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Effect of Faults on Kalman Filter of State Vectors in Linear Systems

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Abstract: Kalman filter (KF) is composed of a set of recursion algorithms which can be used to estimate the optimal state of the linear system, and widely used in the control system, signal processing and other fields. In the practical application of the KF, it is an unavoidable problem that how faults or anomalies are infectious to the estimation value of state vectors in the linear system, which must be paid much attention to and solved down. In this paper, the effect of sensor faults and control input anomalies on the Kalman filtering values of state vectors is discussed, the transmission relationship is established to analyze the estimation deviation of state vectors which comes from pulse or step faults/anomalies, and a sufficient condition is deduced for the convergence of the estimation deviation of state vectors; Four different system models with 3-dimension state vector and 2-dimension observation vector are selected for simulation calculation and comparative analysis, simulation results show that sensor faults and control input anomalies in linear systems may cause significant deviations in the estimation value of state vectors for a long time, and there are distinct differences in the estimation value of state vectors. The research results provide a certain theoretical reference for us to analyze system fault types and to identify fault.

Keywords: System Faults, Kalman Filter, Control Input Anomalies, Sensors Faults

1. Introduction

The Kalman filter (KF) is a time-domain filtering algorithm which can achieve the optimal estimation value of state vectors based on the state-space model of the linear system [1]. The KF is suitable for estimating the change of state vectors online in multivariable time-varying systems, and is widely used in process automation, dynamic system control, spacecraft monitoring and control, mechanical and electrical engineering, etc. [2-3], and has a foundational significance in the modern control theory, real-time signal processing and other fields [4].

During the actual running of the linear system, due to the complex operating environment, working conditions and other factors, it usually appears faults and anomalies, which will affect the stability and reliability of systems. The common faults in linear systems are sensor faults [5-7], actuator faults [8] and so on. The occurrence of different faults will have a serious impact on the system function and even cause the incalculable losses [9]. Many scholars have conducted a lot of research on such issues deeply, and there are a larger number of research results in the literature. Based on the KF, different neural networks are used to diagnose sensor faults of aerospace control systems [10-12]; Aretakis uses geometric pattern recognition technology and KF algorithm to solve the problem of slow drift in sensors [13]; a set of linear Kalman filter are used to diagnose sensor faults after linearization at operating point of the system [14]; a novel Kalman filter is designed to diagnose multi-sensor faults when systems existing colored noise [15]; in order to solve the non-Gaussian distribution problem of wind speed and measured noise in wind power generation systems, a novel filtering algorithm is proposed to identify and isolate the sensor fault [16]; a set of extended Kalman filter are used to diagnose attitude sensor faults [17]. Aiming to the problem of actuator faults in control systems, an improved multiple fading factor strong-tracking nonlinear filter algorithm is proposed to diagnose stuck-at faults and swing faults of the actuator [18]; in order to solve the problem of flywheel faults in satellite attitude control systems, a two-stage EKF algorithm is designed [19]; the actuator and sensor fault are regarded as system states, system states are optimized through the optimal and robust three-stage Kalman filter, and finally achieve the reconstruction of system faults [20]. In practical application of systems, it is often affected by external interference or random noise; therefore, stochastic systems with unknown inputs have gradually attracted much attention [21]. A two-level Kalman filter with unknown inputs is proposed to decouple and estimate states and unknown inputs [22]; the unbiased minimum variance estimation of linear systems with unknown disturbances is studied, and a unknown input Kalman filter (UIKF) is designed [23]; based on the UIKF, the necessary and sufficient condition are deduced for the stability and convergence [24]; a second-order Kalman filter is designed, and its order is used to estimate system states and unknown inputs [25]; based on the UIKF method [23-24], a recursive filter is designed to estimate system states and unknown inputs at same time, and the two are interrelated [26].

Many of the above research methods concerning sensor or actuator faults and control inputs are based on the Kalman filtering algorithm for fault identification, isolation and diagnosis, or decoupling and estimating the unknown input, however, there is not deeply analyze and discuss the effect of faults on the KF of state vectors in linear systems. In view of this, this article selects a linear system as an object, taking sensor measured faults and control input anomalies as examples, theoretical analysis and simulation calculation method are used to analyze and discuss the possible influence of system faults on the KF of state vectors.

2. Effect of Sensor Measured Faults on the KF of State Vectors

A linear system is described by the following state-space model [27]:

$$\begin{cases} X_{k+1} = A_k X_k + B_{k+1} U_{k+1} + \varepsilon_k \\ Y_k = C_k X_k + \eta_k \end{cases} \quad (1)$$

Where, $A_k \in R^{n\times n}$, $B_k \in R^{n\times p}$, $C_k \in R^{m\times n}$

Assuming that the $\{\varepsilon_k\}$ is Gaussian zero-mean with covariance Q and the $\{\eta_k\}$ is Gaussian zero-mean with covariance R independent of $\{\varepsilon_k\}$, the optimal estimation value of state vectors $\hat{X}$ and the error covariance matrix P can be written as [27]:

$$\begin{aligned} \hat{X}_{k+1|k+1} &= A_k \hat{X}_{k|k} + B_{k+1} U_{k+1} \\ &+ K_{k+1}\left(Y_{k+1} - C_{k+1}\left(A_k \hat{X}_{k|k} + B_{k+1} U_{k+1}\right)\right) \end{aligned} \quad (2)$$

$$P_{k+1|k+1} = \left(I_n - K_{k+1} C_{k+1}\right)\left(A_k P_{k|k} A_k^T + Q\right) \quad (3)$$

And the filtering gain is

$$K_{k+1} = \left(A_k P_{k|k} A_k^T\right) C_k^T \left(C_{k+1}\left(A_k P_{k|k} A_k^T\right) C_k^T + R\right)^{-1} \quad (4)$$

Clearly, in the recursive process of the KF, the fault data in sensor measurement outputs and the abnormal data in control inputs can affect the estimation value of state vectors, and different types of faults or anomalies may have different effects on the filtering result. In this section, we will analyze the effect of sensor faults on the estimation value of state vectors in terms of pulse faults and step faults.

2.1. Effect of Pulse Faults on the KF of State Vectors

Assuming that the sensor appear pulse fault at time k_0, and the value of the fault is $\vec{a}$, meanwhile, the measurement data is given by $Y_{k_0}^a = Y_{k_0} + \vec{a}$, and the present estimation value of the state $\hat{X}_{k_0|k_0}^a$ can be obtained by formula (2)

$$\begin{aligned} \Delta\hat{X}_{k_0|k_0}^a &= A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0} U_{k_0} \\ &+ K_{k_0}\left(Y_{k_0}^a - C_{k_0}\left(A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0} U_{k_0}\right)\right) \end{aligned} \quad (5)$$

The present estimation deviation of the state $\Delta\hat{X}_{k_0|k_0}$ is given by

$$\begin{aligned} \Delta\hat{X}_{k_0|k_0} &= \hat{X}_{k_0|k_0}^a - \hat{X}_{k_0|k_0} \\ &= A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0} U_{k_0} \\ &\quad + K_{k_0}\left(Y_{k_0}^a - C_{k_0}\left(A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0} U_{k_0}\right)\right) \\ &\quad - \begin{pmatrix} A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0} U_{k_0} \\ + K_{k_0}\left(Y_{k_0} - C_{k_0}\left(A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0} U_{k_0}\right)\right) \end{pmatrix} \\ &= K_{k_0}\left(Y_{k_0}^a - Y_{k_0}\right) = \left(K_{k_0}\vec{a}\right) \end{aligned} \quad (6)$$

Apparently, the pulse fault in sensors may result in the estimation deviation of the state. If the fault disappears after time k_0, in order to analyze the effect of pulse faults on the estimation value of subsequent states, the estimation deviation $\Delta\hat{X}_{k_0+1|k_0+1}$ at time k_0+1 is calculated

$$\begin{aligned} \Delta\hat{X}_{k_0+1|k_0+1} &= \hat{X}_{k_0+1|k_0+1}^a - \hat{X}_{k_0+1|k_0+1} \\ &= A_{k_0}\hat{X}_{k_0|k_0}^a + B_{k_0+1} U_{k_0+1} \\ &\quad + K_{k_0+1}\left(Y_{k_0+1} - C_{k_0+1}\begin{pmatrix} A_{k_0}\hat{X}_{k_0|k_0}^a \\ + B_{k_0+1} U_{k_0+1} \end{pmatrix}\right) \\ &\quad - \begin{pmatrix} A_{k_0}\hat{X}_{k_0|k_0} + B_{k_0+1} U_{k_0+1} \\ + K_{k_0+1}\left(Y_{k_0+1} - C_{k_0+1}\begin{pmatrix} A_{k_0}\hat{X}_{k_0|k_0} \\ + B_{k_0+1} U_{k_0+1} \end{pmatrix}\right) \end{pmatrix} \\ &= A_{k_0}\left(\hat{X}_{k_0|k_0}^a - \hat{X}_{k_0|k_0}\right) - \\ &\quad K_{k_0+1} C_{k_0+1} A_{k_0}\left(\hat{X}_{k_0|k_0}^a - \hat{X}_{k_0|k_0}\right) \\ &= \left(A_{k_0} - K_{k_0+1} C_{k_0+1} A_{k_0}\right)\left(\hat{X}_{k_0|k_0}^a - \hat{X}_{k_0|k_0}\right) \\ &= \left(A_{k_0} - K_{k_0+1} C_{k_0+1} A_{k_0}\right)\left(K_{k_0}\vec{a}\right) \end{aligned} \quad (7)$$

Introducing a symbol variance

$$M_{k_0} = \left(A_{k_0} - K_{k_0+1} C_{k_0+1} A_{k_0}\right)$$

the formula (7) can be rewritten as

$$\Delta \hat{X}_{k_0+1|k_0+1} = M_{k_0}\left(K_{k_0}\vec{a}\right)$$

similarly, we can calculate the estimation deviation

$$\Delta \hat{X}_{k_0+2|k_0+2}$$

at time k_0+2.

$$\begin{aligned}\Delta \hat{X}_{k_0+2|k_0+2} &= \hat{X}^a_{k_0+2|k_0+2} - \hat{X}_{k_0+2|k_0+2} \\ &= \left(A_{k_0+1} - K_{k_0+2} C_{k_0+2} A_{k_0+1}\right)\begin{pmatrix}\hat{X}^a_{k_0+1|k_0+1} \\ -\hat{X}_{k_0+1|k_0+1}\end{pmatrix} \\ &= \left(A_{k_0+1} - K_{k_0+2} C_{k_0+2} A_{k_0+1}\right) M_{k_0}\left(K_{k_0}\vec{a}\right) \\ &= M_{k_0+1} M_{k_0}\left(K_{k_0}\vec{a}\right)\end{aligned} \quad (8)$$

And so forth, the estimation deviation $\Delta \hat{X}_{k_0+i|k_0+i}$ at time k_0+i is given by

$$\begin{aligned}\Delta \hat{X}_{k_0+i|k_0+i} &= \hat{X}^a_{k_0+i|k_0+i} - \hat{X}_{k_0+i|k_0+i} \\ &= M_{k_0+i-1} M_{k_0+i-2} \cdots M_{k_0+1} M_{k_0}\left(K_{k_0}\vec{a}\right) \\ &= \left(\Pi_{j=1}^{i} M_{k_0+j-1}\right)\left(K_{k_0}\vec{a}\right)\end{aligned} \quad (9)$$

The formula (9) gives the relationship between the pulse fault and the estimation value of subsequent states, it can be seen that the change of the coefficient matrix $\left(\Pi_{j=1}^{i} M_{k_0+j-1}\right)$ may directly affect the change of the estimation value of subsequent states. In order to further analyze the change of the estimation deviation $\Delta \hat{X}_{k_0+i|k_0+i}$, and discuss its convergence, we can make the following definitions.

Definition 1: M is an n-order square matrix, and its eigenvalues are $\lambda_i\left(i = 1,2,\cdots,n\right)$, defining the maximum absolute value of eigenvalues as the spectral radius of M, and it is marked as [28]

$$\rho(M) = \max\left\{|\lambda_1|, |\lambda_2|, \cdots, |\lambda_n|\right\}$$

Definition 2: $M \in C^{m\times n}$, $\|M\|$ is called the matrix norm of M if it meets following four conditions [28]:

(1) $\|M\| \geq 0$ with equality if and only if $M = 0$;

(2) $\|\alpha M\| = |\alpha| \cdot \|M\| \quad (\alpha \in C)$;

(3) $\|M + N\| \leq \|M\| + \|N\| \quad \left(N \in C^{m\times n}\right)$;

(4) $\|MN\| \leq \|M\| \cdot \|N\| \quad \left(N \in C^{m\times n}\right)$.

Lemma 1: $M \in C^{m\times n}$, for any positive number ξ, there is a matrix norm $\|\cdot\|_M$ that makes [28]

$$\|M\|_M \leq \rho(M) + \xi$$

Theorem 1: for the linear system (1), the sufficient condition for the convergence of the state filtering difference caused by pulse faults of sensors is that the spectral radius of factor matrices M_{k_0+i-1} contained in the coefficient matrix

$$\left(\Pi_{j=1}^{i} M_{k_0+j-1}\right) \text{ is } \rho\left(M_{k_0+i-1}\right) < 1\left(i = 1,2,\cdots\right).$$

Proof: as a known condition

$$\rho\left(M_{k_0+i-1}\right) < 1\left(i = 1,2,\cdots\right)$$

for a positive number

$$\xi = \frac{1}{2}\left[1 - \rho\left(M_{k_0+i-1}\right)\right] > 0$$

there is a matrix norm $\|\cdot\|_M$ that makes

$$\left\|M_{k_0+i-1}\right\|_M \leq \rho\left(M_{k_0+i-1}\right) + \xi = \frac{1}{2}\left[1 + \rho\left(M_{k_0+i-1}\right)\right] < 1$$

according to the condition (3) in definition 2, it can be obtained as

$$\left\|\Pi_{j=1}^{i} M_{k_0+j-1}\right\|_M \leq \left\|M_{k_0+i-1}\right\|_M \cdots \left\|M_{k_0}\right\|_M$$

because of

$$\left\|M_{k_0+i-1}\right\|_M < 1$$

so

$$\lim_{i\to+\infty}\left(\left\|\Pi_{j=1}^{i} M_{k+j-1}\right\|_M\right) = 0$$

thus

$$\lim_{i\to+\infty}\left(\Pi_{j=1}^{i} M_{k+j-1}\right) = 0$$

and so

$$\lim_{i\to+\infty}\left(\Pi_{j=1}^{i} M_{k+j-1}\right)\left(B_k\vec{a}\right) = 0$$

immediately

$$\lim_{i\to+\infty}\Delta \hat{X}_{k+i|k+i} = 0$$

Therefore, theorem 1 holds.

Based on the above analysis, we can see that under a certain condition, the estimation value of state vectors can ignore the effect of the estimation deviation caused by pulse faults of sensors after a sufficiently long time in the future.

2.2. Effect of Step Faults on the KF of State Vectors

If the sensor appears a step fault, how will it affect the KF of state vectors? Assuming that the sensor appears a step fault with a duration of (k_0, k_0+i), and the value of the fault is $\vec{a}$, based on the formula (6), the estimation deviation $\Delta\hat{X}_{k_0+1|k_0+1}$ at time k_0+1 is calculated

$$\begin{aligned}\Delta\hat{X}_{k_0+1|k_0+1} &= \hat{X}^a_{k_0+1|k_0+1} - \hat{X}_{k_0+1|k_0+1} \\ &= A_{k_0}\hat{X}^a_{k_0|k_0} + B_{k_0+1}U_{k_0+1} + \\ &\quad K_{k_0+1}\left(Y^a_{k_0+1} - C_{k_0+1}\begin{pmatrix} A_{k_0}\hat{X}^a_{k_0|k_0} \\ +B_{k_0+1}U_{k_0+1}\end{pmatrix}\right) \\ &\quad -\begin{pmatrix} A_{k_0}\hat{X}_{k_0|k_0} + B_{k_0+1}U_{k_0+1} + \\ K_{k_0+1}\left(Y_{k_0+1} - C_{k_0+1}\begin{pmatrix} A_{k_0}\hat{X}_{k_0|k_0} \\ +B_{k_0+1}U_{k_0+1}\end{pmatrix}\right)\end{pmatrix} \\ &= \left(A_{k_0} - K_{k_0+1}C_{k_0+1}A_{k_0}\right)\left(\hat{X}^a_{k_0|k_0} - \hat{X}_{k_0|k_0}\right) + \\ &\quad K_{k_0+1}\left(Y^a_{k_0+1} - Y_{k_0+1}\right) \\ &= \left(A_{k_0} - K_{k_0+1}C_{k_0+1}A_{k_0}\right)\left(K_{k_0}\vec{a}\right) + \left(K_{k_0+1}\vec{a}\right) \\ &= M_{k_0}\left(K_{k_0}\vec{a}\right) + \left(K_{k_0+1}\vec{a}\right) \\ &= \left(M_{k_0}K_{k_0} + K_{k_0+1}\right)\vec{a}\end{aligned} \tag{10}$$

Similarly, we can calculate the estimation deviation $\Delta\hat{X}_{k_0+2|k_0+2}$ at time k_0+2.

$$\begin{aligned}\Delta\hat{X}_{k_0+2|k_0+2} &= \hat{X}^a_{k_0+2|k_0+2} - \hat{X}_{k_0+2|k_0+2} \\ &= \left(A_{k_0+1} - K_{k_0+2}C_{k_0+2}A_{k_0+1}\right)\left(\hat{X}^a_{k_0+1|k_0+1} - \hat{X}_{k_0+1|k_0+1}\right) \\ &\quad + K_{k_0+2}\left(Y^a_{k_0+2} - Y_{k_0+2}\right) \\ &= \left(A_{k_0+1} - K_{k_0+2}C_{k_0+2}A_{k_0+1}\right)\left(\left(M_{k_0}K_{k_0} + K_{k_0+1}\right)\vec{a}\right) \\ &\quad + \left(K_{k_0+2}\vec{a}\right) \\ &= M_{k_0+1}\left(M_{k_0}K_{k_0} + K_{k_0+1}\right)\vec{a} + \left(K_{k_0+2}\vec{a}\right) \\ &= \left(M_{k_0+1}M_{k_0}K_{k_0} + M_{k_0+1}K_{k_0+1} + K_{k_0+2}\right)\vec{a}\end{aligned} \tag{11}$$

And so forth, the state filtering difference $\Delta\hat{X}_{k_0+i|k_0+i}$ at time k_0+i can be given by

$$\begin{aligned}\Delta\hat{X}_{k_0+i|k_0+i} &= \hat{X}^a_{k_0+i|k_0+i} - \hat{X}_{k_0+i|k_0+i} \\ &= M_{k_0+i-1}\begin{pmatrix} M_{k_0+i-2}\cdots M_{k_0}K_{k_0} + \cdots + \\ M_{k_0+i-2}K_{k_0+i-2} + K_{k_0+i-1}\end{pmatrix}\vec{a} \\ &\quad + K_{k_0+i}\vec{a} \\ &= \begin{pmatrix} M_{k_0+i-1}M_{k_0+i-2}\cdots M_{k_0}K_{k_0} + \cdots + \\ M_{k_0+i-1}K_{k_0+i-1} + K_{k_0+i}\end{pmatrix}\vec{a} \\ &= \left(\sum_{c=1}^{i}\Pi^{i}_{b=c}M_{k_0+b-1}K_{k_0+c-1} + K_{k_0+i}\right)\vec{a}\end{aligned} \tag{12}$$

The effect of step faults in sensors on the KF of state vectors may result in the additive estimation deviation during the fault time. If the step fault disappears after time k_0+i, in order to analyze the effect of step faults on the estimation value of subsequent states, we can calculate the estimation deviation $\Delta\hat{X}_{k_0+i+1|k_0+i+1}$ at time k_0+i+1.

$$\begin{aligned}\Delta\hat{X}_{k_0+i+1|k_0+i+1} &= \hat{X}^a_{k_0+i+1|k_0+i+1} - \hat{X}_{k_0+i+1|k_0+i+1} \\ &= A_{k_0+i}\hat{X}^a_{k_0+i|k_0+i} + B_{k_0+i+1}U_{k_0+i+1} + \\ &\quad K_{k_0+i+1}\begin{pmatrix} Y_{k_0+i+1} - \\ C_{k_0+i+1}\begin{pmatrix} A_{k_0+i}\hat{X}^a_{k_0+i|k_0+i} + \\ B_{k_0+i+1}U_{k_0+i+1}\end{pmatrix}\end{pmatrix} \\ &\quad -\begin{pmatrix} A_{k_0+i}\hat{X}_{k_0+i|k_0+i} + B_{k_0+i+1}U_{k_0+i+1} + \\ K_{k_0+i+1}\begin{pmatrix} Y_{k_0+i+1} - \\ C_{k_0+i+1}\begin{pmatrix} A_{k_0+i}\hat{X}_{k_0+i|k_0+i} + \\ B_{k_0+i+1}U_{k_0+i+1}\end{pmatrix}\end{pmatrix}\end{pmatrix} \\ &= \left(A_{k_0+i} - K_{k_0+i+1}C_{k_0+i+1}A_{k_0+i}\right)\cdot \\ &\quad \left(\hat{X}^a_{k_0+i|k_0+i} - \hat{X}_{k_0+i|k_0+i}\right) \\ &= M_{k_0+i}\begin{pmatrix} \sum_{c=1}^{i}\Pi^{i}_{b=c}M_{k_0+b-1}K_{k_0+c-1} \\ + K_{k_0+i}\end{pmatrix}\vec{a}\end{aligned} \tag{13}$$

Similarly, we can calculate the estimation deviation $\Delta\hat{X}_{k_0+i+2|k_0+i+2}$ at time k_0+i+2.

$$\begin{aligned}\Delta\hat{X}_{k_0+i+2|k_0+i+2} &= \hat{X}^a_{k_0+i+2|k_0+i+2} - \hat{X}_{k_0+i+2|k_0+i+2} \\ &= \left(A_{k_0+i+1} - K_{k_0+i+2}C_{k_0+i+2}A_{k_0+i+1}\right)\cdot \\ &\quad \left(\hat{X}^a_{k_0+i+1|k_0+i+1} - \hat{X}_{k_0+i+1|k_0+i+1}\right) \\ &= M_{k_0+i+1}M_{k_0+i}\cdot \\ &\quad \left(\sum_{c=1}^{i}\Pi^{i}_{b=c}M_{k_0+b-1}K_{k_0+c-1} + K_{k_0+i}\right)\vec{a}\end{aligned} \tag{14}$$

And so forth, the estimation deviation $\Delta\hat{X}_{k_0+i+j|k_0+i+j}$ at time k_0+i+j can be given by

$$\begin{aligned}\Delta\hat{X}_{k_0+i+j|k_0+i+j} &= \hat{X}^a_{k_0+i+j|k_0+i+j} - \hat{X}_{k_0+i+j|k_0+i+j} \\ &= M_{k_0+i+j-1}\cdots M_{k_0+i}\cdot \\ &\quad \left(\sum_{c=1}^{i}\Pi^{i}_{b=c}M_{k_0+b-1}K_{k_0+c-1} + K_{k_0+i}\right)\vec{a} \\ &= \left(\prod_{s=i+1}^{i+j}M_{k_0+s-1}\right)\cdot \\ &\quad \left(\sum_{c=1}^{i}\Pi^{i}_{b=c}M_{k_0+b-1}K_{k_0+c-1} + K_{k_0+i}\right)\vec{a}\end{aligned} \tag{15}$$

The relationship between the step fault in sensors and the estimation value of subsequent states is given by (15), i.e., the product of the coefficient matrix $\left(\Pi^{i+j}_{s=i+1}M_{k_0+s-1}\right)(i = 1,2,\cdots,N; j = 1,2,\cdots)$ and the additive estimation deviation $\left(\sum_{c=1}^{i}\Pi_{b=c}^{i} M_{k_0+b-1}K_{k_0+c-1} + K_{k_0+i}\right)\vec{a}$, in essence, similar to formula (9), therefore, we can see that under a certain condition, the estimation value of state vectors can ignore the effect of the additive estimation deviation caused by step faults of sensors after a sufficiently long time in the future.

3. Effect of Control Input Anomalies on the KF of State Vectors

For a linear system, due to internal interference, not only unpredictable faults occur in the sensor, but also the control input will have an abnormal phenomenon. Therefore, in this section, we will analyze the effect of control input anomalies on the KF of state vectors in terms of pulse abnormal inputs and step abnormal inputs.

3.1. Effect of Pulse Abnormal Inputs on the KF of State Vectors

Supposing the control input $\{U_k\}$ appear a pulse abnormal variance $\vec{a}$ at time k_0, and it can be marked as $U_{k_0}^a = U_{k_0} + \vec{a}$, at the same time, from formula (1), we can calculate the present state $X_{k_0}^a = X_{k_0} + B_{k_0}\vec{a}$ and measurement data $Y_{k_0}^a = Y_{k_0} + C_{k_0}B_{k_0}\vec{a}$, so, the present estimation value of the state $\hat{X}_{k_0|k_0}^a$ is given by formula (2)

$$\begin{aligned}\Delta\hat{X}_{k_0|k_0}^a &= A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0}U_{k_0}^a \\ &+ K_{k_0}\left(Y_{k_0}^a - C_{k_0}\left(A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0}U_{k_0}^a\right)\right)\end{aligned} \tag{16}$$

The present estimation deviation $\Delta\hat{X}_{k_0|k_0}$ can be given by

$$\begin{aligned}\Delta\hat{X}_{k_0|k_0} &= \hat{X}_{k_0|k_0}^a - \hat{X}_{k_0|k_0} \\ &= A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0}U_{k_0}^a + \\ &\quad K_{k_0}\left(Y_{k_0}^a - C_{k_0}\left(A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0}U_{k_0}^a\right)\right) \\ &\quad -\begin{pmatrix} A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0}U_{k_0} + \\ K_{k_0}\left(Y_{k_0} - C_{k_0}\left(A_{k_0-1}\hat{X}_{k_0-1|k_0-1} + B_{k_0}U_{k_0}\right)\right)\end{pmatrix} \\ &= \left(B_{k_0} - K_{k_0}C_{k_0}B_{k_0}\right)\left(U_{k_0}^a - U_{k_0}\right) + \\ &\quad K_{k_0}\left(Y_{k_0+1}^a - Y_{k_0+1}\right) \\ &= \left(B_{k_0} - K_{k_0}C_{k_0}B_{k_0}\right)\vec{a} + K_{k_0}C_{k_0}B_{k_0}\vec{a} \\ &= \left(B_{k_0}\vec{a}\right)\end{aligned} \tag{17}$$

Clearly, the pulse abnormal inputs may result in the estimation deviation of state vectors. If the pulse abnormal input disappears after time k_0, in order to analyze the effect of pulse abnormal inputs on the estimation value of subsequent states, we can calculate the estimation deviation $\Delta\hat{X}_{k_0+1|k_0+1}$ at time k_0+1 (present state $X_{k_0+1}^a = X_{k_0+1} + A_{k_0}B_{k_0}\vec{a}$ and measurement data $Y_{k_0+1}^a = Y_{k_0+1} + C_{k_0+1}A_{k_0}B_{k_0}\vec{a}$)

$$\begin{aligned}\Delta\hat{X}_{k_0+1|k_0+1} &= \hat{X}_{k_0+1|k_0+1}^a - \hat{X}_{k_0+1|k_0+1} \\ &= A_{k_0}\hat{X}_{k_0|k_0}^a + B_{k_0+1}U_{k_0+1} + \\ &\quad K_{k_0+1}\left(Y_{k_0+1}^a - C_{k_0+1}\left(A_{k_0}\hat{X}_{k_0|k_0}^a + B_{k_0+1}U_{k_0+1}\right)\right) \\ &\quad -\begin{pmatrix} A_{k_0}\hat{X}_{k_0|k_0} + B_{k_0+1}U_{k_0+1} + \\ K_{k_0+1}\left(Y_{k_0+1} - C_{k_0+1}\left(A_{k_0}\hat{X}_{k_0|k_0} + B_{k_0+1}U_{k_0+1}\right)\right)\end{pmatrix} \\ &= \left(A_{k_0} - K_{k_0+1}C_{k_0+1}A_{k_0}\right)\left(\hat{X}_{k_0|k_0}^a - \hat{X}_{k_0|k_0}\right) + \\ &\quad K_{k_0+1}\left(Y_{k_0+1}^a - Y_{k_0+1}\right) \\ &= \left(A_{k_0} - K_{k_0+1}C_{k_0+1}A_{k_0}\right)B_{k_0}\vec{a} + \\ &\quad K_{k_0+1}C_{k_0+1}A_{k_0}B_{k_0}\vec{a} \\ &= A_{k_0}B_{k_0}\vec{a}\end{aligned} \tag{18}$$

Similarly, we can calculate the estimation deviation $\Delta\hat{X}_{k_0+2|k_0+2}$ at time k_0+2 (present state $X_{k_0+2}^a = X_{k_0+2} + A_{k_0+1}A_{k_0}B_{k_0}\vec{a}$ and measurement data $Y_{k_0+2}^a = Y_{k_0+2} + C_{k_0+2}A_{k_0+1}A_{k_0}B_{k_0}\vec{a}$)

$$\begin{aligned}\Delta\hat{X}_{k_0+2|k_0+2} &= \hat{X}_{k_0+2|k_0+2}^a - \hat{X}_{k_0+2|k_0+2} \\ &= \left(A_{k_0+1} - K_{k_0+2}C_{k_0+2}A_{k_0+1}\right)\begin{pmatrix}\hat{X}_{k_0+1|k_0+1}^a \\ -\hat{X}_{k_0+1|k_0+1}\end{pmatrix} + \\ &\quad K_{k_0+2}\left(Y_{k_0+2}^a - Y_{k_0+2}\right) \\ &= \left(A_{k_0+1} - K_{k_0+2}C_{k_0+2}A_{k_0+1}\right)A_{k_0}B_{k_0}\vec{a} + \\ &\quad K_{k_0+2}C_{k_0+2}A_{k_0+1}A_{k_0}B_{k_0}\vec{a} \\ &= A_{k_0+1}A_{k_0}B_{k_0}\vec{a}\end{aligned} \tag{19}$$

And so forth, the estimation deviation $\Delta\hat{X}_{k_0+i|k_0+i}$ at time k_0+i can be given by

$$\begin{aligned}\Delta\hat{X}_{k_0+i|k_0+i} &= \hat{X}_{k_0+i|k_0+i}^a - \hat{X}_{k_0+i|k_0+i} \\ &= A_{k_0+i-1}A_{k_0+i-2}\cdots A_{k_0+1}A_{k_0}B_{k_0}\vec{a} \\ &= \left(\Pi_{j=1}^{i}A_{k_0+j-1}\right)\left(B_{k_0}\vec{a}\right)\end{aligned} \tag{20}$$

The formula (20) gives the relationship between the pulse abnormal input and the estimation value of subsequent states, it can be seen that the change of the coefficient matrix $\left(\Pi_{j=1}^{i}A_{k_0+j-1}\right)$ may directly affect the estimation value change of subsequent states, and different from formal (9), the effect of pulse abnormal inputs on the estimation value of state vectors is only related to model parameters $\{A_k, B_k\}$. According to the analysis of formula (9), we can see that under a certain condition, the estimation value of state vectors can ignore the effect of the estimation deviation caused by pulse abnormal inputs after a sufficiently long time in the future.

3.2. Effect of Step Abnormal Inputs on the KF of State Vectors

Supposing the control input appears a step abnormal input with a duration of (k_0, k_0+i), and its value is $\vec{a}$, based on formula (17), we can calculate the estimation deviation $\Delta\hat{X}_{k_0+1|k_0+1}$ at time k_0+1 (present state

$$X^a_{k_0+1} = X_{k_0+1} + \left(A_{k_0}B_{k_0} + B_{k_0+1}\right)\vec{a}$$

and measurement data

$$Y^a_{k_0+1} = Y_{k_0+1} + C_{k_0+1}\left(A_{k_0}B_{k_0} + B_{k_0+1}\right)\vec{a}$$

$$\begin{aligned}\Delta\hat{X}_{k_0+1|k_0+1} &= \hat{X}^a_{k_0+1|k_0+1} - \hat{X}_{k_0+1|k_0+1}\\ &= A_{k_0}\hat{X}^a_{k_0|k_0} + B_{k_0+1}U^a_{k_0+1} +\\ &\quad K_{k_0+1}\left(Y^a_{k_0+1} - C_{k_0+1}\left(A_{k_0}\hat{X}^a_{k_0|k_0} + B_{k_0+1}U^a_{k_0+1}\right)\right)\\ &\quad -\begin{pmatrix}A_{k_0}\hat{X}_{k_0|k_0} + B_{k_0+1}U_{k_0+1} +\\ K_{k_0+1}\left(Y_{k_0+1} - C_{k_0+1}\left(A_{k_0}\hat{X}_{k_0|k_0} + B_{k_0+1}U_{k_0+1}\right)\right)\end{pmatrix}\\ &= \left(A_{k_0} - K_{k_0+1}C_{k_0+1}A_{k_0}\right)\left(\hat{X}^a_{k_0|k_0} - \hat{X}_{k_0|k_0}\right) +\\ &\quad \left(B_{k_0+1} - K_{k_0+1}C_{k_0+1}B_{k_0+1}\right)\left(U^a_{k_0+1} - U_{k_0+1}\right) +\\ &\quad K_{k_0+1}\left(Y^a_{k_0+1} - Y_{k_0+1}\right)\\ &= \left(A_{k_0} - K_{k_0+1}C_{k_0+1}A_{k_0}\right)B_{k_0}\vec{a} +\\ &\quad \left(B_{k_0+1} - K_{k_0+1}C_{k_0+1}B_{k_0+1}\right)\vec{a} +\\ &\quad K_{k_0+1}C_{k_0+1}\left(A_{k_0}B_{k_0} + B_{k_0+1}\right)\vec{a}\\ &= \left(A_{k_0}B_{k_0} + B_{k_0+1}\right)\vec{a}\end{aligned} \tag{21}$$

Similarly, we can calculate the estimation deviation $\Delta\hat{X}_{k_0+2|k_0+2}$ at k_0+2 (present state

$$X^a_{k_0+2} = X_{k_0+2} + \left(A_{k_0+1}A_{k_0}B_{k_0} + A_{k_0+1}B_{k_0+1} + B_{k_0+2}\right)\vec{a}$$

and measurement data

$$Y^a_{k_0+2} = Y_{k_0+2} + C_{k_0+2}\left(A_{k_0+1}A_{k_0}B_{k_0} + A_{k_0+1}B_{k_0+1} + B_{k_0+2}\right)\vec{a}$$

$$\begin{aligned}\Delta\hat{X}_{k_0+2|k_0+2} &= \hat{X}^a_{k_0+2|k_0+2} - \hat{X}_{k_0+2|k_0+2}\\ &= \left(A_{k_0+1} - K_{k_0+2}C_{k_0+2}A_{k_0+1}\right)\left(A_{k_0}B_{k_0} + B_{k_0+1}\right)\vec{a} +\\ &\quad \left(B_{k_0+2} - K_{k_0+2}C_{k_0+2}B_{k_0+2}\right)\vec{a}\\ &\quad + K_{k_0+2}C_{k_0+2}\left(A_{k_0+1}A_{k_0}B_{k_0} + A_{k_0+1}B_{k_0+1} + B_{k_0+2}\right)\vec{a}\\ &= \left(A_{k_0+1}A_{k_0}B_{k_0} + A_{k_0+1}B_{k_0+1} + B_{k_0+2}\right)\vec{a}\end{aligned} \tag{22}$$

And so forth, the estimation deviation $\Delta\hat{X}_{k_0+i|k_0+i}$ at time k_0+i can be given by

$$\begin{aligned}\Delta\hat{X}_{k_0+i|k_0+i} &= \hat{X}^a_{k_0+i|k_0+i} - \hat{X}_{k_0+i|k_0+i}\\ &= \begin{pmatrix}A_{k_0+i-1}\cdots A_{k_0+1}A_{k_0}B_{k_0} + \cdots +\\ A_{k_0+i-1}B_{k_0+i-1} + B_{k_0+i}\end{pmatrix}\vec{a}\\ &= \left(\sum_{c=1}^{i}\Pi_{b=c}^{i}A_{k_0+b-1}B_{k_0+c-1} + B_{k_0+i}\right)\vec{a}\end{aligned} \tag{23}$$

The effect of step abnormal inputs on the estimation value of state vectors may result in the superposition estimation deviation during the fault time. If the step abnormal input disappears after time k_0+i, in order to analyze the effect of step abnormal inputs on the estimation value of subsequent states, we can calculate the estimation deviation $\Delta\hat{X}_{k_0+i+1|k_0+i+1}$ at time k_0+i+1.

$$\begin{aligned}\Delta\hat{X}_{k_0+i+1|k_0+i+1} &= \hat{X}^a_{k_0+i+1|k_0+i+1} - \hat{X}_{k_0+i+1|k_0+i+1}\\ &= A_{k_0+i}\hat{X}^a_{k_0+i|k_0+i} + B_{k_0+i+1}U_{k_0+i+1} +\\ &\quad K_{k_0+i+1}\left(Y^a_{k_0+i+1} - C_{k_0+i+1}\begin{pmatrix}A_{k_0+i}\hat{X}^a_{k_0+i|k_0+i} +\\ B_{k_0+i+1}U_{k_0+i+1}\end{pmatrix}\right)\\ &\quad -\begin{pmatrix}A_{k_0+i}\hat{X}_{k_0+i|k_0+i} + B_{k_0+i+1}U_{k_0+i+1} +\\ K_{k_0+i+1}\left(Y_{k_0+i+1} - C_{k_0+i+1}\begin{pmatrix}A_{k_0+i}\hat{X}_{k_0+i|k_0+i} +\\ B_{k_0+i+1}U_{k_0+i+1}\end{pmatrix}\right)\end{pmatrix}\\ &= \left(A_{k_0+i} - K_{k_0+i+1}C_{k_0+i+1}A_{k_0+i}\right)\begin{pmatrix}\hat{X}^a_{k_0+i|k_0+i} -\\ \hat{X}_{k_0+i|k_0+i}\end{pmatrix} +\\ &\quad K_{k_0+i+1}\left(Y^a_{k_0+i+1} - Y_{k_0+i+1}\right)\\ &= A_{k_0+i}\left(\sum_{c=1}^{i}\Pi_{b=c}^{i}A_{k_0+b-1}B_{k_0+c-1} + B_{k_0+i}\right)\vec{a}\end{aligned} \tag{24}$$

Similarly, we can calculate the estimation deviation $\Delta\hat{X}_{k_0+i+2|k+i+2}$ at time k_0+i+2.

$$\begin{aligned}\Delta\hat{X}_{k_0+i+2|k_0+i+2} &= \hat{X}^a_{k_0+i+2|k_0+i+2} - \hat{X}_{k_0+i+2|k_0+i+2}\\ &= \left(A_{k_0+i+1} - K_{k_0+i+2}C_{k_0+i+2}A_{k_0+i+1}\right)\cdot\\ &\quad \left(\hat{X}^a_{k_0+i+1|k_0+i+1} - \hat{X}_{k_0+i+1|k_0+i+1}\right)\\ &\quad + K_{k_0+i+2}\left(Y^a_{k_0+i+2} - Y_{k_0+i+2}\right)\\ &= A_{k_0+i+1}A_{k_0+i}\begin{pmatrix}\sum_{c=1}^{i}\Pi_{b=c}^{i}A_{k_0+b-1}B_{k_0+c-1}\\ + B_{k_0+i}\end{pmatrix}\end{aligned} \tag{25}$$

And so forth, the estimation deviation $\Delta\hat{X}_{k_0+i+j|k_0+i+j}$ at time k_0+i+j can be given by

$$\begin{aligned}\Delta\hat{X}_{k_0+i+j|k_0+i+j} &= \hat{X}^a_{k_0+i+j|k_0+i+j} - \hat{X}_{k_0+i+j|k_0+i+j}\\ &= A_{k_0+i+j-1}\cdots A_{k_0+i}\cdot\\ &\quad \left(\sum_{c=1}^{i}\Pi_{b=c}^{i}A_{k_0+b-1}B_{k_0+c-1} + B_{k_0+i}\right)\vec{a}\\ &= \left(\prod_{s=i+1}^{i+j}A_{k_0+s-1}\right)\cdot\\ &\quad \left(\sum_{c=1}^{i}\Pi_{b=c}^{i}A_{k_0+b-1}B_{k_0+c-1} + B_{k_0+i}\right)\vec{a}\end{aligned} \tag{26}$$

Formula (26) gives the relationship between the step abnormal input and the estimation value of subsequent states, i.e., the product of the coefficient matrix $\left(\Pi_{s=i+1}^{i+j}A_{k_0+s-1}\right)(i = 1,2,\cdots,N;\ j = 1,2,\cdots)$ and the superposition estimation deviation $\left(\sum_{c=1}^{i}\Pi_{b=c}^{i}A_{k_0+b-1}B_{k_0+c-1} + B_{k_0+i}\right)\vec{a}$, and it is only related to model parameters $\{A_k,\ B_k\}$. Based on the analysis of formula (15), we can see that under a certain condition, the estimation value of state vectors can ignore the effect of the superposition estimation deviation caused by step abnormal inputs after a sufficiently long time in the future.

4. Simulation Calculation and Result Analysis

Taking the linear dynamic system (1) as an object, selecting four different state-space models with 3-dimension state vector and 2-dimension observation vector, Monte Carlo method is used to simulate and analyze the effect of system faults on the KF of state vectors.

(a) Model A (controllability and observability systems)

$$\begin{cases} X_{k+1} = \begin{bmatrix} 0.99 & 0.1 & 0 \\ 0 & 0.99 & -0.1 \\ 0 & 0 & 0.99 \end{bmatrix} X_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U_{k+1} + \varepsilon_k \\ Y_k = \begin{bmatrix} 0.9 & 0 & -0.9 \\ 0 & 0.05 & 0 \end{bmatrix} X_k + \eta_k \end{cases} \tag{27}$$

(b) Model B (controllability and unobservability systems)

$$\begin{cases} X_{k+1} = \begin{bmatrix} 0.99 & 0.1 & 0 \\ 0 & 0.99 & -0.1 \\ 0 & 0 & 0.99 \end{bmatrix} X_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U_{k+1} + \varepsilon_k \\ Y_k = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.05 & 0 \end{bmatrix} X_k + \eta_k \end{cases} \tag{28}$$

(c) Model C (uncontrollability and observability systems)

$$\begin{cases} X_{k+1} = \begin{bmatrix} 0.99 & 0.1 & 0 \\ 0 & 0.99 & -0.1 \\ 0 & 0 & 0.99 \end{bmatrix} X_k + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U_{k+1} + \varepsilon_k \\ Y_k = \begin{bmatrix} 0.9 & 0 & -0.9 \\ 0 & 0.05 & 0 \end{bmatrix} X_k + \eta_k \end{cases} \tag{29}$$

(d) Model D (uncontrollability and unobservability systems)

$$\begin{cases} X_{k+1} = \begin{bmatrix} 0.99 & 0 & 0 \\ 0 & 0.99 & 0 \\ 0 & 0 & 0.99 \end{bmatrix} X_k + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U_{k+1} + \varepsilon_k \\ Y_k = \begin{bmatrix} 0.9 & 0 & -0.9 \\ 0 & 0.05 & 0 \end{bmatrix} X_k + \eta_k \end{cases} \tag{30}$$

Where,

$$\varepsilon_k \sim N(0,Q) \quad \eta_k \sim N(0,R)$$

$$Q = diag[0.5^2 \quad 0.1^2 \quad 0.6^2]$$

$$R = diag[0.01^2 \quad 0.1^2]$$

Under the above four different state-space models, sensor faults and control input anomalies are simulated respectively, compared with the normal estimation value of state vectors, the estimation deviation are obtained, and draw three component change curves to analyze and discuss simulation results. When the system is controllable and observable, pulse faults in sensors can lead to appear pulse-type deviations at the fault time, and will still affect the estimation value of subsequent states. The component X_1 becomes larger and larger as the time goes, the component X_2 and X_3 may start to convergence at some time, but they may not converge to the zero value. Step faults in sensors can lead to appear superposition estimation deviations, and will still affect the estimation value of subsequent states. Similarly, the component X_1 becomes larger and larger as the time goes, the component X_2 and X_3 may start to convergence at some time, but they may not converge to the zero value.

When the system is controllable and not observable, pulse faults in sensors can lead to appear pulse-type estimation deviations at the fault time, and will still affect the estimation value of subsequent states. The component X_1 and X_2 become larger and larger as the time goes and the component X_1 is greater, the component X_3 may converge to a certain value after a long time. Step faults in sensors can lead to appear superposition estimation deviations, and will still affect the estimation value of subsequent states. The component X_1 becomes larger and larger as the time goes, the component X_2 and X_3 may start to convergence after a long time, but the former may converge to a certain value, the latter may converge to the zero value.

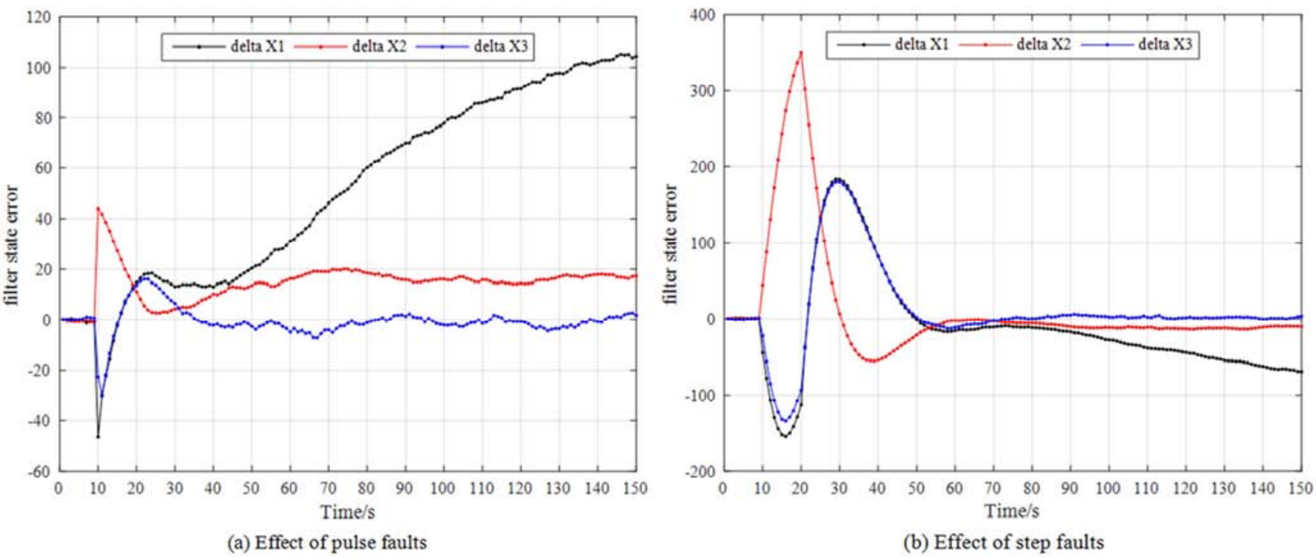
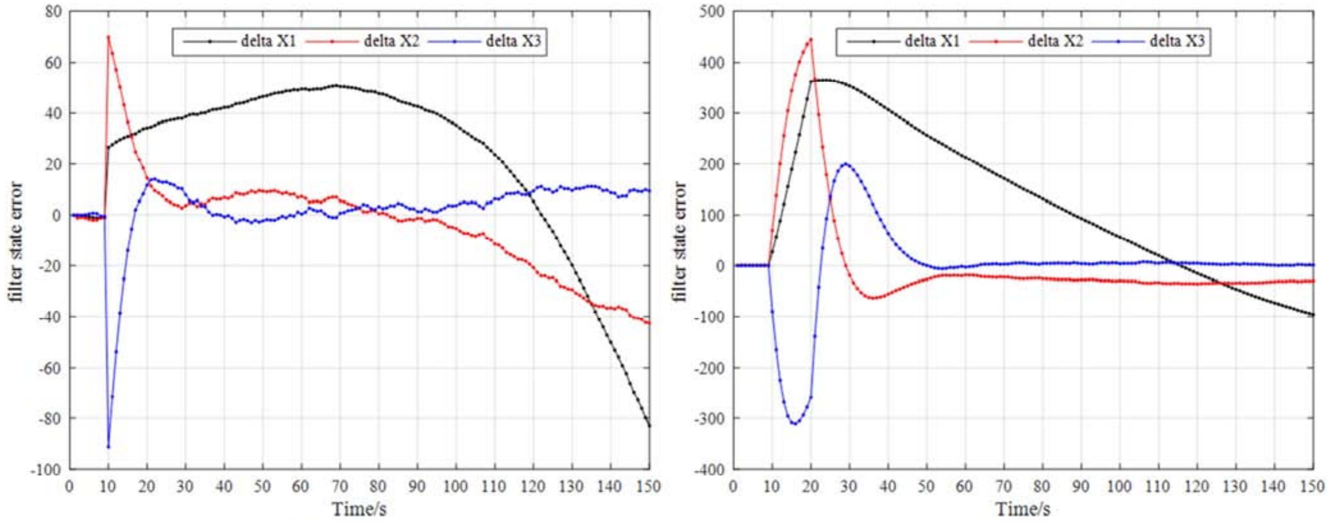


Figure 1. *The estimation deviation of state vectors in sensor faults under model A.*

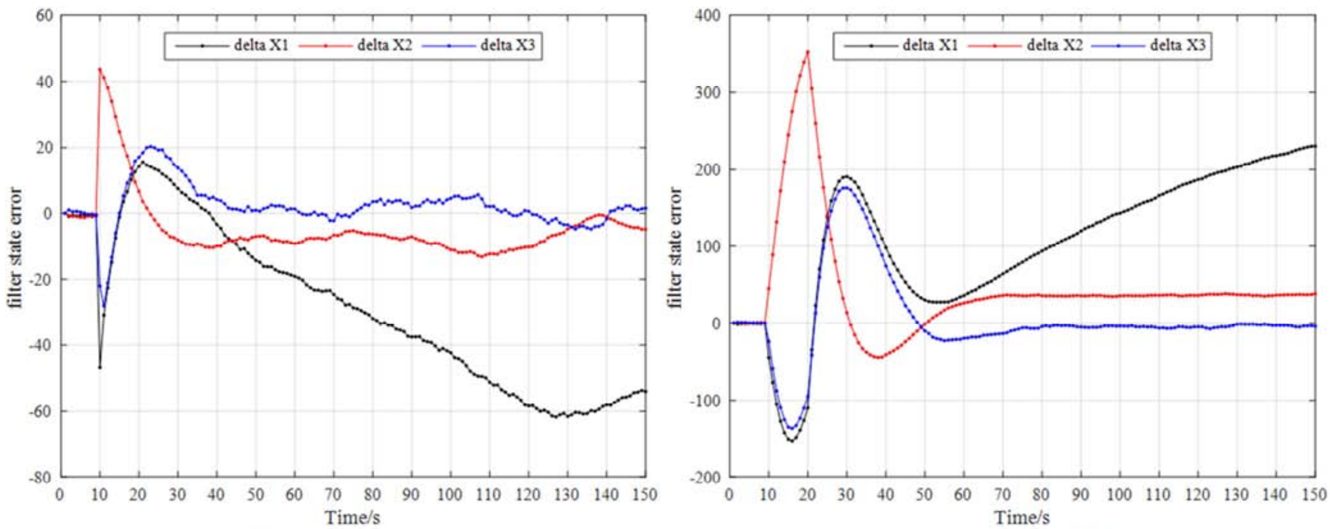


(a) Effect of pulse faults (b) Effect of step faults

Figure 2. *The estimation deviation of state vectors in sensor faults under model B.*

When the system is not controllable and observable, pulse faults in sensors can lead to appear pulse-type estimation deviations at the fault time, and will still affect the estimation value of subsequent states. The component X_1 becomes larger and larger as the time goes, the component X_2 and X_3 may converge to a certain value after a long time. Step faults in sensors can lead to appear superposition estimation deviations, and will still affect the estimation value of subsequent states. The component X_1 becomes larger and larger as the time goes, the component X_2 and X_3 may start to convergence after a long time, but the former may converge to a certain value, the latter may converge to the zero value.

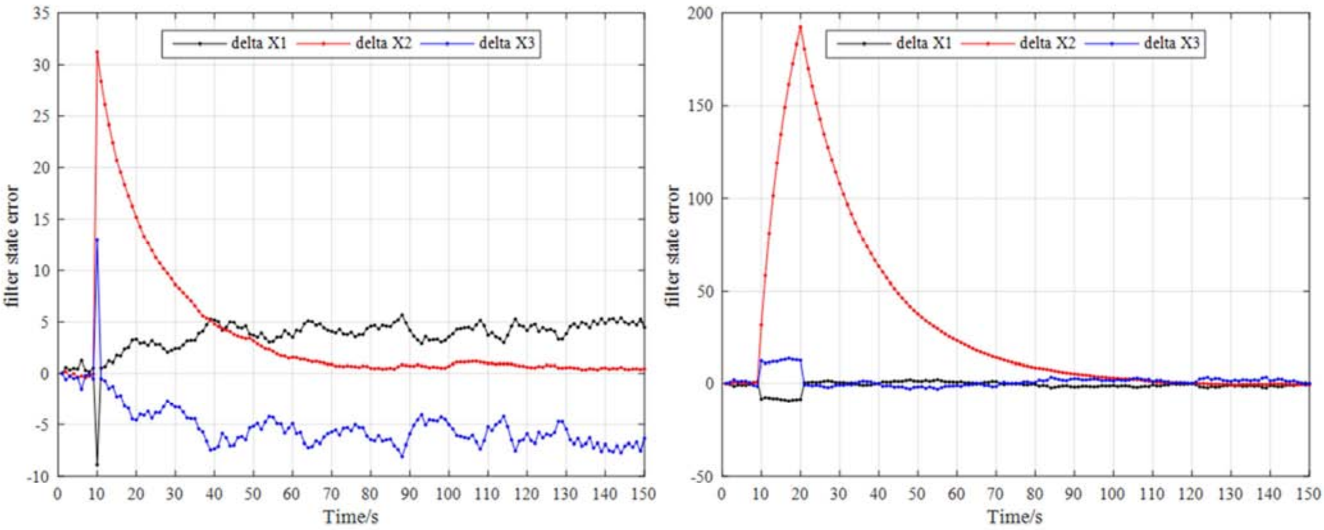
When the system is not controllable and not observable, pulse faults in sensors can lead to appear pulse-type estimation deviations at the fault time, and will still affect the estimation value of subsequent states. The component X_1, X_2 and X_3 may start to convergence after a long time, but component X_1 and X_3 may converge to a certain value, component X_2 may converge to the zero value. Step faults in sensors can lead to component X_2 appear superposition estimation deviations, and the component X_2 will start to converge to the zero value after a long time, the component X_1 and X_3 may converge to the zero value quickly after a step change.



(a) Effect of pulse faults (b) Effect of step faults

Figure 3. *The estimation deviation of state vectors in sensor faults under model C.*

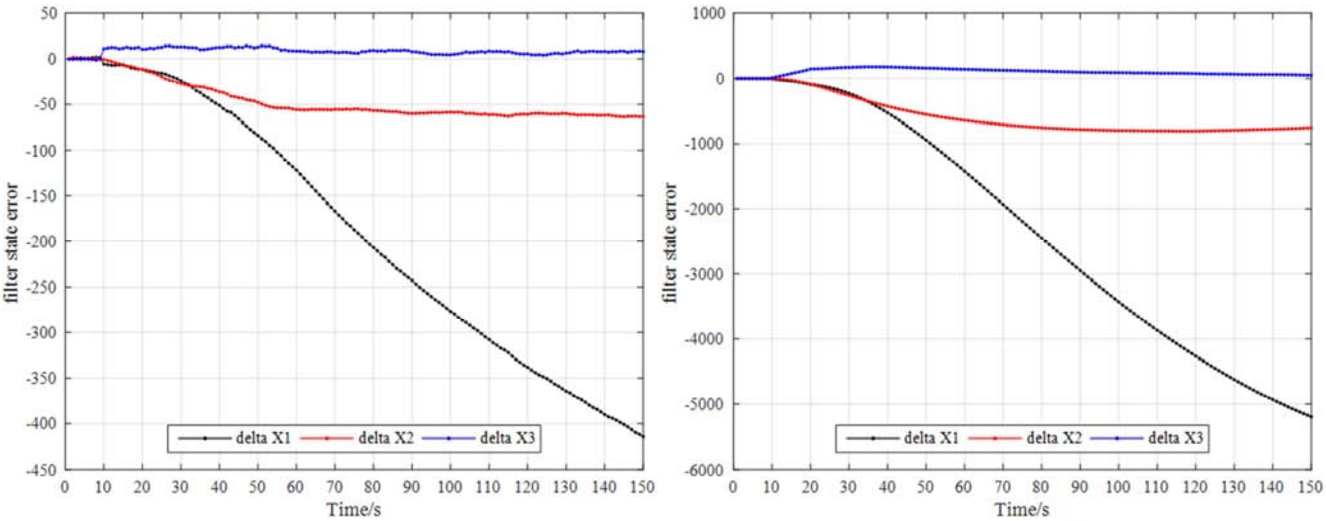
Comprehensive analysis of Figures 1 to 4 show that sensor faults can result in pulse or superposition estimation deviations of state vectors, and will continue to affect the filtering result for a long time. Whether the convergence of the estimation deviation is occur or not, which is not necessarily related to the controllability and observability of linear systems, even if convergence occurs, it may converge to a certain value, and not necessarily converge to the zero value.



(a) Effect of pulse faults (b) Effect of step faults

Figure 4. *The estimation deviation of state vectors in sensor faults under model D.*

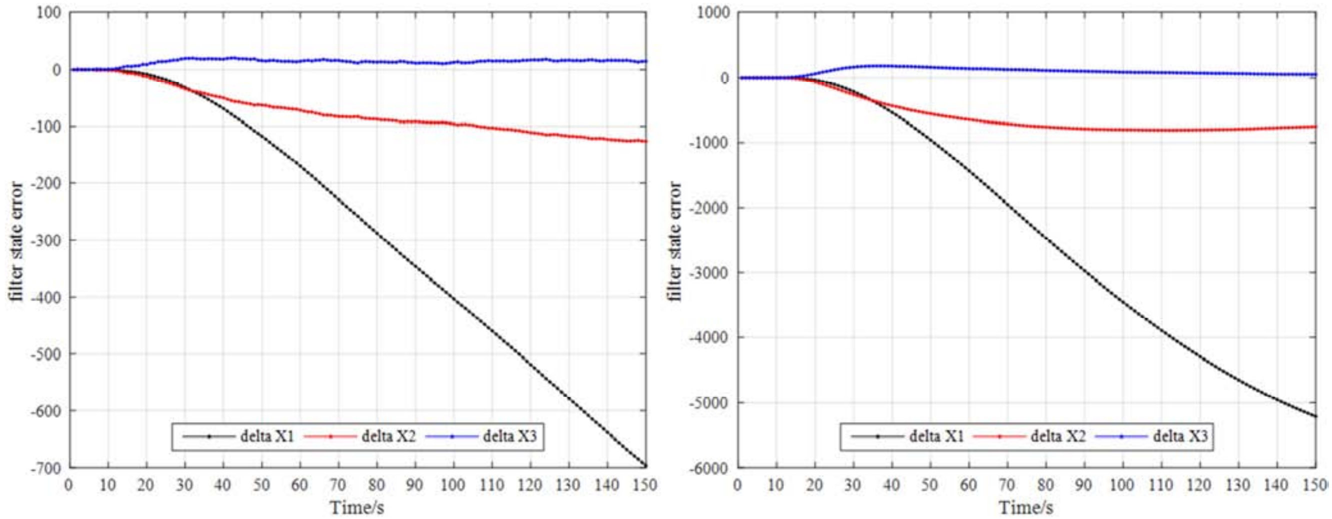
When the system is controllable and observable, pulse abnormal inputs can result in the estimation deviation of state vectors, and will still affect the estimation value of subsequent states. The component X_1 becomes larger and larger as the time goes and finally divergence, component X_2 may start to converge to a certain value after a long time, and component X_3 may tend to smoothly converge to a certain value after anomalies disappear. Step abnormal inputs can lead to the component X_3 appear superposition estimation deviations, and will start to converge to the zero value after a long time, the component X_1 becomes larger and larger as the time goes, and finally divergence, the component X_2 may converge to a certain value after a period of changes.



(a) Effect of pulse abnormal inputs (b) Effect of step abnormal inputs

Figure 5. *The estimation deviation of state vectors in control input anomalies under model A.*

When the system is controllable and not observable, pulse abnormal inputs can result in the estimation deviation of state vectors, and will still affect the estimation value of states for a long time. The component X_1 becomes larger and larger as the time goes and finally divergence, component X_2 and X_3 may tend to converge to a certain value after increasing for a long time. Step abnormal inputs can result in the superposition estimation deviation of state vectors, and will still affect the filtering result. The component X_1 becomes larger and larger as the time goes and finally divergence, component X_2 and X_3 may converge to a certain value after increasing for a long time.

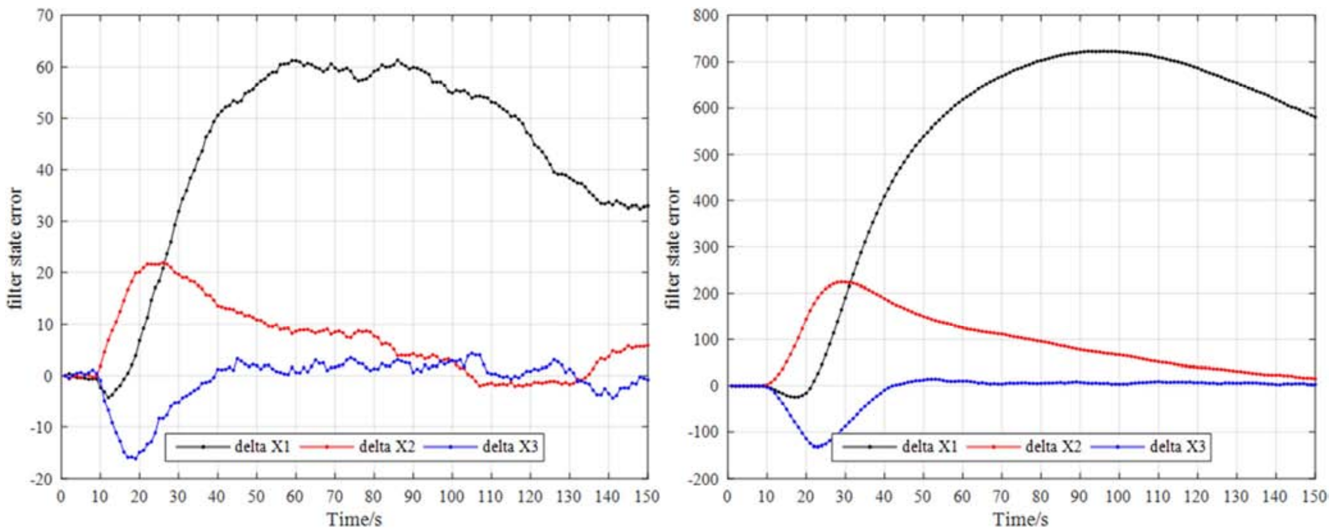


(a) Effect of pulse abnormal inputs (b) Effect of step abnormal inputs

Figure 6. *The estimation deviation of state vectors in control input anomalies under model B.*

When the system is not controllable and observable, pulse abnormal inputs can result in the estimation deviation of state vectors, and will still affect the filtering result for a long time. The component X_1 and X_2 finally converge to different values after long time changes, and component X_3 may converge to the zero value after long time changes. Step abnormal inputs can result in the superposition estimation deviation of state vectors. The component X_1 starts to decrease after increasing for a long time and does not necessarily converge to a certain value, the component X_2 and X_3 may converge to the zero value after increasing for a long time, and the convergence rate of the difference component X_2 is much slower than the difference component X_3.

When the system is not controllable and not observable, pulse abnormal inputs can result in the estimation deviation of state vectors, and will still affect the filtering result for a long time. The component X_1, X_2 and X_3 start to convergence after a long time, the component X_1 and X_3 may converge to the zero value, and component X_2 may converge to a certain value. Step abnormal inputs can result in the superposition estimation deviation of state vectors. The component X_2 starts to decrease after increasing for a long time and finally may divergence, component X_1 and X_3 may converge to different values after long time changes.



(a) Effect of pulse abnormal inputs (b) Effect of step abnormal inputs

Figure 7. *The estimation deviation of state vectors in control input anomalies under model C.*

From Figures 5 to 8, we can see that control input anomalies may result in superposition estimation deviations of state vectors, and will continue to affect the filtering result for a long time. Whether the convergence of the estimation deviation is occur or not, which is not necessarily related to the controllability and observability of linear systems, it may finally divergence, even if convergence occurs, it may not necessarily converge to the zero value.

Comparative analysis of Figure 1 to Figure 8 show that the effect of sensor faults and control input anomalies on the KF of state vectors will be distinct different. Sensor faults can result in distinct pulse spikes or damped oscillations estimation deviation of state vectors, as well as persistent step estimation deviations, meanwhile, it can be seen that the convergence speed and time of the estimation deviation caused by sensor faults may be faster and shorter.

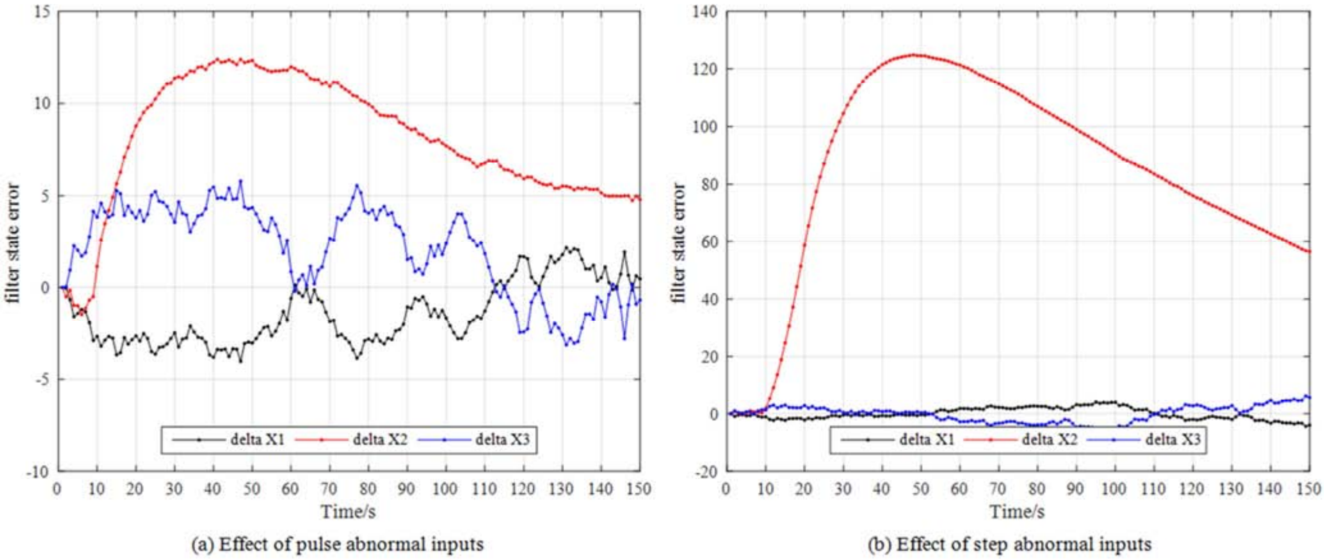


Figure 8. *The estimation deviation of state vectors in control input anomalies under model D.*

5. Conclusions

In this paper, the analysis method of theoretical derivation and model simulation calculation is combined to analyze and discuss the effect of faults or anomalies on the KF of state vectors in linear systems. In theory, taking sensor faults and control input anomalies as examples, the influence relationship between pulse and step faults (anomalies) and the estimation value of state vectors are deduced and established respectively, and the sufficient conditions for the convergence of the filtering result in the presence of faults or anomalies are given; in simulation, selecting a linear system state-space model with three-dimension state vector and two-dimension observation vector as an object, four different structure models are used to simulate and analyze the effect of sensor faults and control input anomalies on the KF, simulation results show that faults and anomalies in linear systems can result in distinct estimation deviations of state vectors for a long time in future, and whether the estimation deviation is convergence or not, which is not necessarily related to the controllability and observability of linear systems, even if convergence occurs, it may converge to a certain value, and does not necessarily converge to the zero value. In term of a same structure system, there are obvious differences in the effect of sensor faults and control input anomalies on the KF of state vectors, relatively speaking, the effect of sensor faults on the KF of state vectors is more obvious, which will result in pulse-type, superposition or step-type estimation deviations of state vectors, and the convergence speed and time of the estimation deviation may be faster and shorter.

The research results in this paper have a certain practical reference value for us to apply the KF algorithm and analyze the stability of the filtering result in the presence of system faults or anomalies, and also provide a theoretical reference for us to use the analysis of the filtering result to achieve fault detection, identification and diagnosis in linear systems.

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