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Gravitational Collapse and Singularities in Some Non-Schwarzschild's Space-Times

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Abstract: Singularities in three non-Schwarzschild space-times: Minkowski, Friedman-Lemaitre-Robertson-Walker and Reissner-Nordstromare investigated. Gravitational collapse in the Schwarzschild solution is obvious and widely studied. However, gravitational collapse should not be limited to Schwarzschild solution only as interesting findings exist in other metric fields. The Ricci curvature scalar for each space-time is evaluated and used in the determination of true curvature singularities. The Ricci scalar has proved to be very effective in determining the presence of singularities or otherwise in space-time geometry. Results indicate that there are inherent singularities in components of space-time in all three cases. Gravitational singularities in Minkowski space are found to be consequences of the choice of coordinate. Minkowksi space possesses only coordinate singularities and no curvature singularity. This differs with Schwarzschild's metric which has true curvature singularity. Friedman-Lemaitre-Robertson-Walker (FLRW) and Reissner-Nordstrom metrics have true curvature singularities. Gravitational collapse in the FLRW metric yields a curvature singularity which shows the universe started a finite time ago. Cosmic strings, white holes and blackholes are deduced from the Reissner-Nordstrom singularities. Reissner-Nordstrom solution show that the addition of small amounts of electric charge or angular momentum could completely alter the nature of the singularity, causing the matter to fall through a 'wormhole' and emerge into another universe. Analysis of gravitational collapse in this article provides one of the most exciting research frontiers in gravitation physics and high energy astrophysics; as the debate on their physical existence persists.

Keywords: Singularities, Non-Schwarzschild, Reissner-Nordstrom, Curvature, Minkowski, Space-Time

1. Introduction

Space time (or gravitational) singularity is defined as a point where the quantities used to measure gravity become infinite. This is independent of the coordinate system used to describe the gravitational field. Alternatively, a gravitational singularity is a location at which all the physical laws are indistinguishable from each other. That is, space and time combine indistinguishably and lose any independent meaning [1].

The physical implications of space-time singularities are eminent and there are sufficient reasons to conclude that the space time of the universe is singular. Several singularity theorems have been established using definitions of singularities based on Path-incompleteness [2]. Using these theorems, it can be shown that if some conditions are satisfied, then singularities cannot be avoided in certain circumstances. Prominent among these conditions is the "positive energy conditions" which asserts that energy can never be negative [3]. Also, it can be deduced from these theorems that the universe started some 13.7 billion years ago with an initial singularity, in the "Big Bang". Furthermore, collapsing matter forms a black hole with a central singularity under certain circumstances [4].

There are many definitions of space-time singularities but the most accepted criterion is based on the fact that incomplete paths exist in some space times [4]. Moreover, definitions of gravitational singularities based on curvature pathology also use the concept of path incompleteness [2]. A path in space-time geometry is defined as a continuous chain of events through space and time. In other words, paths are possible trajectories of particles and observers in space time ("world-lines"). Paths are made up of events occupied by objects throughout their lifetimes. A particle or observer following a path is said to "run out of the world" after finite time if the path is incomplete and in-extendible [2]. Such a particle will hurtle into the tear in the fabric of space-time and vanish. Alternatively, such an observer could leap out of the tear of the universe to follow a different path [5].

The case of gravitational collapse in Schwarzschild metric is widely studied and accepted [6]. However, the study of gravitational collapse should not be limited to Schwarzschild solution only. This paper explores singularities in three other exact Non-Schwarzschild solutions (Minkowski, Friedman-Lemaitre-Robertson-Walker and Reissner-Nordstrom).

The Minkowski (flat) space-time is the first and easiest exact solution for empty space with zero energy momentum tensor. If we take the four coordinates as u^1, u^2, u^3, u^4, then the metric in this space time can be expressed as

$$ds^2 = (du^4)^2 + (du^3)^2 + (du^2)^2 - (du^1)^2 \tag{1}$$

The universe is known from astronomical observations to be approximately spherically symmetric about every point in space [7]. The most suitable metric in this case is the Friedman-Lemaitre-Robertson-Walker (FLRW) metric which is at the foundation of modern Cosmology. The FLRW metric describes the space-time of a homogeneous and isotropic Universe [8]. It is usually written in spherical polar coordinates as

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{2}$$

where $a(t)$ is the time dependent scale factor.

The space-time exterior toa spherically symmetric electrically charged star is described by the solution of coupled Einstein-Maxwell equations called the Reissner-Nordstrom [9]. The metric is given as;

$$ds^2 = -\Delta dt^2 + \Delta^{-1}dr^2 + r^2 d\Omega^2 \tag{3}$$

where;

$$\Delta = 1 - \frac{2m}{r} + \frac{q^2}{r^2} \tag{4}$$

with$m$as the mass of spherical body and$q$ is total electric charge.

Other space-times include Nariai and anti-Nariai, Anti-de Sitter, Plebanski-Hacyan, Melvin, Schwarzschild-Melvin, Vaidya, Weyl, Zipoy-Voorhes, Bonnor-Swaminarayan, Bicak-Hoenselaers-Schmidt, Plebanski-Demianski etc [10]

2. Theoretical Methods

2.1. Sampling

In this article, the concept of singularities and gravitational collapse is studied in three space-times namely;

i Minkoswki Space-time
ii Friedman-Lemaitre-Robertson-Walker Space-time
iii Reissner-Nordstrom Space-time

These space-times were randomly selected from many non-Schwarzschild space-times.

2.2. Determination of Singularities

In general relativity theory, it is imperative to distinguish between curvature singularities and coordinate singularities. A curvature singularity is a genuine space-time singularity which causes the components of space-time to diverge. In contrast, a coordinate singularity is a mere artifact of the choice of coordinate. At a coordinate singularity, the curvature of space-time is perfectly fine but the metric components diverge owing to a bad choice of coordinates. The curvature is measured by the Riemann tensor and it is quite difficult to say where a tensor becomes infinite since its components are coordinate dependent. Scalar invariants can be constructed from the Riemann curvature metric tensor. Since these are coordinate-independent, it will be meaningful to say that they become infinite. These scalars include the Ricci scalar, Weyl Scalar and Kretschmann scalar.

The Riemann curvature tensor is a function of the affine connection which is itself dependent on the metric tensor. Therefore one may look to the condition $g_{00} = 0$ for space-time singularities. However, the definition that gravitational collapse occurs at $g_{00} = 0$ is strictly geometric [9, 10]. Therefore, only a scalar invariant can be used to determine true curvature singularities. In this article, the Ricci scalar is used to determine space-time singularities.

2.3. Visualizing Space-Time Curvature

Visualization of curved 4-D space-times is a challenging task. An extremely useful, widely and commonly used method of space-time visualization is that of Penrose diagrams. It was initially developed to compare space-times with Minkowski space-time. The structure of the Penrose diagram allows us to subdivide conformal infinity into a few different regions. In order to visualize or capture the global and causal structure of a space-time, a coordinate transformation is found such that conformal infinity lies at a finite coordinate distance and the radial light rays are always at 45^0 [11]. With this conformal infinity can be subdivided into a few finite regions such as future time-like infinity, spatial infinity, future null infinity and past null infinity [9]. Penrose diagrams of some solutions of Einstein's equations will be drawn.

3. Results and Discussion

3.1. Gravitational Collapse in Minkowski Space-Time

The covariant and contravariant metric tensors of the Minkowski space-time (equation 1) are given respectively as

$$g_{11} = g_{22} = g_{33} = 1, \quad g_{00} = -1 \tag{5}$$

and

$$g^{11} = g^{22} = g^{33} = 1, \quad g^{00} = -1 \tag{6}$$

It can be seen from equations (5) and (6) that all non-zero parts of the metric tensors are constant and hence the affine coefficients and Riemann tensor vanishes. The implication of these results is that the curvature scalars also vanish and hence confirms the fact that the Minkowski space covers the entire manifold. Hence, it has no physical singularity. However, another form of the metric can be obtained by using spherical polar coordinates (t, r, θ, ϕ) which are related to (u^1, u^2, u^3, u^4) as in equation (7)

$$u^3 = r\cos\theta, u^2 = r\sin\theta\cos\phi, \; u^1 = r\sin\theta\sin\phi, \quad u^4 = t \tag{7}$$

with

$$0 \le r < \infty, 0 \le \theta \le \pi, 0 \le \phi < 2\pi - \infty < t < \infty \tag{8}$$

Then, it can be shown that the metric in equation 1 takes the form:

$$ds^2 = -dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + dt^2 \tag{9}$$

In this case, the metric has apparent singularities at $r = 0$ and at $\sin\theta = 0$. These singularities exist because spherical polar coordinates are not admissible at this point. It is imperative to note that, although this case is easily recognizable, it is not always easy to conclude that an apparent singularity in the metric is just due to a bad choice of coordinates.

An interesting representation of Minkoswki space time is presented as [11]

$$ds^2 = \left(dt^i\right)^2 - \left(dr^i\right)^2 - \sin^2 r^i\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{10}$$

Apparent (or removable) singularities exist in the metric at $r^1 = 0$ and $r^2 = \pi$ (the zeros of $\sin^2 r^i$). It has also been shown that the singularities can be removed by the transformation of these points to local coordinates in some regions [4]. The Penrose diagram of the singularity-free Minkowski space time is presented in Figure 1.

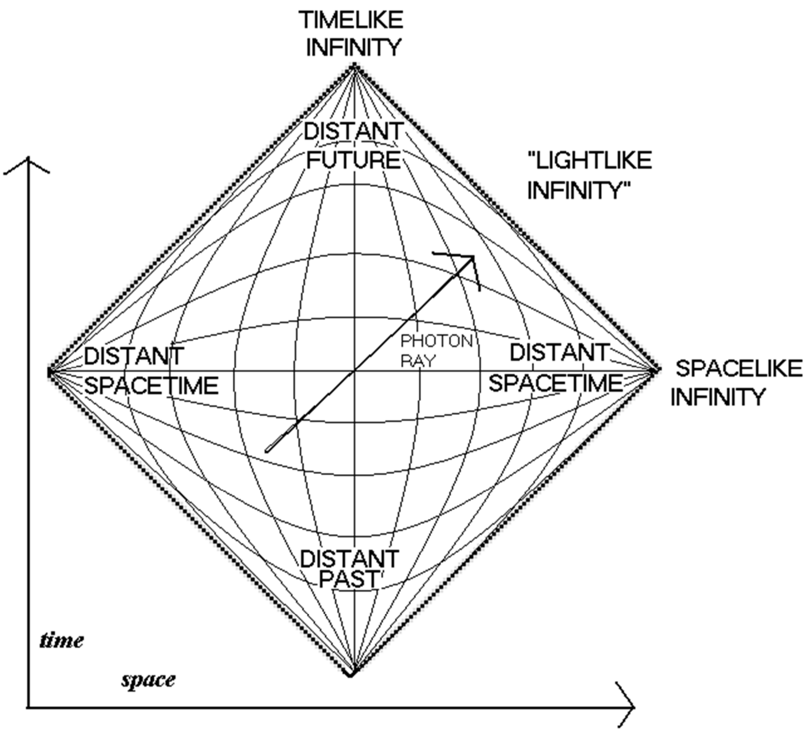

Figure 1. *Penrose diagram of the singularity-free Minkowski space-time [12].*

Thus, gravitational collapse and singularity in Minkowski space is just a consequence of the choice of coordinates. Hence, Minkowksi space possesses only coordinate singularities and no curvature singularity. This is contrary to Schwarzschild's metric where a true curvature singularity is obtained at $r = 0$ [13]. The curvature scalars vanish and the simplicity of the metric makes it easy for many coordinate transformations. A conical singularity is obtained by representing the Minkowski space-time in cylindrical coordinate [10]. It has been successfully proved that the resulting space-time is a simple model for a cosmic string. Observational evidences such as gravitational lensing of galaxies and observation of quasars recently confirmed the existence of cosmic strings [14]. Cosmic strings are 1-dimensional topological defects believed to have formed during the early stages of the evolution of the universe when phase transitions and symmetry breaking were taking place [15]. Also, cosmic strings can be used in mining energy from a blackhole [16].

3.2. Gravitational Collapse in Robertson Walker Metric

The non-zero covariant components of equation (2) are;

$$g_{00}=1, g_{11}=\frac{a^2}{1-kr^2}, g_{22}=a^2r^2, g_{33}=a^2r^2\sin^2\theta \tag{11}$$

This is obviously a diagonally symmetric space-time and therefore the contravariant metric tensor is found using the orthogonality relation and yields;

$$g^{00}=1,\quad g^{11}=\frac{1-kr^2}{a^2},\quad g^{22}=\frac{1}{a^2r^2},\quad g^{33}=\frac{1}{a^2r^2\sin^2\theta} \tag{12}$$

The affine coefficients or Christoffel symbols of the second kind are obtained using [17] as

$$\Gamma^{\sigma}_{\mu\lambda}=\frac{1}{2}g^{\sigma\upsilon}\left(g_{\mu\upsilon,\lambda}+g_{\upsilon\lambda,\mu}-g_{\mu\lambda,\upsilon}\right) \tag{13}$$

$$\begin{gathered}
\Gamma^0_{11}=\frac{a\dot{a}}{1-kr^2},\ \Gamma^0_{22}=a\dot{a}r^2,\ \Gamma^0_{33}=a\dot{a}r^2\sin^2\theta,\\
\Gamma^1_{10}=\Gamma^1_{01}=\frac{\dot{a}}{a},\ \Gamma^1_{11}=\frac{kr}{1-kr^2},\ \Gamma^1_{22}=-r\left(1-kr^2\right)\\
\Gamma^1_{33}=-r\left(1-kr^2\right)\sin^2\theta,\ \Gamma^2_{20}=\Gamma^2_{02}=\frac{\dot{a}}{a},\ \Gamma^2_{21}=\Gamma^2_{12}=\frac{1}{r},\ \Gamma^2_{33}=-\sin\theta\cos\theta,\\
\Gamma^3_{30}=\Gamma^3_{03}=\frac{\dot{a}}{a},\ \Gamma^3_{31}=\Gamma^3_{13}=\frac{1}{r},\ \Gamma^3_{23}=\Gamma^3_{32}=\cot\theta
\end{gathered} \tag{14}$$

The Ricci tensor is obtained using the formula

$$R_{ab}=\Gamma^c_{ab,c}-\Gamma^c_{ac,b}+\Gamma^d_{ab}\Gamma^c_{cd}-\Gamma^d_{ac}\Gamma^c_{bd} \tag{15}$$

as

$$R_{00}=-3\frac{\ddot{a}}{a} \tag{16}$$

$$R_{11}=\frac{2\dot{a}+a\ddot{a}+2k}{1-kr^2} \tag{17}$$

$$R_{22}=r^2\left(2\dot{a}^2+a\ddot{a}+2k\right) \tag{18}$$

$$R_{33}=r^2\sin^2\theta\left(a\ddot{a}+2k+2\dot{a}^2\right) \tag{19}$$

The Ricci scalar, R is obtained from the contraction of the Ricci tensor as

$$\begin{aligned}
R&=g^{00}R_{00}-g^{11}R_{11}-g^{22}R_{22}-g^{33}R_{33}\\
&=-6\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)
\end{aligned} \tag{20}$$

It can be depicted from the Ricci curvature scalar that a curvature singularity exist at the point $a(t)=0$ where the scalar diverges. Hence, gravitational collapse occurs at the curvature singularity, $a=0$ for the FLRW space-time. This clearly indicates that the universe started a finite time ago from the FLRW singularity.

Observation of distant galaxies indicates that galaxies are receding and hence the Universe is expanding [18, 19]. If we trace backwards in time the expansion of the universe, then the singularity of the FLRW space-time at $a=0$ represents the Big Bang. This shows that the universe was created from a singular point and was not an explosive outburst of matter. This confirms the fact stated by Hawkings that "singularities are inevitable in solutions which satisfy certain reasonable global conditions and in which the energy-momentum tensor satisfies a reasonable inequality" [7].

The Friedman equation for the energy density was derived for the FLRW spacetime as [20]

$$\rho=\frac{3}{8\pi G}\left[\frac{\dot{a}^2-k}{a^2}\right] \tag{21}$$

At the singularity, $a(t)=0$, the density becomes infinite. It is highly believed that the perfect symmetric nature of FLRW universes accounted for this singularity. Singularity theorems show that any universe satisfying the conditions: $\rho\geq 0$ and $p\geq 0$ must have originated at a singularity [9]. The energy density will become extremely high as $a(t)\to 0$, and in such a regime, a description of nature cannot be obtained from classical general relativity. It is hoped that a well developed theory of quantum gravity can be able to describe nature in such cases. Many questions about the beginning of time are serious concerns for the contemporary scientific world. Lots of arguments and counter-arguments have been put forth for and against the notion of time having a beginning [5, 7, 21-23]. Until a quantum gravity theory is put forth to explain what happened at the Big Bang, it is only reasonable, in the meantime, to view this singularity as a perfect description of the beginning of the Universe at the

Big Bang. The singularity of the Schwarzschild space-time is space-like unlike that in FLRW which is time-like. Hence, while the Schwarzschild metric singularity describes a blackhole, the FLRW singularity describes the evolution of the Universe

3.3. Gravitational Collapse in Reissner-Nordstrom Space-Time

Event horizons and singularities in this metric are more complex compared to that in Schwarzschild metric. This is due to the presence of a charge term, q, in the space-time. The components of the covariant metric tensor in equation (3) are:

$$g_{00} = 1 - \frac{2m}{r} + \frac{q^2}{r^2} \tag{22}$$

$$g_{11} = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} \tag{23}$$

$$g_{22} = r^2 \tag{24}$$

$$g_{33} = r^2 \sin^2\theta \tag{25}$$

The contravariant metric tensor components obtained using the orthogonality condition are given as

$$g^{00} = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} \tag{26}$$

$$g^{11} = 1 - \frac{2m}{r} + \frac{q^2}{r^2} \tag{27}$$

$$g^{22} = \frac{1}{r^2} \tag{28}$$

$$g^{33} = \frac{1}{r^2 \sin^2\theta} \tag{29}$$

The non-zero coefficients of affine connections are calculated as equations (30)-(38)

$$\Gamma^0_{01} = \frac{1}{2} g^{00}\left(g_{00,1}\right) = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1}\left(\frac{mr - q^2}{r^3}\right) \tag{30}$$

$$\Gamma^1_{11} = \frac{1}{2} g^{11}\left(g_{11,1}\right) = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1}\left(\frac{mr - q^2}{r^3}\right) \tag{31}$$

$$\Gamma^1_{00} = \frac{1}{2} g^{11}\left(g_{00,1}\right) = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)\left(\frac{mr - q^2}{r^3}\right) \tag{32}$$

$$\Gamma^1_{22} = \frac{1}{2} g^{11}\left(-g_{22,1}\right) = -r\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) \tag{33}$$

$$\Gamma^1_{33} = \frac{1}{2} g^{11}\left(-g_{22,1}\right) = -r\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)\sin^2\theta \tag{34}$$

$$\Gamma^2_{12} = \frac{1}{2} g^{22}\left(g_{22,1}\right) = \frac{1}{r} \tag{35}$$

$$\Gamma^2_{33} = -\frac{1}{2} g^{22}\left(g_{33,2}\right) = -\sin\theta\cos\theta \tag{36}$$

$$\Gamma^3_{13} = -\frac{1}{2} g^{33}\left(g_{33,1}\right) = \frac{1}{r} \tag{37}$$

$$\Gamma^3_{23} = -\frac{1}{2} g^{33}\left(g_{33,2}\right) = \cot\theta \tag{38}$$

The Ricci tensor is given as equations (39)-(42)

$$R_{00} = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)\left(\frac{-2mr + 3q^2}{r^4}\right) \tag{39}$$

$$R_{11} = 2\left(\frac{mr - q^2}{r^3}\right)^2\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-2} - \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1}\left(\frac{-2mr + 3q^2}{r^4}\right) - \frac{2}{r}\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1}\left(\frac{mr - q^2}{r^3}\right) \tag{40}$$

$$R_{22} = -\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) - 2r\left(\frac{mr - q^2}{r^3}\right) + 1 \tag{41}$$

$$R_{33} = \frac{m}{r}\sin^2\theta \tag{42}$$

The Ricci scalar is calculated as

$$R = g^{00}R_{00} - g^{11}R_{11} - g^{22}R_{22} - g^{33}R_{33} = \frac{-2mr + 2q^2}{r^2} \tag{43}$$

Equation (43) shows that at $r = 0$ the Ricci scalar diverges and gravitational collapse occurs. Hence for this metric a singularity exist at $r = 0$ and to have a clear understanding of this singularity, it is imperative to study the presence or otherwise of event horizons around the singularity. Suppose $g_{00} = 0$ then using equation (22) we can write

$$r^2 - 2mr + q^2 = 0 \tag{44}$$

or

$$r_{\pm} = m \pm \sqrt{m^2 - q^2} \tag{45}$$

The solutions in (45) indicate coordinate singularities of the metric

Case one: Imaginary Solution ($m^2 < q^2$)

In this case, the coefficient $\left(1-\frac{2m}{r}+\frac{q^2}{r^2}\right)$ in the metric is never zero and always positive. Also, this metric is completely regular in the (t,r,θ,ϕ) coordinates from the singularity at $r=0$ and all over the space-time. There is no hindrance or obstruction on the path of an observer moving to the singularity and returning to report what was observed as there is no event horizon. This type of singularity is said to be naked as it is not shielded by an event horizon. Figure 2 shows that both the time-like and space-like geodesics directly touch the singularity at $r=0$ because there is no event horizon.

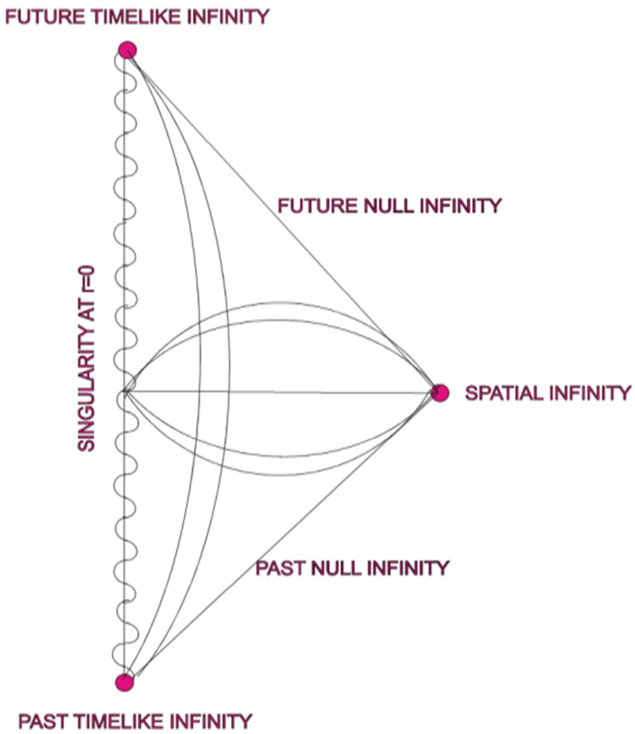

Figure 2. *Penrose diagram for the naked singularity solution of the Reissner-Nordstrom Metric [7].*

Case two ($m^2 > q^2$)

This is the situation which is expected when the gravitational collapse is real. In this case, the total energy is greater than the electromagnetic field energy. $\left(1-\frac{2m}{r}+\frac{q^2}{r^2}\right)$ is greater than zero at extremely large or small values of r and inside the two vanishing points defined by equation (45). It is less than zero. There are coordinate singularities of the metric at r_+ and r_- which canbe removed by a change or transformation of coordinates. The singularity at $r=0$ is covered by event horizons and hence this solution is a black hole solution of the Reissner-Nordstrom metric. Figure 3 shows the Penrose diagram.

The salient features of Figure 3 include:

White holes: these are hypothetical features of the universe. White holes are regarded as the opposite of black holes. Whereas black holes do not allow anything to escape from their surfaces, white holes allow matter and energy to erupt from it, though nothing can get inside [25].

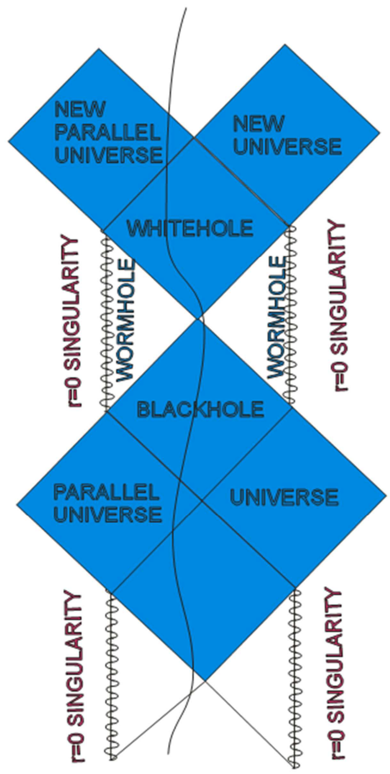

Figure 3. *Penrose diagram representing real solution of Reissner-Nordstrommetric [24].*

Worm holes: These arespace-time theoretical passages that can create short cuts for long journeys across the universe. It is also called Einstein-Rosen bridges as they were proposed by Albert Einstein and Nathan Rosen in 1935 [25]. Worm holes basically create short cuts between two points in space-time, thus theoretically reducing distance and travel time.

Here, unlike in a naked singularity, the inner event horizons prevent the geodesics from directly travelling to the singularity at*r=0*.

Case three ($m^2 = q^2$)

This case describes an extremal blackhole and is therefore called the extremal Reissner-Nordstrom solution. The mass is exactly balanced by the charge. It is a very unstable case because adding small matter will reduce it to case two. Black holes in Reissner-Nordstrom metric are charged unlike those in Schwarzschild's metric. Also, while a singularity exist for both solutions at $r=0$, Reissner-Nordstrom singularity is covered by two event horizons whereas Schwarzschild singularity has one event horizon.

Gravitational collapse of an object leads to occurrences of singularities which can be classified into blackholes and naked singularity. A blackhole singularity is hidden from external observers by the event horizon while a naked singularity is visible to external observers as they are not shielded by event horizons. Thus, a region of space-time which is invisible to an asymptotic observer is called a black hole. It has a boundary that seals off part of space-time from the outside.

The occurrence of naked singularities is reflected in the Reissner-Nordstrom solution (case one). This solution violates the cosmic censorship conjecture of Roger Penrose which states that “the gravitational collapse of an astrophysical body should not lead to the formation of naked singularities” [26]. This indicates that a black hole hiding within an event horizon is created when a massive star collapses. However, this remains a conjecture and event horizons of collapsing stars are yet to be demonstrated [27].

The formation and existence of space-time singularities and collapse are thus predicted from the General Theory of Relativity. However, the singularity is not restricted to the black hole region only. General Relativity is unable to predict the sequence of events as the star collapses. The question remains whether it is the singularity or the horizon that comes first. This is a very crucial puzzle of black hole physics today and attracts extensive research.

A special case of the solution $m^2 < q^2$ exist for which $m = 0$ and describes the gravitational field in the vicinity of a massless charged object. This is obviously unrealistic and corroborates the claims that this solution is unphysical [10]. Suppose, particles or objects (such as electrons) are considered classically as point particles (having masses and charges), then they would satisfy $q^2 > m^2$ by a significant margin. This is because gravitational interactions are completely negligible compared with electromagnetic or Coulomb interactions. It has been rightly pointed out that there is a possibility that a large number of gravitationally collapsed objects of mass greater than or equal to 10^{-5}g are formed when there are fluctuations in the early universe [28]. These objects can possess electric charge of up to ± 30 electron units and trace distinctpaths or tracks in bubble chambers forming atoms with protons or orbiting electrons. Although, electrons are basically quantum mechanical objects and cannot be studied using classical general relativity theory, the suggestion that particles can be modeled after the naked singularity solution of the Reissner-Nordstrom space-time is, however, not out of place. Researches into low mass stars are currently being pursued vigorously.

4. Conclusion

Gravitational collapse and singularities are unquestionably an integral part of our current theory of gravity (general relativity). Curvature scalars are the only true indicators of true curvature singularities. It is evident that gravitational collapse of the solutions to Einstein equations can result to the creation of black holes and naked singularities, whereas the gravitational collapse of the FLRW leads to the Big Bang singularity describing the evolution of the Universe. The conditions for collapse in Minkowski, Friedman-Lemaitre-Robertson-Walker and Reisnner-Nordstrom (R-D) space-times were studied. The Ricci scalar proved very effective in determining the presence of singularity or otherwise of a space-time. In its natural form, the Minkowski space is singularity-free while the FLRW and R-N space-times have singularities at $a = 0$ and $r = 0$ respectively (points where the Ricci scalar of each solution diverges). In the case of a collapsing star that is spherically symmetric and uncharged, all matter strikes the singularity. On the contrary, Reissner-Nordstrom solution shows that if a small amount of angular momentum or electric charge is added or removed, it will completely alter the nature of the singularity, causing matter to go through a ‘wormhole’ and comes out in another universe. The existence of naked singularities defies the cosmic censorship conjecture that forbids the existence of naked singularities and raises questions on the credibility of the General Theory of Relativity. Gravitational collapse in three out of many solutions to Einstein’s equation has been studied in this article. More solutions can be studied for possible interpretations of singularities and gravitational collapse.

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