



The Geometric Thinking Levels of Pre-service Teachers in Ghana

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Abstract: Teachers' geometrical competencies are very critical to the effective teaching of the subject. This study focused on the van Hiele Levels of geometric thinking reached by Ghanaian pre-service teachers before leaving for their Student Internship Programme (Teaching Practice) at the basic schools. In all, 300 second year pre-service teachers from 4 Colleges of Education were involved in this study. These pre-service teachers were given the van Hiele Geometry Test adapted from the 'Cognitive Development and Achievement in Secondary School Geometry Test' items during their second year, first semester. The results showed that 16.33% of pre-service teachers attained van Hiele Level 0 (i.e. the Pre-recognition Level or Level for those who have not yet attained any van Hiele Level), 27% of pre-service teachers attained Level 1, 32% attained Level 2 while 17.67% of pre-service teachers attained Level 3. However, only 6% and 1% of Pre-service Teachers attained Levels 4 and 5 respectively. These results show that majority (75.33%) of pre-service teachers' van Hiele Levels are lower than that expected of their future Junior High School 3 learners. This suggests that most of the pre-service teachers' geometry knowledge is not sufficient to teach at basic schools.

Keywords: Van Hiele Levels, Geometric Thinking, College of Education Geometry, Pre-service Teachers, Ghana

1. Introduction

In Ghana, the key institutions that train pre-service teachers for basic schools are the Colleges of Education. Pre-service Teachers are students being trained to become first time professional teachers. The subject of teacher knowledge has been a key issue in mathematics education over the years. Several researchers have indicated that teachers at all levels need experiences studying geometry in order to attain the content knowledge necessary to be effective instructors [12] [18] [19]. Apart from the field of Mathematics, geometry is important in other curriculum areas such as Science, Geography, Art, Design and Technology [32]. It is therefore not surprising that one of the aims of teaching Mathematics in Ghana is to develop an understanding of geometric concepts and relationships [22].

The College of Education geometry in Ghana is in two different aspects; content aspects, which is studied in first year and methodology aspects, studied in second year. Geometry in the first year content course covers areas such as

lines and angles, polygons, congruent and similar triangles, geometrical constructions including loci, circle theorems, two and three-dimensional shapes, movement geometry and co-ordinate geometry. In the method course, geometry covers areas such as developing ideas about shape and space, teaching measurements, teaching geometrical constructions and teaching rigid motion [22]. Geometry has a separate subject status and forms a considerable amount of the content of College of Education mathematics curriculum in Ghana. It is a branch of mathematics that deals with the study of shape and space. Without spatial ability, students cannot fully appreciate the natural world [32].

The study of geometry contributes to helping students develop the skills of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof. Geometric representations can also be used to help students make sense of other areas of Mathematics: fractions and multiplication in arithmetic, the relationships between the graphs of functions (of both two and three variables), and graphical representations of data in statistics [17]. Thus, it is imperative

for pre-service teachers to attain a sufficient geometric thinking level so that their subject matter knowledge in other areas of Mathematics is enhanced for positive impact on their future basic school learners.

However, a lot of concerns have been raised about the levels of students' geometric thinking in Ghanaian schools, especially at the basic school level [3] [4] [6]. At the College of Education level, the Chief Examiner's annual reports for End-of-Second Semester Mathematics Examination in geometry, in the years 2011 and 2012 revealed that the pre-service teachers' presentations of solutions to most of the 2 and 3-dimensional geometrical problems were poor and majority of them had problems solving questions involving the concepts of exterior and interior angles of polygons and their properties, among other concepts [23] [24]. In 2013 and 2014, the examiner's report once again revealed candidates' lack of adequate knowledge in geometry and application of geometric concepts [25] [26]. The Conference Board of the Mathematical Sciences (CBMS) notes that learning of geometry is usually confronted by conceptual difficulties [8]. Teaching and learning of geometry still remain as one of the most disappointing experiences in many schools across nations [17].

Throughout the history of modern educational system, there have always been students who have had difficulties and thus, fallen behind others in the field of Mathematics especially in geometry. This has encouraged teachers to experiment with new methods of teaching in an attempt to understand and correct this imbalance. A range of models to describe learners' spatial sense and thinking have also been proposed and researched and these include Piaget and Inhelder's Topological Primacy Thesis [20] van Hiele's Levels of Geometric Thinking [28] and Cognitive Science Model [11]. However, the theoretical framework on geometrical thinking proposed by the van Hieles tended to have attracted more attention than many others in terms of giving an accurate description of students' geometric thinking and also impacting on geometry classroom instructional practices. Although the van Hiele theory was primarily aimed at improving teachers' as well as learners' understanding of geometrical concepts, it also appealed as an ideal model for use as a theoretical framework as well as a frame of reference to link geometry to educational principles [10].

[29] argues that the quality of instruction has one of the greatest influences on the students' acquisition of geometry knowledge in mathematics classes and that the students' progress from one level to the next in geometry depends on the quality of instruction more than other factors, such as biological maturation or students' age. Moreover, there are many other factors, such as knowledge of teachers, gender, task difficulty, environment, curriculum etc. appearing to play vital roles on student achievement and motivation in the mathematics classroom [13] [12] [21]. However, in view of the fact that students spend most of their time in the schools it is logical to say that the teacher is one of the most important factors in student learning and thus, teacher's

mathematical and pedagogical content knowledge play vital roles in impacting positively on students motivation and mathematics learning. Furthermore, according to Stipek cited in [12] teachers' content knowledge plays prominent roles in students' performance, and the pre- and in-service school teachers' inadequate geometry knowledge might be another important factor behind students' poor performance in geometry. This statement is consistent with the argument made by [18] and [19] who stated that content knowledge in geometry among pre-service teachers is not sufficient.

Studies have shown that learners who have not attained a van Hiele Level 3 before taking a secondary school (Senior High School) geometry course have a low chance of success [19] [27]. Therefore, attainment of Level 3 upon completion of elementary and middle school (i.e. Junior High School) is desirable [27] [9]. Several researchers have also affirmed the validity of the existence of the first four van Hiele Levels in high school geometry courses [9] [19] [27]. Therefore, it is expected that pre-service teachers attain at least, van Hiele Level 4 of geometric thinking prior to the completion of their Senior High School programme as well as their first year geometry course in College of Education. It is also logical to assume that for basic school learners to attain these respective levels of geometric thinking, their prospective teachers need to have attained a level of geometric thinking at or above these levels in order to assist them by providing appropriate scaffolding and learning experiences. These arguments might be clearly explained by finding the van Hiele Levels of pre-service teachers in geometry since the van Hiele theory has been a facilitator for much of the renewed interest in geometry.

1.1. Theoretical Framework

In the field of geometry, the best and most well-defined theory for students' levels of thinking is based on the van Hiele theory [1] [31] [32]. The theory emerged from the separate doctoral works of a husband-and-wife team of Dutch Mathematics educators, Dina van Hiele-Geldof and Pierre van Hiele, which were completed simultaneously at the University of Utrecht, Netherlands in 1957 [27]. The couple did research in the late 1950s on thought and concept development in geometry among school children. Since Dina died shortly after finishing her dissertation, it was her husband, Pierre who clarified, amended, and advanced the theory. The theory enables insight into why many students encounter difficulties in their geometry courses, particularly with formal proofs. The van Hiele theory comprises three main aspects, namely: Levels of geometric thinking, properties of the Levels and phases of learning which offers a model of teaching that teachers could apply in order to promote their learners' levels of understanding in geometry [9] [15] [29].

The van Hiele theory originally consists of five sequential and hierarchical discrete Levels of geometric thought namely: *Recognition*, *Analysis*, *Order (Informal Deduction)*, *Deduction*, and *Rigor* [27]. There are two different numbering schemes that are commonly used to describe the

van Hiele Levels: Level 0 through to 4, and Level 1 through to 5. Originally the van Hieles numbering scheme used Level 0 through to 4, however, Americans [16] [27] and van Hiele's [29] [30] more recent writings make use of the Level 1 through to 5 numbering scheme instead. This according to [16] allows for a sixth Level, Pre-recognition Level (i.e. Level for learners who have not yet achieved even the basic Level 1) to be called Level 0. This study used the Level 1 to 5 numbering scheme to allow utilization of Level 0.

The van Hiele Levels can be described as follows:

Level 1: Recognition (or visual level)

At this Level learners use visual perception and nonverbal thinking. They recognize figures by appearance alone "and compare the figures with their prototypes or everyday things ("it looks like a door"), categorize them ("it is / it is not a..."). They use simple language [31]". Learners at this Level do not identify the properties of geometric figures [30].

Level 2: Analysis (or descriptive level)

At this Level, "figures are the bearers of their properties. A figure is no longer judged because it looks like one but rather because it has certain properties [30]". Learners start analyzing and naming properties of geometric figures but they do not understand the interrelationship between different types of figures, and they also cannot fully understand or appreciate the uses of definitions at this level [16].

Level 3: Order (or informal deduction level)

Learners at this Level are able to see the interrelationship between different types of figures. They can create meaningful definitions and give informal arguments to justify their reasoning at this Level. Logical implications and class inclusions, such as squares being a type of rectangle, are understood [13] [16].

Level 4: Deduction

At this Level learners can give deductive geometric proofs. They understand the role of definitions, theorems, axioms and proofs. Learners at this Level can supply reasons for statements in formal proofs [13] [31].

Level 5: Rigor

Learners at this Level "understand the formal aspects of deduction, such as establishing and comparing mathematical systems [16]". Here, learners learn that geometry needs to be understood in the abstract; see the "construction" of geometric systems. Learners at this level should understand that other geometries exist and that what is important is the structure of axioms, postulates, and theorems [9].

1.2. Problem Statement

A teacher is viewed as someone that should possess specific and adequate content knowledge. [5] indicated that in various countries the need to improve the experience of classroom mathematical learning through the development of teachers' knowledge of mathematics and knowledge of pedagogy is still relevant. This is because the level of understanding that learners achieve for any concept is limited by the level of understanding of their teacher [2] [13]. [21] work focused on elements of teacher knowledge (content and pedagogical content knowledge) with teacher knowledge

being one of the factors that influence teacher behaviour. This has led to a growing recognition of the need for more research studies on teacher knowledge. However, the majority of the prior studies have focused on number concepts [5] [7] [14] and studies regarding geometry are limited.

The van Hiele theory has been applied to many curricula to improve geometry classroom instruction in many developed nations but in Ghana, the literature appears to suggest that there has been little investigation involving the van Hiele theory. Thus, very little studies have applied the van Hiele theory to determine the level of geometric conceptualization of Ghanaian pre-service teachers and also to improve geometry instruction. Meanwhile, there is evidence that many students in Ghana encounter severe difficulties with school geometry [6]. Thus, this study was designed to fill this void.

1.3. Purpose of Study and Research Question

The purpose of this study was to investigate the van Hiele Levels of geometric thinking reached by Ghanaian second year pre-service teachers before leaving for their Student Internship Programme (Teaching Practice) at the basic schools. The researchers therefore, seek to address the issue of whether pre-service teachers possess enough understanding of geometry to teach the subject well. In pursuance of this purpose, the following research question was formulated to guide the study: Which stages of van Hiele Levels of geometric thinking do Ghanaian pre-service teachers reach in their study of geometry before leaving College of Education?

2. Method

2.1. Research Design

The researchers employed mainly the survey approach using test. The survey in this study was used for descriptive purposes. The researchers aimed at getting an accurate description of the van Hiele Levels of geometric thinking reached by pre-service teachers before leaving for their Student Internship Programme (Teaching Practice) at the basic schools.

2.2. Participants and Setting

Convenience sampling was used to select four Colleges of Education in the Ashanti, Central and Greater-Accra Regions of Ghana. The researchers are of the view that these Colleges of Education were ideal for this study because the pre-service teachers in these Colleges of Education are admitted from all over the ten regions in Ghana. This has enriched the sample used for the study in terms of pre-service teachers' abilities, cultural and social backgrounds. The sample used therefore represents the characteristics of Ghanaian pre-service teachers in any part of the country who had spent at least a year studying geometry in the College. Stratified random sampling was then used to select 300 second year pre-service

teachers from these four Colleges.

2.3. Instrument

In order to address the research question in this study, the participating pre-service teachers were given van Hiele Geometry Test (VHGT) to identify their geometric thinking levels. The VHGT was taken from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project, developed by [27] which found the van Hiele theory as a good predictor of students' success in geometry courses. The test consists of 25-item multiple choice test and is organized sequentially in blocks of five Items (Part A). Items 1-5 deal with identification, naming and comparing of geometric shapes such as squares, rectangles and rhombi and measure students understanding at Level 1. Items 6-10 deal with recognizing and naming properties of geometric figures and measure students understanding at Level 2. Items 11-15 deal with logical order of the properties of figures previously identified, and the relationships between these properties, these measure students understanding at Level 3. Items 16-20 deal with questions that require students to understand the significance of deduction and the role of postulates, axioms, theorems and proof, these also measure students understanding at Level 4. While items 21-25 deal with the formal aspects of deduction and measure student understanding at Level 5 [19]. The researchers included a second part (Part B of the VHGT) consisting of 3 items where participants were expected to provide written responses. This was designed to further explore the problem-solving abilities of the pre-service teachers. These items included some commonly found in texts and examination papers set for these learners. Item 1 required the pre-service teachers to calculate a missing value in a given geometrical shape; item 2 also required the pre-service teachers to find the surface area of a geometric figure; and item 3 required the pre-service teachers to write a complete proof of a theorem in geometry giving reasons.

Administration and Grading of the VHGT

The VHGT was administered in the second year, first semester and written by all second year pre-service teachers who were participating in the study. All participants' answer sheets from VHGT were read and scored by the researchers. Scoring of the part A of the VHGT was done as indicated below;

First grading method: Each correct response to the 25-item multiple-choice test was assigned 1 point. Hence, each Pre-service Teacher's score ranges from 0–25 marks.

Second grading method: the second method of grading the VHGT was based on "3 of 5 correct" success criterion suggested by [27]. By this criterion, if a Pre-service Teacher answered correctly at least 3 out of the 5 items in any of the 5 subtests within the VHGT, the Pre-service Teacher was considered to have mastered that level. Using this grading system developed by [27], the pre-service teachers were assigned weighted sum scores in the following manner:

- (1) 1 point for meeting criterion on items 1-5 (Level-I, Recognition);

- (2) 2 points for meeting criterion on items 6-10 (Level-II, Analysis);
- (3) 4 points for meeting criterion on items 11-15 (Level-III, Ordering);
- (4) 8 points for meeting criterion on items 16-20 (Level-IV, Deduction);
- (5) 16 points for meeting criterion on items 21-25 (Level-V, Rigor).

Thus, the maximum point obtainable by any Pre-service Teacher was $1 + 2 + 4 + 8 + 16 = 31$ points. The method of calculating the weighted sum makes it possible for a person to determine upon which van Hiele Level the criterion has been met from the weighted sum alone. For example, a score of 7 indicates that the learner met the criterion at Levels I, II and III (i.e. $1 + 2 + 4 = 7$). The second grading system served the purpose of assigning the learners into various van Hiele Levels based on their responses.

According to [27], there are two different cases that can be used in assigning Levels to students namely, the Classical Case and the Modified Case. The study employed the modified case in assigning Levels to pre-service teachers. [27] posited that;

The assigning of Levels in either the classical or modified case requires that a student's responses satisfy Property 1 of the Levels, i.e., that the student at level n satisfy the criterion not only at that level but also at all preceding Levels. Thus a student who satisfies the criterion at Levels 1, 2 and 5, for instance, would have a weighted sum of $1 + 2 + 16$ or 19 points, would have no classical van Hiele Level, but would be assigned the modified van Hiele Level 2. A student who satisfies the criterion at Level 3 only would not be assigned either a classical or modified van Hiele Level. Neither of these students would be said to fit the classical van Hiele model.

The first case, identified as the Classical case, is based on there being five distinct Levels. The second case, identified as the Modified case, is based on four distinct Levels. The decision to use the Modified Case to identify the van Hiele Level of the test subjects was based on the fact that the "modified van Hiele Levels fit more students more consistently than the classical van Hiele Levels [27]" also it gives a higher percentage of subjects that could be analyzed. For the Part B, each of the 3 items was assigned 10 points. Thus, pre-service teachers' scores ranged between 0 and 30 marks.

2.4. Analysis of Data

This study aims to determine the van Hiele geometric thinking Levels of the participating pre-service teachers. To analyze data, descriptive statistics were used in an attempt to understand, interpret and describe the experiences of the research participants in terms of their levels of geometric conceptualization. In specific terms, various descriptive statistics such as frequency distribution, percentages, chart and measures of central tendency, were used to analyse, describe and compare the quantitative data in this study.

3. Results

3.1. Pre-service Teachers' Performance in Part A of the VHGT

Table 1 presents the overall pre-service teachers' performance on each item of the part A in the VHGT. As can be seen in Table 1, each van Hiele Level (VHL) had five items with five multiple choice options. However, some pre-service teachers did not choose any of the options for some items. This made the researchers include an additional option (a "blank" option) in this table. For each item, the number in bold corresponds to the right option and also represents the total number of pre-service teachers who answered that item correctly.

Table 1. Van Hiele Geometry Test (Part A): Item Analysis for each Level.

	Choice items	A	B	C	D	E	Blank
Level 1	1	0	278	0	14	8	0
	2	2	0	50	236	2	10
	3	26	4	250	0	16	4
	4	14	198	20	60	4	4
	5	28	0	146	110	12	4
Level 2	6	36	92	126	28	2	16
	7	46	4	30	8	192	20
	8	80	22	86	34	50	28
	9	14	8	214	12	48	4
	10	36	20	72	116	20	36
Level 3	11	64	70	56	40	56	14
	12	70	134	30	18	38	10
	13	30	4	4	28	230	4
	14	48	6	102	18	112	14
	15	26	120	12	62	66	14
Level 4	16	76	60	48	24	36	56
	17	132	26	52	52	14	24
	18	60	52	56	72	28	32
	19	144	56	40	12	20	28
	20	72	44	50	74	30	30
Level 5	21	80	34	52	36	42	56
	22	68	26	48	90	32	36
	23	140	48	18	30	38	26
	24	28	30	30	148	34	30
	25	32	84	100	24	28	32

*The figures in bold correspond to the right options and also represent the total number of pre-service teachers who answered that item correctly. n= 300.

3.1.1. Performance on Subtest 1: Van Hiele Level 1

The pre-service teachers performed well only in the first four items of subtest 1. Table 1 shows that 278 (92.67%), 236 (78.67%), 250 (83.33%), 198 (66%) of the pre-service teachers managed to correctly answer items 1, 2, 3 and 4 respectively, compared to item 5, 110 (36.67%) which was not very encouraging. Figure 1 is an item from Subtest 1. The correct answer for this item is choice D. Table 1 shows that only 110 (36.67%) of the pre-service teachers had this item correct, that is, knew that all the given quadrilaterals can be referred to as parallelograms. This clearly indicates that 190 (63.33%) of the pre-service teachers who participated in this research study lack knowledge of "class inclusion".

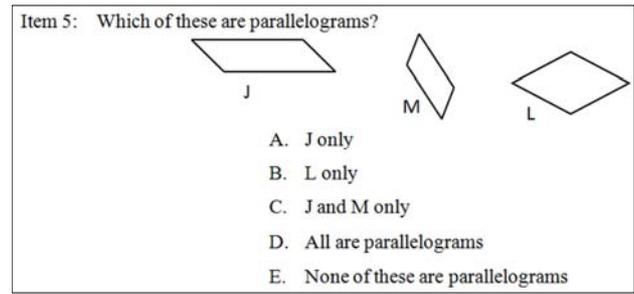


Figure 1. Sample Item in Subtest 1.

3.1.2. Performance on Subtest 2: Van Hiele Level 2

Pre-service teachers did quite well on items 7 and 9. Out of the 300 pre-service teachers, 192 (64%) and 214 (71.33%) respectively answered items 7 and 9 correctly. However, pre-service teachers' performance on items 6, 8 and 10 was not satisfactory. From a total of 300 pre-service teachers only 92 (30.67%), 80 (26.67%) and 116 (38.67%) of the pre-service teachers were able to answer questions on these items respectively.

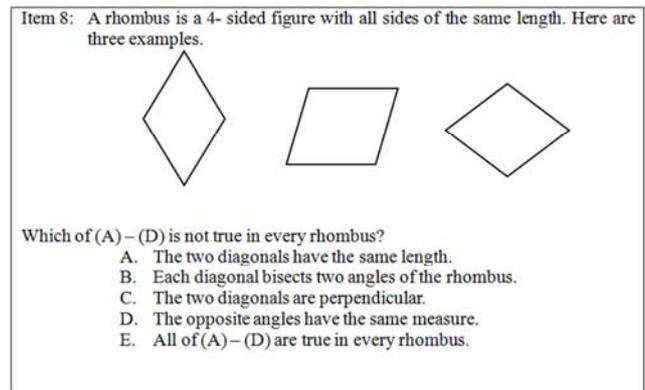


Figure 2. Sample Item in Subtest 2.

Figure 2 is an item from Subtest 2. The correct answer for this item is choice A. Table 1 shows that only 80 (26.67%) of pre-service teachers had this question correct. A total of 73.33% of the pre-service teachers answered this item wrongly. This shows pre-service teachers' lack of knowledge about the properties of a rhombus.

3.1.3. Performance on Subtest 3: Van Hiele Level 3

Subtest 3 is about learners knowing the interrelationship between different types of figures. The performance of the pre-service teachers for Subtest 3 was generally not encouraging. Table 1 shows that 56 (18.67%), 134 (44.67%), 30 (10%), 48 (16%) and 120 (40%) of the pre-service teachers correctly answered items 11, 12, 13, 14 and 15 respectively, which was not encouraging. The performance of pre-service teachers on item 13 was abysmally poor. This item is presented in Figure 3.

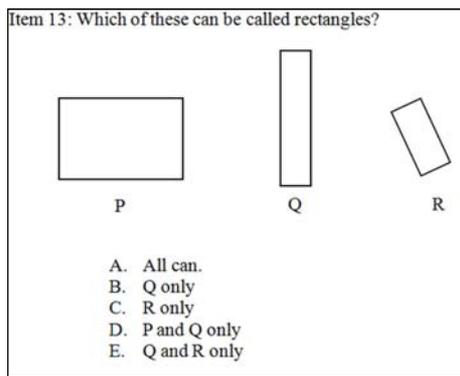


Figure 3. Sample Item in Subtest 3.

The correct choice for item 13 in Figure 3 is A. However, only 30 (10%) of pre-service teachers had this item correct. This clearly shows that majority (90%) of the pre-service teachers did not know that rectangles have common properties with squares, in order words all squares are rectangles. This implies that pre-service teachers have difficulties in understanding “class inclusion”.

3.1.4. Performance on Subtest 4: Van Hiele Level 4

Subtest 4 is about learners being able to give deductive geometric proofs, understanding the role of definitions, theorems, axioms and proofs. Learners at this Level should be able to supply reasons for statements in formal proofs. This is the Level of development that high school students need to be prior to completion of high school. However, the performance of the pre-service teachers for Subtest 4 was generally very poor. Table 1 shows that 48 (16%), 52 (17.33%), 72 (24%), 12 (4%) and 72 (24%) of the pre-service teachers managed to correctly answer items 16, 17, 18, 19 and 20 respectively which was abysmally poor. This generally indicates that the pre-service teachers have difficulties understanding simple deductive geometric proofs, understanding the role of definitions, theorems, axioms and proofs.

3.1.5. Performance on Subtest 5: Van Hiele Level 5

Subtest 5 is about learners being able to work in a variety of axiomatic systems that is, being able to study non-Euclidean geometries comparing different systems and also seeing geometry in the abstract. Similarly, Table 1 indicates that 16 (5.33%), 18 (6%), 16 (5.33%), 16 (5.33%), and 10 (3.33%) of the pre-service teachers in the control group managed to correctly answer items 21, 22, 23, 24 and 25 respectively.

Even though some pre-service teachers were able to answer some items in subtests 4 and 5 correctly, the number of Pre-service Teachers attaining Level 4 and Level 5 in the VHGT was very small.

3.2. Performance on Part B of the VHGT

Table 2 summarizes the overall performance of pre-service teachers in the section B part of the VHGT. There were 3 test items; item 1 was on triangles, properties of parallel lines and

transversal, item 2 was on area of two-dimensional shapes while item 3 was a short proof on congruent triangles. The responses of pre-service teachers who demonstrated good knowledge and provided the right responses for the items were described as correct. Responses of pre-service teachers who attempted items but did not get the total marks allotted per test item were described as partially correct, while the responses that exhibited lack of knowledge about the items were described as completely wrong. However, few pre-service teachers did not attempt some of the items at all; these were described as “blank”.

Table 2. Van Hiele Geometry Test (Part B): Item Analysis.

	Correct	Partially Correct	Completely Wrong	Blank
Item	(%)	(%)	(%)	(%)
1	88(29.33)	110(36.67)	88(29.33)	14(4.67)
2	30(10)	190(63.33)	70(23.33)	10(3.33)
3	26(8.67)	66(22)	198(66)	10(3.33)

* n= 300

The results in Table 2 show that the pre-service teachers performed well only in the first item. Majority of the pre-service teachers (110) representing 36.67% had item 1 partially correct while 88 pre-service teachers representing 29.33% had item 1 correct. The performance of pre-service teachers in item 2 was not encouraging; out of a total of 300 pre-service teachers only 30 pre-service teachers representing 10% answered this item correctly. Again, pre-service teachers’ performance in item 3 was extremely poor; majority (198) pre-service teachers representing 66% had this item completely wrong. This again revealed pre-service teachers difficulties in understanding simple deductive geometric proofs, understanding the role of simple definitions, theorems, axioms and proofs.

3.3. Overall Scores of Pre-service Teachers in the VHGT

Table 3 presents the overall scores of pre-service teachers in both Parts of the VHGT. The minimum score pre-service teachers obtained in the VHGT was 28%, while the maximum score was 69%. The mean score of pre-service teachers was 49.25% while the standard deviation was 5.01.

Table 3. Means, Standard Deviations, Minimum and Maximum VHGT Scores of Pre-service Teachers.

N	Mean	Stand Dev	Maximum	Minimum
150	49.25	5.01	69	28

3.4. Levels Reached by Pre-service Teachers in the VHGT

The purpose of this study was to find out the the van Hiele Levels of geometric thinking reached by Ghanaian pre-service teachers’ before leaving for their Student Internship Programme (Teaching Practice) at the basic schools. The bar chart in Figure 4 provides a visual confirmation of the van Hiele Levels of geometric thinking attained by these pre-service teachers.

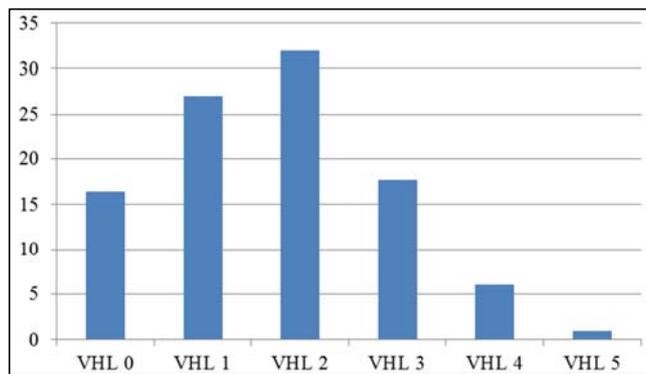


Figure 4. A Bar Chart showing Pre-service Teachers van Hiele Levels of Geometric Thinking.

As shown in Figure 4, 16.33% of pre-service teachers attained VHL 0 (i.e. the Pre-recognition Level or Level for those who have not yet attained any van Hiele Level). For VHL 1, 27% of pre-service teachers attained that Level. 32% of the pre-service teachers attained VHL 2. In addition, 17.67% of pre-service teachers attained VHL 3. However, only 6% and 1% of pre-service teachers attained VHL 4 and 5 respectively.

4. Discussion

The results of the VHGT revealed that 27% of the pre-service teachers attained van Hiele Level 1, 27% reached level 2 and 17.67% reached Level 3 by the theory. Again, only 6% and 1% of pre-service teachers attained Level 4 and Level 5 in the VHGT respectively. In other words, majority of pre-service teachers showed only the first three reasoning stages described by the van Hiele Levels in different percentiles. However, 16.33% of the pre-service teachers did not attain any of the van Hiele Levels suggesting that these pre-service teachers who are about to leave for their Student Internship Programme (Teaching Practice) at the basic schools are at the Pre-recognition Level. These findings concur with those of previous research studies [12] [18] [19]. The findings indicate that majority of pre-service teachers were found to be operating at the basic van Hiele Levels (i.e. Levels 1 and 2) as well as the pre-recognition level, and that a very small number of pre-service teachers operated at van Hiele Levels 3, 4 and 5. This is problematic, since Level 4 skills are desirable prior to the completion of Senior High School and thus, required to successfully begin College of Education geometry.

From the item-by-item analysis, it was evident that the pre-service teachers could identify plane shapes by mere visualization and also identify the properties of these plane shapes, which is a van Hiele Levels 1 and 2 geometric competences respectively. However, the pre-service teachers' responses to items in subtest 3 suggested that majority of them could not identify the interrelationship between different types of figures. Pre-service teachers could not create meaningful definitions and give informal arguments to justify their reasoning. In addition, logical implications and class inclusions, such as squares being a type of rectangle were not

understood by most of these pre-service teachers. This finding is also consistent with [6] observation that class inclusion which theoretically according to van Hiele belongs to Level 3, was frequently found most difficult among learners. Also, only few (6% and 1% respectively) of the pre-service teachers exhibited geometric reasoning at van Hiele Levels 4 and 5. This suggests that teaching and learning in geometry is mainly focused on the basic van Hiele Levels (i.e. van Hiele Levels 1 and 2), with a small amount of geometry work being done at the advanced Levels (i.e. Levels 3, 4 and 5).

5. Limitations of the Study

From a total of 38 public Colleges of Education in Ghana, only 4 Colleges of Education were selected for this study. Though this was compensated for by the strategic location of the Colleges to attract students from several regions of the country, it is still difficult to generalize the findings for the whole country. Also, the range of activities or tasks that the pre-service teachers were tested on was limited due to time constraints. So too was the range of instruments used. Further investigations with larger groups and a wider range of activities and instruments might yield different results.

6. Conclusion

This study was an attempt to measure the van Hiele Levels of geometric thinking among pre-service teachers in Ghana. It specifically sought to find out the stages of the van Hiele Levels of understanding Ghanaian second year pre-service teachers reach in their study of geometry before leaving for their Student Internship Programme (Teaching Practice) at the basic schools. In all, 300 second year pre-service teachers from 4 Colleges of Education were involved in this study. These pre-service teachers were given the van Hiele Geometry Test (VHGT) from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) test items. The results show that majority (59%) of the pre-service teachers attained the basic van Hiele Levels 1 (Recognition) and 2 (Analysis). In addition, 16.33% attained Level 0 (Pre-recognition), a Level of thinking which is not even expected from the Junior High School learner. It can be counter argued, that much depends on whether, during their Senior High School years and first year geometry course in College of Education, the pre-service teachers were taught such concepts as identifying the interrelationship between different types of figures, creating meaningful definitions, giving informal arguments to justify their reasoning, class inclusions and simple deductive geometric proofs to enable them operate at higher van Hiele Levels.

In order to teach geometry successfully at the basic school level, the expected geometric reasoning stage for the basic school teachers is Level 3 (Order) or above [9] [19] [27]. However, this study found that 75.33% of Pre-service basic school teachers' van Hiele Levels was below Level 3 (Order). This indicates that these pre-service teachers, about to leave for their Student Internship Programme (Teaching

Practice) in the basic schools, demonstrate a van Hiele Level that is lower than that expected of their target audience. These findings are alarming, and raise a concern about how to break the cycle of limited geometric understanding.

Recommendations

From the findings of this study, it is recommended that;

- (1) Pre-service Teachers geometry course should be revised in terms of content and scope and these courses may be reorganized according to the geometrical thinking levels of van Hiele.
- (2) Geometry instruction should be supportive and appropriate to the van Hiele geometrical thinking levels. This should involve more hands-on investigations that will actively engage the learners. Geometry instructors should ensure that learners understand and know the properties of all geometric shapes as well as their interrelationships to enable them establish class inclusion, which according to this study is sorely lacking. More work need to be done on class inclusions and simple deductive geometric proofs to enable pre-service teachers operate at van Hiele Levels higher than that expected of their future basic school learners.

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