_0^\infty \cos\left(\frac{u^3}{3} + u\eta\right)du$ so-called Airy's

function.

Normalizing wave function $\Psi_2(\eta)$ by a rule of the continuous spectrum functions normalization [11]:

$$\int_{-\infty}^{+\infty} \Psi_2^*(\eta)\Psi_2(\eta')dq = \delta(\eta' - \eta) \tag{44}$$

where $\delta(\eta' - \eta)$ there is delta - function we find:

$$A = \left(\frac{1}{2\pi^{\frac{5}{2}} \hbar^2 j^{\frac{1}{2}}} \right)^{\frac{1}{3}} \tag{45}$$

Taking into account in a vector - potential space full electron energy actually is energy of its ring currents we accept for calculation $E \pm jq \approx \pm jq$.

On Figure 3 the graph of Airy's function plotted at a sign minus in the formula (40) i.e. at $|\Psi_1| = \exp(-jq)$ or $\theta = 180^0$ is shown.

State $|\Psi_1| = \exp(jq)$ or $\theta = 0$ there is unstable since at increase q an exponent quickly increases. This state is inlikely because a direction of electronic current density j and a direction of a vector - potential A (hence, and the generalized coordinate q) are determined in stationary conditions by a direction of a uniform vector of magnetic field H, Figure 2.

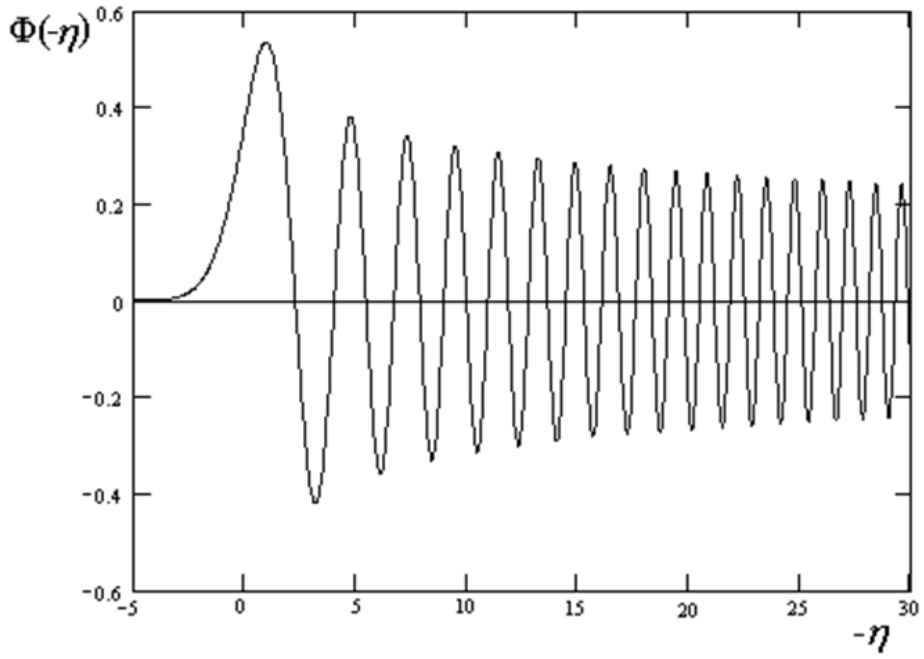

Figure 3. *The graph of Airy's function determining the wave function of the electron magnetic moment.*

As can see from the graph the wave function $\Psi_2(\eta)$ of the magnetic moment in a vector - potential space has the oscillatory character. Apparently it also reflects occurrence of oscillations (Dirac's "jitter") in an electron surface. Due to quantum waves on the electron surface on its surface there are quantum currents, and hence the electron magnetic moment (and also its spin in Euclidian space).

6. Interaction of a Photon and Atom

The formula (34) can be considered as the first Bohr's condition in space of vector - potential for change of energy at transition of atom from one state in another. As against the Bohr's condition for Euclidian spaces the energies of atom states differ on an integer.

The second Bohr's condition can be received only using some additional information on atom. For a finding of this condition the formula (34) we shall write down as:

$$\Delta E_n = \sum_{i=1}^{n} \Delta E_i = n \tag{46}$$

where ΔE_i there is energy of transition from i-th power level on $i-1$ power level.

The additional information on atom we shall enter assuming existence in it n constant ring currents which energies in a vector – potential space look like (37). For i-th atom current according to (37) we have:

$$\Delta E_i = j_i \Delta q_i \tag{47}$$

Substituting (47) in (46) we shall find the integrated sum:

$$\sum_{i=1}^{n} j_i \Delta q_i = n \tag{48}$$

Passing to integral on a closed current cycle (trajectory) we shall find the second Bohr's condition in Sommerfeld's form [11]:

$$\oint j dq = n \tag{49}$$

Let's notice the role of an electron impulse in a vector - potential space plays the current density $\boldsymbol{j}$.

Meaning the given analogy it is possible to write down the Heisenberg's uncertainty principle in space of vector - potential:

$$\Delta j \cdot \Delta q \geq 1 \tag{50}$$

where Δj there is uncertainty of a current density, Δq - uncertainty of the generalized coordinate (vector - potential).

The atom energy at the lowermost power level at quantum number $n = 0$ can be found under the formula (33) $E_0 = \hbar\delta - \frac{c^2}{8\pi}$. Near to atom there is a process of self-action of a vector - potential owing to what there is a uncertainty of energy ΔE_0. Occurrence of this uncertainty in a vector - potential space there is similarly to Lamb's effect in the Euclidian space [2].

Let's estimate uncertainty of energy ΔE_0. Energy of process of a magnetic field self-action in classical electrodynamics is determined the term a kind $\frac{e^2}{2mc^2}\mathbf{A}^2 = \frac{1}{2} r_e A^2$ in atom Hamiltonian, where e there is an electron charge, m - its mass, r_e - so-called classical electron radius [2].

Let's assume on a pulsation of the vector - potential $d\mathbf{A}$ arisen near to atom has effect the field with a vector - potential $\boldsymbol{A}$. Consequently the energy of interaction of a field pulsation and the itself field looks like $\frac{1}{2} r_e A dA$. Passing in a vector - potential space we have $\frac{1}{2} r_e c^2 q dq$, where r_e - in this case the constant which has been written down in this space.

Occurrence of a field pulsation near to atom it is a random the process initiated by atom. Therefore, the element of energy of self-action dE_0 needs to be written down as:

$$dE_0 = \frac{1}{2} r_e c^2 \left|\Psi_{20}\right|^2 q dq \tag{51}$$

where the wave function Ψ_{2n} is determined by the formula (28) attributed to atom. Taking into account $H_0 = 1$ [11] in the conditional time moment $t = 0$ we find $\Psi_{20} = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{q^2}{4\pi\hbar^2}\right)$.

Hence, the formula (51) looks like:

$$dE_0 = \frac{1}{2\pi} r_e c^2 \exp\left(-\frac{q^2}{2\pi\hbar^2}\right) q dq \tag{52}$$

The full energy of self-action of a field near to atom is integration (52):

$$dE_0 = \frac{1}{2\pi} r_e c^2 \int_{-\infty}^{\infty} \exp\left(-\frac{q^2}{2\pi\hbar^2}\right) q dq = r_e c^2 \hbar^2 \tag{53}$$

Thus the full energy of a field near to atom at a zero power level is equal:

$$E_0 + dE_0 = \hbar\delta - \frac{c^2}{8\pi} + r_e c^2 \hbar^2 = \hbar\left(\delta + \Delta\delta\right) - \frac{c^2}{8\pi} \tag{54}$$

where the size $\Delta\delta = r_e c^2 \hbar$ there is Lamb's shift of frequency in a vector - potential space.

7. Multiphoton System in a Vector - Potential Space

To find the wave function for multiphoton system in a vector - potential space inconveniently since it is necessary to know a multisoliton solution of the Schrodinger's equation (8). Therefore we shall be limited to finding of the set n photons energy average value in a vector - potential space.

Average value of the system n photons energy can be calculated under the formula similar [2]:

$$\overline{\varepsilon} = \frac{\left|\Psi_1\right|^2 \varepsilon_1 + \left|\Psi_2\right|^2 \varepsilon_2 + \ldots + \left|\Psi_n\right|^2 \varepsilon_n}{\left|\Psi_1\right|^2 + \left|\Psi_2\right|^2 + \ldots + \left|\Psi_n\right|^2} \tag{55}$$

where $\varepsilon_i = \hbar\delta_i$ there is energy i-th photon in system, $\left|\Psi_i\right|^2$ - probability of that photon has energy ε_i.

The amplitude of a photon wave function according to (17) is equal:

$$\left|\Psi_i\right| = \exp\left(\frac{c^2}{8\pi} - \hbar\delta_i + \frac{1}{2}\right) \exp\left[-\frac{\left(\mathbf{q} - \mathbf{c}t\right)^2}{8\pi\hbar^2}\right] = B \exp\left(-\hbar\delta_i\right) \tag{56}$$

where $B = \exp\left(\frac{c^2}{8\pi} + \frac{1}{2}\right) \exp\left[-\frac{\left(\mathbf{q} - \mathbf{c}t\right)^2}{8\pi\hbar^2}\right]$.

We substitute the formula (56) in (55). Using $\delta_i = i\delta_1$ where δ_1 there is minimal frequency of the photon system we shall find average value of the photon system energy on coordinate $\mathbf{q} = const$ at the moment of time t:

$$\bar{\varepsilon} = \frac{B^2 \sum_{i=1}^{n} \hbar\delta_i \exp(-2\hbar\delta_i)}{B^2 \sum_{i=1}^{n} \exp(-2\hbar\delta_i)} = \frac{\hbar\delta_1 \frac{d}{d(\hbar\delta_1)} \int \left(\sum_{i=1}^{n} i \exp(-2\hbar i\delta_1) \right) d(\hbar\delta_1)}{\sum_{i=1}^{n} \exp(-2\hbar i\delta_1)} =$$
$$= \frac{\hbar\delta_1 \frac{d}{d(\hbar\delta_1)} \left(-\frac{1}{2} \sum_{i=1}^{n} \exp(-2\hbar i\delta_1) \right)}{\sum_{i=1}^{n} \exp(-2\hbar i\delta_1)} = \hbar\delta_1 \frac{d}{d(\hbar\delta_1)} \ln \left(\sum_{i=1}^{n} \exp(-2\hbar i\delta_1) \right) \quad (57)$$

Using the formula of the geometrical progression sum with a denominator $\exp(-2\hbar\delta_1)$ we shall find:

$$\sum_{i=1}^{n} \exp(-2\hbar i\delta_1) = \exp(-2\hbar\delta_1) \frac{1-\exp(-2\hbar n\delta_1)}{1-\exp(-2\hbar\delta_1)} \quad (58)$$

Substituting (58) in (57) we shall find:

$$\bar{\varepsilon} = \hbar\delta_1 \left(1 + \frac{d}{d(-2\hbar\delta_i)} \ln(1-\exp(-2\hbar n\delta_1)) - \frac{d}{d(-2\hbar\delta_i)} \ln(1-\exp(-2\hbar\delta_1)) \right) =$$
$$= \hbar\delta_1 \left(1 - \frac{n\exp(-2\hbar n\delta_1)}{1-\exp(-2\hbar n\delta_1)} + \frac{\exp(-2\hbar\delta_1)}{1-\exp(-2\hbar\delta_1)} \right) = \frac{\hbar\delta_1}{1-\exp(-2\hbar\delta_1)} - \frac{n\hbar\delta_1 \exp(-2\hbar n\delta_1)}{1-\exp(-2\hbar n\delta_1)} \quad (59)$$

For $n \to \infty$ the formula (59) becomes simpler:

$$\bar{\varepsilon} = \frac{\hbar\delta_1}{1-\exp(-2\hbar\delta_1)} = \hbar\delta_1 + \frac{\hbar\delta_1}{\exp(2\hbar\delta_1)-1} \quad (60)$$

However there is also an essential difference. In a vector - potential space we do not connect system of quantums with set of the quantum oscillators [12]. Therefore there is no zero energy of a photon in the basic condition and in the formula (60) is added composed $\hbar\delta_1$ - the minimal energy of a photon in system.

As shown in [4] the energy of a photon (including average) includes the energy of vacuum equal $\frac{1}{2}$. For small energies it is possible to accept $\exp(-2\hbar\delta_1) \approx 1-2\hbar\delta_1$. Hence $\bar{\varepsilon} = \frac{1}{2}$. The clean average energy of a photon is equal:

$$\bar{\varepsilon}_p = \frac{\hbar\delta_1}{1-\exp(-2\hbar\delta_1)} - \frac{1}{2} \quad (61)$$

If photons are absent $\bar{\varepsilon}_p = 0$.

8. Conclusion

To investigate interaction of a photon and a material particle, for example, electron it is necessary to consider them in the uniform reference system moving with a light velocity. But the material particle cannot be investigated in Euclidian space in the reference system moving with a light velocity. This is opposed by the special relativistic theory. It is necessary to pass to other space where existence of a reference system of the cooperating particles moving with a light velocity is possible. One of such spaces is the space of a vector - potential. In this space the volumetric density of photon energy, its velocity and the density of electron and atom ring currents are exist and preserved.

Consideration of process of a photon and electron interaction in space of a vector - potential shows:

a. For an opportunity of an electron and photon interaction the electron should represent quantum oscillator i.e. to "jitter" (as Dirac) with a discrete set of energies.

b. Dirac's "jitter" of an electron occurs with a light velocity in vacuum.

It is found the electron energy in vacuum in space of a vector - potential.

The electron magnetic moment (and hence its spin) is determined by electronic ring currents. On the basis of dependence of the electron magnetic moment energy from the generalized coordinate and density of electronic ring currents the Schrodinger's equation for wave function of the electron magnetic moment (spin) is received. The solution of this equation shows that this wave function is connected to Airy's function, and has essentially oscillatory character in a vector – potential space.

The principle of transition to model of a photon and atom interaction is shown. Values of energy which the atom can have are found. The condition of an atom currents quantization in a vector - potential space as Sommerfeld's condition is submitted. Also a principle of the Heisenberg's uncertainty in this space is submitted. To that Lamb's shift of frequency equal in a vector - potential space it is shown.

An average energy of the photons in a vector - potential space is found. Thus it is shown that a hypothesis about a photon as quantum oscillator is superfluous.

References

[1] Berestetskij V. B., Lifshits E. M., Pitaevskij L. P. "Quantum Electrodynamics," *Science*, Moscow, 1989. Pp. 356, 404.

[2] Levich V. G. "Theoretic Physics Course," V. 1, *FIZMATGIS*, Moscow, 1962. Pp. 165, 200, 230. V. 2. Pp. 320, 321, 646, 734, 796.

[3] Chandrasekhar Roychoudhuri, Krasklauer A. F., Katherine Creath. The Nature of Light. What is Photon? *CRC Press. Taylor & Francis Group*. Boca Raton, London, New York, 2008. 456 p.

[4] Volobuev A. N. "Analysis of a Vector - Potential Representation of Electromagnetic Radiation Quantum," *Engineering Physics*. V. 2, No. 4, 2017. Pp. 54-59.

[5] Bogolubov N. N., Shirkov D. V., "Quantum Fields," *FIZMATLIT*, Moscow, 2005. Pp. 24, 27.

[6] Landau L. D., Lifshits E. M. "Theory of Field," *Science*, Moscow, 1967. Pp. 45, 69, 109.

[7] Mathematical physics. The encyclopedia. Under edit. Faddeev L. D. *Scientific publishing house "Big Russian encyclopedia"*, Moscow, 1998. P. 385.

[8] Schrödinger, E. Über die kräftefreie Bewegung in der relativistischen Quantenmechanik / Schrödinger E. // *Sitzungsber. Peuss. Akad. Wiss*. Phys. Math. Kl. - 1930. - Vol. 24. - Pp. 418-428.

[9] Vonsovskij S. V., Svirsky M. S. "Paradox Klein and jittering movement of an electron in a field with constant scalar potential," Moscow, 1993, *Uspekhi Fizicheskikh Nauk*, V. 163, No. 5, pp. 115-118.

[10] Dirac P. A. M. "Recollections of an Exciting Era," *Science*, Moscow, 1990. Pp. 203 - 206.

[11] Landau L. D., Lifshits E. M. "Quantum Mechanics," *FIZMATLIT*, Moscow, 2004, p. 35, 98, 104, 216, 779, 781.

[12] Feynman R., Leighton R., Sands M. "The Feynman Lectures on Physics," V. 3, 4. *World*, Moscow, 1976. P. 290.