**International Journal of Applied Mathematics and Theoretical Physics**
2018; 4(4): 91-97
http://www.sciencepublishinggroup.com/j/ijamtp
doi: 10.11648/j.ijamtp.20180404.11
ISSN: 2575-5919 (Print); ISSN: 2575-5927 (Online)

# A Relativistic Consideration of Kinematic Magnetic and Electric Fields

**Vladimir Alexandr Leus*, Stephen Taylor**

Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool, UK

**Email address:**
vladalex@liv.ac.uk (V. A. Leus)
*Corresponding author

**To cite this article:**
Vladimir Alexandr Leus, Stephen Taylor. A Relativistic Consideration of Kinematic Magnetic and Electric Fields. *International Journal of Applied Mathematics and Theoretical Physics*. Vol. 4, No. 4, 2018, pp. 91-97. doi: 10.11648/j.ijamtp.20180404.11

**Received**: October 22, 2018; **Accepted**: November 8, 2018; **Published**: December 26, 2018

**Abstract:** Kinematic fields arise due to a uniform movement (constant velocity) of a permanent magnet or an electric charge. Previous experimental and theoretical results for the classical approximation demonstrate that kinematic fields do not propagate in a wave-like manner, but move like a rigid body synchronously with their source. In this paper a further analysis of kinematic fields, taking into account special relativity theory is presented. Despite the appearance of a new feature, the previous conclusions are upheld for the relativistic case. A complete mathematical study irrefutably proves the non-wave nature of the field movement along with its carrier.

**Keywords:** Moving Permanent Magnet, Moving Charge, Relative Motion, Faraday's Law, Ampere-Maxwell Law, Lorentz Force and Biot-Savart Force, Special Relativity, Wave Equation

## 1. Introduction

It is well known that the field of a point charge, moving in the laboratory at a speed $v$, is not Coulomb-like for a stationary observer (see, for example, [1] chapter 8, entitled "Relativity and electricity"). This moving electric field (termed kinematic field) is weakened in front and behind the charge (i.e. in the direction of motion) and is stronger at right angles to the direction of motion. As a consequence of the charge motion a kinematically induced magnetic field is created. Experimental proof was presented firstly by Rowland [2] and later more precisely corroborated in series of careful experiments by Eihenwald [3]. This electro-kinematic phenomenon, termed the Rowland-Eihenwald (RE) effect, has a symmetrical counterpart. The magneto-kinematic effect – motion of a magnetic field with a moving permanent magnet, similarly creates a kinematically induced electric field. This was first demonstrated for rotary magnetic field motion by Zajev and Dokuchajev [4] (the ZD-effect). This has been confirmed more recently [5, 6] and also established in the case of the rectilinear motion of a permanent magnet [7].

The present paper deals with kinematic fields and with kinematically induced fields. They arise from electrostatic field and magneto-static field when their sources (charged body, permanent magnet) are moving in laboratory at speed $v < c$. Previous work for the non-relativistic case demonstrated that kinematic fields do not satisfy the wave equation [8, 9]. As a consequence therefore the uniform motion in space of a charge (or permanent magnet) does not result in propagation of radiation. Kinematic fields transfer energy in space "… *not like a ripple on the river, but like the water itself in the stream*" [9]. Kinematic fields are therefore quite different in nature and behaviour to electromagnetic (EM) waves.

It is well known that Maxwell's equations are entirely compatible with special relativity theory (SRT) [10, pp. 553-560; 11, pp. 461-469]. For a thorough examination of kinematic phenomena the role of the gamma-factor term, $\gamma$, in Maxwell's equations should be considered. In the following sections an appropriate quantitative analysis for kinematic fields is presented with due regard to SRT, pertaining specifically to the electrical and magnetic cases.

## 2. Fields of Kinematical Origin Induced by a Moving Magnetized Sphere

Consider a sphere of diameter $d$, with a uniform magnetization $\mathbf{M}$ of magnitude $M$ and parallel to the $z$ axis, in vacuum (figure 1). Let $(r, \theta, \varphi)$ be the system of spherical coordinates with polar axis directed along the vector **M** and with origin in the centre of the sphere. This is a unique case when the magnetic field in the external space has a purely dipole character with dipole moment $\mathbf{p}_m = \pi d^3\,\mathbf{M}/6$ . It may be expressed through simple elementary functions [10, p.183]. The components of the magnetic flux density (SI - units) are:

$$B_r = p\frac{cos\theta}{r^3}, B_\theta = \frac{p}{2}\frac{sin\theta}{r^3}, B_\varphi = 0,$$

where $p = \mu\mu_0 p_m/2\pi$ is a constant numeric coefficient depending on using system of units. The projections of the vector **B** on axes of Cartesian (rectangular) coordinates are:

$$B_x = 3p\frac{\sin\theta\cos\theta\cos\varphi}{r^3}, B_y = 3p\frac{\sin\theta\cos\theta\, sin\,\varphi}{r^3}, B_z = p(2 - 3sin^2\theta).$$

The length of a radius-vector to an arbitrary point $A$ is $r = \sqrt{x^2 + y^2 + z^2}$ and the usual trigonometry functions may be expressed as:

$$sin\,\theta = \frac{\sqrt{x^2 + y^2}}{r}, cos\,\theta = \frac{z}{r}, sin\,\varphi = \frac{y}{\sqrt{x^2 + y^2}}, cos\,\varphi = \frac{x}{\sqrt{x^2 + y^2}}.$$

Substitution of these expressions into Cartesian projections of the vector **B** gives the following:

$$B_x = \frac{3p}{2}\frac{xz}{r^5}, B_y = \frac{3p}{2}\frac{yz}{r^5}, B_z = \frac{p}{2}\frac{2z^2 - x^2 - y^2}{r^5}. \quad (1)$$

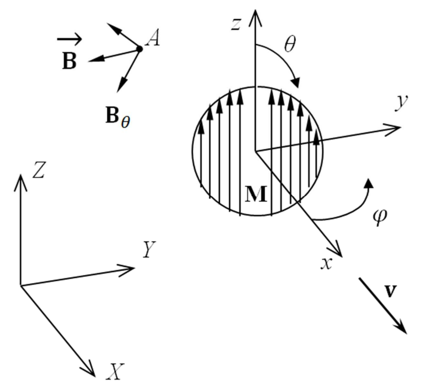


***Figure 1.*** *Magnetized sphere moves along X-axis.*

Let the frame of reference, where the magnetised sphere is at rest, has the axes $(x, y, z)$ in parallel with the axes $X, Y, Z$ of the laboratory reference frame (figure 1). Assuming the sphere to be moving relative to the laboratory reference frame along the $X$ axis at a constant velocity $\mathrm{v} = (v, 0, 0)$. Unlike the case considered in [12], a motion directed transversally relative to the magnetization **M** is seen here. The magnetised sphere provides a magnetic field with space-variable flux density **B**. The Lorentz transform [1, p. 237] gives the following connections between the values in both systems:

$$x = \gamma(X - vt), y = Y, z = Z\,;\ B_X = B_x\,, B_Y = \gamma B_y\,, B_Z = \gamma B_z\,. \quad (2)$$

Here the relativistic factor $\gamma = (1 - v^2/c^2)^{\,1/2}$, where $c$ is speed of light. Substitution of expressions (2) into equation (1) leads to components of the magnetic vector **B** in any point $(X, Y, Z)$ of the laboratory system corresponding to the point $(x, y, z)$ of the moving one:

$$B_X = \frac{3p\gamma}{2}\frac{(X - vt)Z}{R^5}, B_Y = \frac{3p\gamma}{2}\frac{YZ}{R^5}, B_Z = \frac{p\gamma}{2}\frac{2Z^2 - \gamma^2(X - vt)^2 - Y^2}{R^5}, \quad (3)$$

where $R = \sqrt{\gamma^2(X - vt)^2 + Y^2 + Z^2}$ . The corresponding electric field $\mathrm{E} = \mathrm{B} \times \mathrm{v}$ of kinematical origin has the following components:

$$E_X = 0, E_Y = vB_Z, E_Z = -vB_Y. \quad (4)$$

Observe that constant factor $p/2$ common for all components may be omitted in the following calculations.

The partial derivatives of the magnetic components (3) in the laboratory system are:

$$\frac{\partial B_X}{\partial X} = 3\gamma Z\frac{Y^2 + Z^2 - 4\gamma^2(X - vt)^2}{R^7}, \frac{\partial B_Y}{\partial Y} = 3\gamma Z\frac{\gamma^2(X - vt)^2 + Z^2 - 4Y^2}{R^7}, \frac{\partial B_Z}{\partial Z} = 3\gamma Z\frac{3\gamma^2(X - vt)^2 + 3Y^2 - 2Z^2}{R^7}\,; \quad (5)$$

$$\frac{\partial B_X}{\partial Y} = -15\gamma\frac{(X - vt)YZ}{R^7}, \frac{\partial B_X}{\partial Z} = 3\gamma(X - vt)\frac{\gamma^2(X - vt)^2 + Y^2 - 4Z^2}{R^7};$$

$$\frac{\partial B_Y}{\partial X} = -15\gamma^3\frac{(X - vt)YZ}{R^7}, \frac{\partial B_Y}{\partial Z} = 3\gamma Y\frac{\gamma^2(X - vt)^2 + Y^2 - 4Z^2}{R^7}; \quad (6)$$

$$\frac{\partial B_Z}{\partial X} = 3\gamma^3(X - vt)\frac{\gamma^2(X - vt)^2 + Y^2 - 4Z^2}{R^7}, \frac{\partial B_Z}{\partial Y} = 3\gamma Y\frac{\gamma^2(X - vt)^2 + Y^2 - 4Z^2}{R^7};$$

$$\frac{\partial B_X}{\partial t} = 3\gamma vZ\frac{4\gamma^2(X - vt)^2 - Y^2 - Z^2}{R^7}, \frac{\partial B_Y}{\partial t} = 15\gamma^3 v\frac{(X - vt)YZ}{R^7}, \frac{\partial B_Z}{\partial t} = 3\gamma^3 v(X - vt)\frac{4Z^2 - \gamma^2(X - vt)^2 - Y^2}{R^7}. \quad (7)$$

From the formulae (5, 6) we have

$$div\,\mathbf{B} = \frac{\partial B_X}{\partial X} + \frac{\partial B_Y}{\partial Y} + \frac{\partial B_Z}{\partial Z} = 0;$$

$$curl\,\mathbf{B} = \left(\frac{\partial B_Z}{\partial Y} - \frac{\partial B_Y}{\partial Z}, \frac{\partial B_X}{\partial Z} - \frac{\partial B_Z}{\partial X}, \frac{\partial B_Y}{\partial X} - \frac{\partial B_X}{\partial Y}\right) = \left(0, 3\gamma(X - vt)\frac{\gamma^2(X-vt)^2 + Y^2 - 4\,Z^2}{R^7}(1-\gamma^2), 15\gamma\frac{(X-vt)YZ}{R^7}\,(1-\gamma^2)\right). \quad (8)$$

The partial derivatives of the induced electric components (4) are:

$$\frac{\partial E_X}{\partial X} = \frac{\partial E_X}{\partial Y} = \frac{\partial E_X}{\partial Z} \equiv 0, \frac{\partial E_Y}{\partial Y} = v\frac{\partial B_Z}{\partial Y}, \frac{\partial E_Z}{\partial Z} = -v\frac{\partial B_Y}{\partial Z}\ ;$$

$$\frac{\partial E_Y}{\partial X} = v\frac{\partial B_Z}{\partial X}, \frac{\partial E_Y}{\partial Z} = v\frac{\partial B_Z}{\partial Z}, \frac{\partial E_Z}{\partial X} = -v\frac{\partial B_Y}{\partial X}, \frac{\partial E_Z}{\partial Y} = -v\frac{\partial B_Y}{\partial Y}; \quad (9)$$

$$\frac{\partial E_X}{\partial t} = 0, \frac{\partial E_Y}{\partial t} = v\frac{\partial B_Z}{\partial t}, \frac{\partial E_Z}{\partial t} = -v\frac{\partial B_Y}{\partial t}. \quad (10)$$

From formulae (6, 9) we have:

$$div\,\mathbf{E} = \frac{\partial E_X}{\partial X} + \frac{\partial E_Y}{\partial Y} + \frac{\partial E_Z}{\partial Z} = 0 + v\frac{\partial B_Z}{\partial Y} - v\frac{\partial B_Y}{\partial Z} = 0 + \frac{3\gamma vY}{R^7}[\gamma^2(X-vt)^2 + Y^2 - 4\,Z^2 - \gamma^2(X-vt)^2 - Y^2 + 4\,Z^2] = 0;$$

$$curl\,\mathbf{E} = \left(\frac{\partial E_Z}{\partial Y} - \frac{\partial E_Y}{\partial Z}, -\frac{\partial E_Z}{\partial X}, \frac{\partial E_Y}{\partial X}\right) = -v\left(\frac{\partial B_Y}{\partial Y} + \frac{\partial B_Z}{\partial Z}, -\frac{\partial B_Y}{\partial X}, -\frac{\partial B_Z}{\partial X}\right)$$
$$= -v\left(3\gamma Z\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7}, 15\gamma^3\frac{(X-vt)YZ}{R^7}, -3\gamma^3(X-vt)\frac{(X-vt)^2 + Y^2 - 4Z^2}{R^7}\right).$$

From comparison with the formulae (7) we find that $curl\,\mathbf{E} = -\,\partial\mathbf{B}/\,\partial t$ in accordance with Faraday's law.

In order to compare $curl\,\mathbf{B}$ with $\partial\mathbf{E}/\,\partial t$, it is necessary first to recalculate the expression $(\gamma^2 - 1)$:

$$\gamma^2 - 1 = \frac{1}{1-v^2/c^2} - 1 = \frac{1-1+v^2/c^2}{1-v^2/c^2} = \frac{v^2/c^2}{1-v^2/c^2} = \gamma^2\frac{v^2}{c^2}. \quad (11)$$

Taking into account the equation (11) we have from (8):

$$(curl\ \mathrm{B})_Y = -3\gamma(X-vt)\frac{\gamma^2(X-vt)^2 + Y^2 - 4\,Z^2}{R^7}\gamma^2\frac{v^2}{c^2}$$
$$= \frac{v}{c^2}\frac{\partial B_Z}{\partial t} = \frac{1}{c^2}\frac{\partial E_Y}{\partial t};$$

$$(curl\ \mathrm{B})_Z = -15\gamma\frac{(X-vt)YZ}{R^7}\gamma^2\frac{v^2}{c^2} = -\frac{v}{c^2}\frac{\partial B_Y}{\partial t} = \frac{1}{c^2}\frac{\partial E_Z}{\partial t};$$

For $(curl\ \mathrm{B})_X = \partial E_X/\partial t\ = 0$, we have the vector form $curl\,\mathbf{B} = \frac{1}{c^2}\partial\mathbf{E}/\,\partial t$ (Ampere-Maxwell law [13, p.101]).

Let us calculate now all second partial derivatives, viewing from the laboratory, in order to verify compatibility with wave equation. The second partial derivatives of the magnetic $X$-component are:

$$\frac{\partial^2 B_X}{\partial X^2} = \frac{\partial}{\partial X}\left(\frac{\partial B_X}{\partial X}\right) = 15\gamma^3(X-vt)Z\frac{4\gamma^2(X-vt)^2 - 3Y^2 - 3\,Z^2}{R^9}\ ;$$

$$\frac{\partial^2 B_X}{\partial Y^2} = \frac{\partial}{\partial Y}\left(\frac{\partial B_X}{\partial Y}\right) = 15\gamma(X-vt)Z\frac{6Y^2 - \gamma^2(X-vt)^2 - Z^2}{R^9}\ ;$$

$$\frac{\partial^2 B_X}{\partial Z^2} = \frac{\partial}{\partial Z}\left(\frac{\partial B_X}{\partial Z}\right) = 15\gamma(X-vt)Z\frac{4Z^2 - 3\gamma^2(X-vt)^2 - 3Y^2}{R^9}.$$

Application of the Laplacian operator to the magnetic component yields the following expression:

$$\nabla^2 B_X = \frac{\partial^2 B_X}{\partial X^2} + \frac{\partial^2 B_X}{\partial Y^2} + \frac{\partial^2 B_X}{\partial Z^2} = 15(\gamma^2 - 1)\gamma(X-vt)Z\frac{4\gamma^2(X-vt)^2 - 3Y^2 - 3\,Z^2}{R^9}. \quad (12)$$

In a similar manner of doing we have

$$\frac{\partial^2 B_Y}{\partial X^2} = 15\gamma^3 YZ\frac{6\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^9}\ ; \frac{\partial^2 B_Y}{\partial Y^2} = 15\gamma YZ\frac{4Y^2 - 3\gamma^2(X-vt)^2 - 3Z^2}{R^9};$$

$$\frac{\partial^2 B_Y}{\partial Z^2} = 15\gamma YZ\frac{4Z^2 - 3\gamma^2(X-vt)^2 - 3Y^2}{R^9}\ ;$$

$$\nabla^2 B_Y = \frac{\partial^2 B_Y}{\partial X^2} + \frac{\partial^2 B_Y}{\partial Y^2} + \frac{\partial^2 B_Y}{\partial Z^2} = 15(\gamma^2 - 1)\gamma YZ\frac{6\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^9}. \quad (13)$$

$$\frac{\partial^2 B_Z}{\partial X^2} = 3\gamma^3 \frac{Y^4 - 4\gamma^4 (X-vt)^4 - 4Z^4 - 3\gamma^2 (X-vt)^2 Y^2 + 27\gamma^2 (X-vt)^2 Z^2 - 3Y^2 Z^2}{R^9};$$

$$\frac{\partial^2 B_Z}{\partial Y^2} = 3\gamma \frac{\gamma^4 (X-vt)^4 - 4Y^4 - 4Z^4 - 3\gamma^2 (X-vt)^2 Y^2 - 3\gamma^2 (X-vt)^2 Z^2 + 27 Y^2 Z^2}{R^9};$$

$$\frac{\partial^2 B_Z}{\partial Z^2} = 3\gamma \frac{3\gamma^4 (X-vt)^4 + 3Y^4 + 8Z^4 + 6\gamma^2 (X-vt)^2 Y^2 - 24\gamma^2 (X-vt)^2 Z^2 - 24 Y^2 Z^2}{R^9};$$

$$\nabla^2 B_Z = \frac{\partial^2 B_Z}{\partial X^2} + \frac{\partial^2 B_Z}{\partial Y^2} + \frac{\partial^2 B_Z}{\partial Z^2} = 3(\gamma^2 - 1)\gamma \frac{Y^4 - 4\gamma^4 (X-vt)^4 - 3\gamma^2 (X-vt)^2 Y^2 + 27\gamma^2 (X-vt)^2 Z^2 - 3Y^2 Z^2 - 4Z^4}{R^9}. \quad (14)$$

From (7) the second partial derivatives of the magnetic vector over time are:

$$\frac{\partial^2 B_X}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial B_X}{\partial t}\right) = 15\gamma^3 v^2 (X-vt) Z \frac{4\gamma^2 (X-vt)^2 - 3Y^2 - 3Z^2}{R^9}; \quad (15)$$

$$\frac{\partial^2 B_Y}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial B_Y}{\partial t}\right) = 15\gamma^3 v^2 Y Z \frac{6\gamma^2 (X-vt)^2 - Y^2 - Z^2}{R^9}; \quad (16)$$

$$\frac{\partial^2 B_Z}{\partial t^2} = 3\gamma^3 v^2 \frac{Y^4 - 4\gamma^4 (X-vt)^4 - 3\gamma^2 (X-vt)^2 Y^2 + 27\gamma^2 (X-vt)^2 Z^2 - 3Y^2 Z^2 - 4Z^4}{R^9}. \quad (17)$$

From comparison (12 – 14) with (15 – 17) correspondently, and taking into account (11) we get the wave equation for magnetic vector **B** of a moving magnetized sphere:

$$\nabla^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = \mathbf{0}. \quad (18)$$

The second partial derivatives of the kinematically induced electric Y-component (4) are:

$$\frac{\partial^2 E_Y}{\partial X^2} = v\frac{\partial^2 B_Z}{\partial X^2}, \frac{\partial^2 E_Y}{\partial Y^2} = v\frac{\partial^2 B_Z}{\partial Y^2}, \frac{\partial^2 E_Y}{\partial Z^2} = v\frac{\partial^2 B_Z}{\partial Z^2},$$

Application of the Laplacian operator to the electric component $E_Y$, is taken from the equation (14):

$$\nabla^2 E_Y = v\nabla^2 B_Z. \quad (19)$$

By analogy

$$\frac{\partial^2 E_Z}{\partial X^2} = -v\frac{\partial^2 B_Y}{\partial X^2}, \frac{\partial^2 E_Z}{\partial Y^2} = -v\frac{\partial^2 B_Y}{\partial Y^2} =, \frac{\partial^2 E_Z}{\partial Z^2} = -v\frac{\partial^2 B_Y}{\partial Z^2},$$

and

$$\nabla^2 E_Z = -v\nabla^2 B_Y. \quad (20)$$

The second partial derivatives over time are:

$$\frac{\partial^2 E_Y}{\partial t^2} = v\frac{\partial^2 B_Z}{\partial t^2}, \frac{\partial^2 E_Z}{\partial t^2} = -v\frac{\partial^2 B_Y}{\partial t^2}. \quad (21)$$

For $E_X \equiv 0$, we have from (19), (20) and (21):

$$\nabla^2 E_X = \frac{\partial^2 E_Y}{\partial t^2} \equiv 0;\ \nabla^2 E_Y = v\nabla^2 B_Z = \frac{v}{c^2}\frac{\partial^2 B_Z}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 E_Y}{\partial t^2};$$

$$\nabla^2 E_Z = -v\nabla^2 B_Y = \frac{-v}{c^2}\frac{\partial^2 B_Z}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 E_Y}{\partial t^2}.$$

The above equalities may be written in short vector form:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mathbf{0}. \quad (22)$$

## 3. Fields of Kinematic Origin Induced by a Moving Charged Sphere

A sphere made of a conductive material is charged with a constant surface density so that the full charge $q$ is uniformly spread over all the surface of the sphere. In this case the electrostatic field in air outside the sphere coincides with the field of a point charge $q$ focused in the centre of the sphere. A point charge $q$ being immobile in the laboratory has electrostatic field described by the Coulomb formula for electric intensity:

$$\mathbf{E} = \frac{q\boldsymbol{r}}{4\pi\varepsilon_0 r^3}.$$

At any point $A$ electric vector **E** is co-linear with the radius- vector at this point (figure 2)

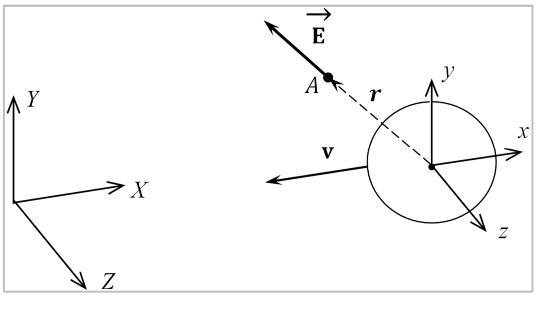


***Figure 2.*** *Charged sphere moves along X-axis.*

If the charge moves with a constant velocity v, a magnetic field appears due to the RE-effect, but the associated electrostatic field is distorted [1]. It may be described in Cartesian coordinates [1, p 242]. If the charged sphere is moving relative to the laboratory reference frame with co-ordinates $X, Y, Z$ along the $X$ axis at a constant velocity v $= (v, 0, 0)$ , there is a dependency in time between coordinates of both systems:

$$x = \gamma(X - vt), y = Y, z = Z\ .$$

So, the electric vector in the laboratory reference frame is described by the equation:

$$\mathbf{E}(X,Y,Z) = \frac{q}{4\pi\varepsilon_0}\frac{\gamma\mathbf{r}}{[\gamma^2\,(X-vt)^2 + Y^2 + Z^2]^{\frac{3}{2}}} = \frac{q}{4\pi\varepsilon_0}\frac{\gamma\mathbf{r}}{R^3}.$$

At any point in the laboratory space at a given moment $t$ in the Cartesian system with an accuracy to the constant factor $q/4\pi\varepsilon_0$, electric vector $\mathbf{E}_{X,Y,Z}$ has the components:

$$E_X = \frac{\gamma(X-vt)}{R^3}\ , E_Y = \frac{\gamma Y}{R^3}\ , E_Z = \frac{\gamma Z}{R^3}. \quad (23)$$

The corresponding magnetic field $\mathbf{B} = \frac{1}{c^2}\,(\mathbf{v}\times\mathbf{E})$ of the electro-kinematical origin (R-E effect) has the following components with accuracy to the factor $q/4\pi\varepsilon_0$ :

$$B_X = 0,\ B_Y = -\frac{v}{c^2}E_Z = -\frac{\gamma v}{c^2}\frac{Z}{R^3},\ B_Z = \frac{v}{c^2}\,E_Y = \frac{\gamma v}{c^2}\frac{Y}{R^3}. \quad (24)$$

The partial derivatives of the electric components (23) are

$$\frac{\partial E_X}{\partial X} = \gamma\frac{Y^2 + Z^2 - 2\gamma^2(X-vt)^2}{R^5}, \frac{\partial E_Y}{\partial Y} = \gamma\frac{\gamma^2(X-vt)^2 + Z^2 - 2Y^2}{R^5},$$

$$\frac{\partial E_Z}{\partial Z} = \gamma\frac{\gamma^2(X-vt)^2 + Y^2 - 2Z^2}{R^5}\ ;$$

$$\frac{\partial E_X}{\partial Y} = -3\gamma\frac{(X-vt)Y}{R^5}, \frac{\partial E_X}{\partial Z} = -3\gamma\frac{(X-vt)Z}{R^5}, \frac{\partial E_Y}{\partial X} = -3\gamma^3\frac{(X-vt)Y}{R^5}\ ;$$

$$\frac{\partial E_Y}{\partial Z} = -3\gamma\frac{YZ}{R^5}, \frac{\partial E_Z}{\partial X} = -3\gamma^3\frac{(X-vt)Z}{R^5}, \frac{\partial E_Z}{\partial Y} = -3\gamma\frac{YZ}{R^5}\ ;$$

$$\frac{\partial E_X}{\partial t} = v\gamma\frac{2\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^5}, \frac{\partial E_Y}{\partial t} = 3v\gamma^3\frac{(X-vt)Y}{R^5}, \frac{\partial E_Z}{\partial t} = 3v\gamma^3\frac{(X-vt)Z}{R^5}. \quad (25)$$

Using the above expressions (25) we have

$$div\,\mathbf{E} = \frac{\partial E_X}{\partial X} + \frac{\partial E_Y}{\partial Y} + \frac{\partial E_Z}{\partial Z} = 0;$$

$$curl\,\mathbf{E} = \left(\frac{\partial E_Z}{\partial Y} - \frac{\partial E_Y}{\partial Z}, \frac{\partial E_X}{\partial Z} - \frac{\partial E_Z}{\partial X}, \frac{\partial E_Y}{\partial X} - \frac{\partial E_X}{\partial Y}\right) = \left(0, 3\gamma(\gamma^2-1)\frac{(X-vt)Z}{R^5}, -3\gamma(\gamma^2-1)\frac{(X-vt)Y}{R^5}\right) = \left(0, \frac{3\gamma^3 v^2}{c^2}\frac{(X-vt)Z}{R^5}, -\frac{3\gamma^3 v^2}{c^2}\frac{(X-vt)Y}{R^5}\right) \quad (26)$$

The partial derivatives of the induced magnetic components (24) are

$$\frac{\partial B_X}{\partial X} = \frac{\partial B_X}{\partial Y} = \frac{\partial B_X}{\partial Z} = \frac{\partial B_X}{\partial t} = 0; \frac{\partial B_Y}{\partial X} = \frac{3\gamma^3 v}{c^2}\frac{(X-vt)Z}{R^5},$$

$$\frac{\partial B_Y}{\partial Y} = \frac{3\gamma v}{c^2}\frac{YZ}{R^5}, \frac{\partial B_Y}{\partial Z} = \frac{\gamma v}{c^2}\frac{2Z^2 - \gamma^2(X-vt)^2 - Y^2}{R^5};$$

$$\frac{\partial B_Z}{\partial X} = -\frac{3\gamma^3 v}{c^2}\frac{(X-vt)Y}{R^5}, \frac{\partial B_Z}{\partial Y} = \frac{\gamma v}{c^2}\frac{\gamma^2(X-vt)^2 + Z^2 - 2Y^2}{R^5}, \frac{\partial B_Z}{\partial Z} = -\frac{3\gamma v}{c^2}\frac{YZ}{R^5};$$

$$\frac{\partial B_Y}{\partial t} = -\frac{3\gamma^3 v^2}{c^2}\frac{(X-vt)Z}{R^5}, \frac{\partial B_Z}{\partial t} = \frac{3\gamma^3 v^2}{c^2}\frac{(X-vt)Y}{R^5}. \quad (27)$$

So we are able to calculate:

$$div\,\mathbf{B} = 0 + \frac{3\gamma v}{c^2}\frac{YZ}{R^5} - \frac{3\gamma v}{c^2}\frac{YZ}{R^5} = 0$$

and

$$curl\,\mathbf{B} = \left(\frac{\gamma v}{c^2}\frac{2\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^5}, \frac{3\gamma^3 v}{c^2}\frac{(X-vt)Y}{r^5}, \frac{3\gamma^3 v}{c^2}\frac{(X-vt)Z}{r^5}\right). \quad (28)$$

The comparison values of the respective components (25) with (28) and (26) with (27) yields the following vector equations:

$$curl\,\mathbf{B} = \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial \mathrm{t}}, curl\,\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}.$$

In this case both Ampere-Maxwell law and Faraday's law are in result.

Let us calculate the second derivatives and check the induced electric vector $\mathbf{E}_{X,Y,Z}$ to meet wave equation.

$$\frac{\partial^2 E_X}{\partial X^2} = 3\gamma^3(X-vt)\frac{2\gamma^2(X-vt)^2 - 3Y^2 - 3\,Z^2}{R^7}, \frac{\partial^2 E_X}{\partial Y^2} = -3\gamma(X-vt)\frac{\gamma^2(X-vt)^2 - 4Y^2 +\, Z^2}{R^7},$$

$$\frac{\partial^2 E_X}{\partial Z^2} = -3\gamma(X-vt)\frac{\gamma^2(X-vt)^2 + Y^2 - 4\,Z^2}{R^7}, \frac{\partial^2 E_X}{\partial t^2} = 3\gamma^3 v^2(X-vt)\frac{2\gamma^2(X-vt)^2 - 3Y^2 - 3Z^2}{R^7}.$$

Applying the Laplacian operator to the component $E_X$ and taking into account the equation (11) we have:

$$\nabla^2 E_X = 3\gamma^3\frac{v^2}{c^2}(X-vt)\frac{2\gamma^2(X-vt)^2 - 3Y^2 - 3\,Z^2}{R^7} = \frac{1}{c^2}\frac{\partial^2 E_X}{\partial t^2}.$$

The second derivatives of the component $E_Y$ are:

$$\frac{\partial^2 E_Y}{\partial X^2} = 3\gamma^3 Y\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7}, \frac{\partial^2 E_Y}{\partial Y^2} = -3\gamma Y\frac{3\gamma^2(X-vt)^2 - 2Y^2 + 3Z^2}{R^7},$$

$$\frac{\partial^2 E_Y}{\partial Z^2} = -3\gamma Y\frac{\gamma^2(X-vt)^2 + Y^2 - 4\,Z^2}{R^7}, \frac{\partial^2 E_Y}{\partial t^2} = 3\gamma^3 v^2 Y\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7}$$

and applying the Laplacian operator to the component $E_Y$ gives:

$$\nabla^2 E_Y = 3\gamma^3\frac{v^2}{c^2}Y\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7} = \frac{1}{c^2}\frac{\partial^2 E_Y}{\partial t^2}.$$

The second derivatives of the component $E_Z$ are:

$$\frac{\partial^2 E_Z}{\partial X^2} = 3\gamma^3 Z\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7}, \frac{\partial^2 E_Z}{\partial Y^2} = -3\gamma Z\frac{\gamma^2(X-vt)^2 - 4Y^2 + Z^2}{R^7},$$

$$\frac{\partial^2 E_Z}{\partial Z^2} = -3\gamma Z\frac{3\gamma^2(X-vt)^2 + 3Y^2 - 2Z^2}{R^7}, \frac{\partial^2 E_Z}{\partial t^2} = 3\gamma^3 v^2 Z\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7},$$

where from

$$\nabla^2 E_Z = 3\gamma^3\frac{v^2}{c^2}Z\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7} = \frac{1}{c^2}\frac{\partial^2 E_Z}{\partial t^2}.$$

All these by the component equalities provide the short vector form:

$$\boldsymbol{\nabla}^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \mathbf{0}. \qquad (29)$$

Starting from a magnetic field $\mathbf{B} = \frac{1}{c^2}(\mathbf{v}\times\mathbf{E})$ of an electro-kinematical origin (24) we have the second derivative of the second component

$$\frac{\partial^2 B_Y}{\partial X^2} = -\frac{v}{c^2}\frac{\partial^2 E_z}{\partial X^2}, \frac{\partial^2 B_Y}{\partial Y^2} = -\frac{v}{c^2}\frac{\partial^2 E_z}{\partial Y^2}, \frac{\partial^2 B_Y}{\partial Z^2} = -\frac{v}{c^2}\frac{\partial^2 E_z}{\partial Z^2},$$

$$\frac{\partial^2 B_Y}{\partial t^2} = -\frac{v}{c^2}\frac{\partial^2 E_z}{\partial t^2} = -3\gamma^3\frac{v^3}{c^2}Z\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7},$$

and the corresponding Laplacian operator is:

$$\nabla^2 B_Y = -\frac{v}{c^2}\nabla^2 E_Z = -\frac{3\gamma^3}{c^2}\frac{v^3}{c^2}Z\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7} = -\frac{1}{c^2}\frac{v}{c^2}\frac{\partial^2 E_z}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 B_Y}{\partial t^2}.$$

By analogy:

$$\frac{\partial^2 B_Z}{\partial X^2} = \frac{v}{c^2}\frac{\partial^2 E_Y}{\partial X^2}, \frac{\partial^2 B_Z}{\partial Y^2} = \frac{v}{c^2}\frac{\partial^2 E_Y}{\partial Y^2}, \frac{\partial^2 B_Z}{\partial Z^2} = \frac{v}{c^2}\frac{\partial^2 E_Y}{\partial Z^2}$$

$$\frac{\partial^2 B_Z}{\partial t^2} = \frac{v}{c^2}\frac{\partial^2 E_Y}{\partial t^2} = 3\gamma^3\frac{v^3}{c^2}Y\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7},$$

and the Laplacian operator is:

$$\nabla^2 B_Z = \frac{v}{c^2}\nabla^2 E_Y = \frac{3\gamma^3}{c^2}\frac{v^3}{c^2}Y\frac{4\gamma^2(X-vt)^2 - Y^2 - Z^2}{R^7} = \frac{1}{c^2}\frac{v}{c^2}\frac{\partial^2 E_Y}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 B_Y}{\partial t^2}$$

As soon as the first component of induced magnetic field is nought, the two above equalities bring to the vector one:

$$\boldsymbol{\nabla}^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} = \mathbf{0}. \qquad (30)$$

# 4. Conclusions

From the above analysis, the following conclusions may be drawn. In the classical case for kinematic fields (resulting from uniform motion of charge or permanent magnet with velocity $v < c$) the wave equation is *not* a valid description of the field motion. If this movement were of wave-like nature, it would satisfy the vector equation:

$$\nabla^2\mathbf{F} - \frac{1}{v^2}\frac{\partial^2\mathbf{F}}{\partial t^2} = \mathbf{0}\,, \tag{31}$$

but as it has been shown above, this is not the case, therefore kinematic field motion *cannot* be described using a wave equation of type (31) and specifically (18), (22), (29), (30). In reality, a kinematically induced field F moves in the laboratory together and synchronously with its carrier at finite speed $v < c$.

In the relativistic case, the kinematic field formally satisfies the wave equation with speed of light as parameter, but the wave equations (18), (22), (29), (30) do not describe the motion of the kinematic fields either because they are moving with a finite velocity $v \neq c$. The previous conclusion that there exist two fundamentally different types of induction, dynamic and kinematic, is further established (see cited pioneering works, where the kinematic aspect of electromagnetism was introduced into scientific usage). Dynamic induction is governed by the wave equation, whereas kinematic induction not at all. The main implication of the above analysis is that relativistic correction does not play a key role in the character of moving electric and magnetic fields provided by the motion of their sources. If a source of a static field moves, the transfer of energy in space is not wave-like. In the case of accelerated motion, the both kinematic and dynamic fields are present, but the fundamental physical difference between them persists.

## Acknowledgements

The authors sincerely thank Mr D. McIntosh for valuable help giving during the preparation and checking of this paper.

## References

[1] A. P. French, S*pecial Relativity.* Thomas Nelson & Sons, Great Britain (1981).

[2] Rowland H., "On the Magnetic Effect of Electric Convection", *American Journal of Science*, Vol. XV, No. 3, 30-33, (1878).

[3] Eihenwald A. A., "*Izbrannye raboty – Selected works*," Physico-matematicheskaja literatura, Moscow, (in Russian), (1956)

[4] Zajev N. E. and Dokuchajev V. I., *Electrotechnika (Electrical Engineering, in Russian)* 11, 64 (1964).

[5] Leus V. A. and Zatolokin V. N. "The magneto-kinematical effect". *IJEEE* 43 (4), 245 (2006).

[6] Leus V. and Taylor S., "On the motion of the field of a permanent magnet", *Eur. J. Phys.,* Vol. 32, No 5, 1179-1192, (2011).

[7] Taylor, S., and Leus, V. A. "The magneto-kinematic effect for the case of rectilinear motion". *Eur. J. Phys.,* Vol. 33, No 4, 837-852, (2012).

[8] Leus V. and Taylor S.., "Experimental Evidence for the Magneto-kinematic Effect", PIERS Proceedings, (Moscow), pp.1040-1048 (August 2012).

[9] Leus, V. A. "The magneto-kinematical and electro-kinematical fields". *Progress In Electromagnetics Research M,* Vol. 32, 27-41, (2013)

[10] Jackson J. D., "*Classical Electrodynamics"* (3th Ed), John Wiley & Sons, New York, (1999).

[11] Grant I. S. and Phillips W. R., "*Electromagnetism"* 2nd edn, John Wiley & Sons, Chichester, (1998).

[12] Leus V. A. "Triplet paradox in special relativity and discrepancy with electromagnetism", *American Journal of Modern Physics*, Vol. 4, № 2-1, (2015).

[13] Daniel Fleisch, *"A student's Guide to Maxwell's Equations",* Cambridge University Press, (2008).