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Growth of Vapour Bubble Flow inside a Symmetric Vertical Cylindrical Tube

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Abstract: The paper introduces the incompressible Newtonian fluid with heat transfer in a vertical cylindrical tube under the assumptions of long wavelength and low Reynolds number. The system of mass, momentum, and energy equations are solved analytically. The velocity and temperature field are obtained for two-phase densities. The growth of vapour bubble and its velocity between two-phase densities are obtained for first time under the effect of Grashof number and constant heating source. The obtained results are compared with experiment and Mohammadein at all model with good agreement.

Keywords: Heat Transfer, Two-Phase Flow, Bubbly Flow, Newtonian Fluid

1. Introduction

The study of temperature field and behavior vapour bubble dynamics between two-phase densities [1-3, 5-7, and 9] are most important physical phenomena because of its necessity in many technical processes. The mixed lubrication, chemical metallurgic, oil and gas processes are applications of heat exchange. It plays a major role in the industry of refrigerators, boilers and nuclear reactors which are used for generation of electrical current. The vapour bubble is considered as a finite sink growing inside a mixture (vapor and superheated liquid).There are three stages for bubble growth, inertial, thermal, and diffusion. In the inertial stage, the bubble nucleus depends strongly on the interfacial mechanical interactions such as acceleration, pressure force, and surface tension forces. The inertial stage takes a few milliseconds and thermal phenomena are negligible, therefore, this stage is called isothermal. In the thermal stage, the radius of the nucleus increases and growth becomes mostly dependent on the supply of heat that is consumed to vaporize the liquid on the bubble's surface. The growth of vapour bubble is affected by heat transfer and pressure changes between two densities. The bubble radius grows within a superheated liquid has been studied by many workers [1-10]. The effect of heat transfer of a Newtonian fluid through asymmetric vertical cylindrical tube is studied by Raoet. al. [10]. The closed form solutions of velocity field and temperature are obtained. The influence of various physical parameters on the flow is observed. The temperature and the heat transfer are discussed through graphs. In Sec. 2, the mathematical model is presented and solving the velocity and temperature distribution, in addition to, the radius of vapour bubble and velocity of bubble are obtained. The discussion and results are presented in Sec. 3. In Sec. 4, the concluded remarks are indicated to the importance of study velocity and temperature distribution around the growing vapour bubble.

1.1. Analysis

Consider the flow of a viscous incompressible Newtonian fluid through a vertical tube. The flow is generated by sinusoidal wave trains propagating with constant speed along the wall of the outer rube. The axisymmetric cylindrical polar coordinate system is chosen such that the coordinate is along the center line of the tube and coordinate along the radial coordinate. The wall of the tube is maintained at a temperature $T_0$and at the center we have used axisymmetric condition on temperature as in Fig. 1; which depicts the physical model of the problem. Where$a$ the radius of the tube is, b is the amplitude of the wave, λ is the wavelength and t

is the time. The flow is unsteady in the fixed frame (z, r). However, in a coordinate system moving with the propagation velocity c.

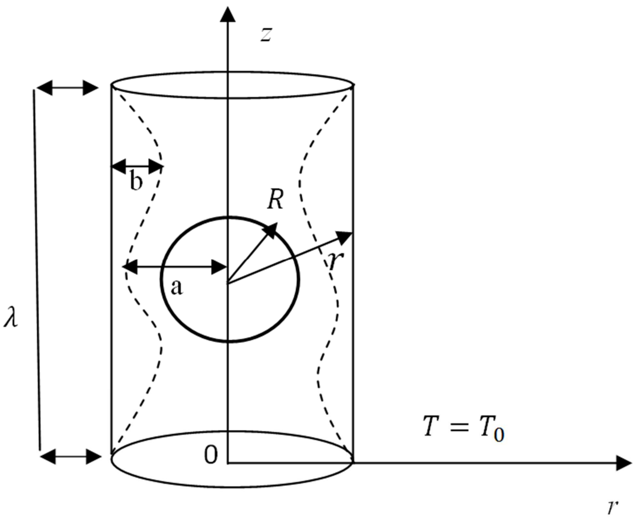

Fig. 1. *Physical Model.*

The equations governing the flow of a Newtonian fluid are the linear momentum, the conservation of mass and the heat equation are given as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u) + \frac{\partial w}{\partial r} = 0 \quad (1)$$

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \quad (2)$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2} + \rho g\alpha(T - T_0) \quad (3)$$

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + Q_0 \quad (4)$$

Introducing the dimensionless variables as follows

$$\bar{r} = \frac{r}{a}, \bar{z} = \frac{z}{\lambda}, \bar{w} = \frac{w}{c}, \bar{u} = \frac{u}{c\delta}, \bar{p} = \frac{pa^2}{\mu c\lambda},$$

$$\bar{\theta} = \frac{T - T_0}{T_0}, \bar{\delta} = \frac{a}{\lambda}, e = \frac{b}{a},$$

$$R_e = \frac{\rho c a}{\mu}, G = \frac{a^3 g\alpha T_0}{\nu^2}, Pr = \frac{\mu c_p}{k}\beta = \frac{a^2 Q_0}{kT_0}, \quad (5)$$

Substituting by the equation (5) into the equations (1-4), we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u) + \frac{\partial w}{\partial r} = 0 \quad (6)$$

$$R_e\delta^3\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{\delta}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \delta^2\frac{\partial^2 u}{\partial z^2} \quad (7)$$

$$R_e\delta\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\delta\partial^2 w}{\partial z^2} + G\theta \quad (8)$$

$$R_e\,Pr\,\delta\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \delta^2\frac{\partial^2 T}{\partial z^2} + \beta. \quad (9)$$

When the wavelength is large $(\delta \ll 1)$, the Reynolds number is quite small $(R_e \to 0)$ and the equations (7-9) becomes

$$\frac{\partial p}{\partial r} = 0 \quad (10)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + G\theta \quad (11)$$

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \beta = 0. \quad (12)$$

The dimensionless volume flow rate in the fixed frame of reference is given by

$$q = 2\int_0^h wr dr \quad (13)$$

the corresponding dimensionless boundary conditions are

$$\frac{\partial w}{\partial r} = 0\ at\ r = 0 \quad (14)$$

$$w = -1\ at\ r = h \quad (15)$$

$$\frac{\partial\theta}{\partial r} = 0\ at\ r = 0 \quad (16)$$

$$\theta = 0\ at\ r = h. \quad (17)$$

Solving Eq. (12) using the Eqs. (16) and (17), we get

$$\theta = \frac{\beta}{4}(h^2 - r^2), \quad (18)$$

substituting by Eq. (18) into the Eq. (11) and solving Eq. (11) with the boundary conditions of Eqs. (14) and (15), we get

$$w = \frac{1}{4}\frac{dp}{dz}(r^2 - h^2) - \frac{G\beta}{4}\left(\frac{h^2r^2}{4} - \frac{r^4}{16}\right) + \frac{3\,G\beta h^4}{64} - 1, \quad (19)$$

The volume flow rate is given by

$$q = -\frac{dp}{dz}\left(\frac{h^4}{8}\right) + \frac{G\,\beta\,h^6}{48} - h^2. \quad (20)$$

From the Eq. (20), we have

$$w = \left(\frac{G\beta h^2}{24} - \frac{2q}{h^4} - \frac{2}{h^2}\right)(r^2 - h^2) - \frac{G\beta}{4}\left(\frac{h^2r^2}{4} - \frac{r^4}{16}\right) + \frac{3\,G\beta h^4}{64} - 1. \quad (21)$$

On the basis of continuity Eq. (1), we find that, the velocity of cylindrical coordinates of the vapour bubble, can be written as

$$w = \frac{\epsilon\,R\,\dot{R}}{r}. \quad (22)$$

From the Eq. (22), we can obtain the velocity of vapour bubble radius in a vertical cylindrical tube as the form

$$\dot{R}(r,t) = \left(\frac{1}{\varepsilon}\right)\left(\frac{G\beta h^2}{24} - \frac{2q}{h^4} - \frac{2}{h^2}\right)(r^2 - h^2) - \frac{G\beta}{4}\left(\frac{h^2r^2}{4} - \frac{r^4}{16}\right) + \frac{3\,G\beta h^4}{64} - 1, \quad (23)$$

$$R(t) = \sqrt{\frac{M}{A}}\tan\left[\tan^{-1}\frac{R_0}{\sqrt{\frac{M}{A}}} - \frac{\sqrt{M\,A}}{\varepsilon}(t - t_0)\right], \quad (24)$$

where,

$$M = -1 - \left(\frac{-2q}{h^2} - 2\right),$$
$$A = \frac{-2q}{h^4} - \frac{2}{h^2},$$
$$\varepsilon = \left(1 + \frac{\rho_v}{\rho_l}\right). \quad (25)$$

1.2. Discussion and Results

The problem of Newtonian flow through a vertical cylindrical tube is described in Fig. 1, the physical problems is described by the system of equations (1-4), and solved analytically to calculate the temperature distribution in Eq. (18), velocity distribution is derived in Eq. (21), and the velocity and radius of vapour bubble are presented in Eqs. (23), (24). The bubble radius is derived in terms of r, z, and t. Heat equation surrounding the vapour bubbly flow is solved. The equation (18) represents temperature distribution in the mixture surrounding the vapour bubble. The equation (23) describes the vapour bubble flow in a vertical cylindrical pipe. The temperature distribution inside a superheated water under atmospheric pressure (P_∞ =247kPa and T_s=400 K). The physical values are calculated by Haar [3] as given by Table 1. Moreover, by using Mathematica program (version 6.0), we get the following graphs that demonstrate the effect of the physical parameters on temperature, velocity distribution and the vapour bubble. Velocity distribution in terms of parameter r for two different values of Non-dimensional heat source parameter β, is obtained in Fig. 2 (a), it is clearly the velocity distribution increases with the increasing of parameter β. The velocity distribution taken the same behavior in the Fig. 2(b) under the effect of Grashof number$G$.This behavior is in agreement with Refs. [10]. In the Fig. 3, we can show the velocity distribution is plotted as a function or r and z. We note that the velocity distribution $w(r, z)$ increases with the increasing of the volume rate flow q .On the other hand, we can show the temperature distribution in terms of parameter r for two different values of amplitude ratio $e$is obtained in Fig. 4, it is observed that the temperature distribution is proportional with amplitude ratio e. The behavior of vapour bubble radius for different values of two phase densities ε is shown in Fig. 5. It is observed that the bubble radius is increasing and shifted for the lower values with increasing of ε . This behavior is agreement with Ref. [7]. In the Fig. 6, the velocity of vapour bubble is plotted as a function of time t with two different values of physical parameter ε, which decreasing with the increasing of time t, and shifted for lower values when the increasing of physical parameter ε.The velocity of vapour bubble is proportionally inversely with the radius of vapour bubble and shifted for the upper values with increasing the Grashof number G, heat source parameter β as shown in Figs. 7, 8, and this is agreement with Ref. [4].The growth of vapour bubble compared with Mohammadein et. al. model is shows in Fig. 9. It is observed that the present model performs at lower values than Mohammadein model with fraction density $\varepsilon = 0.4$ and initial velocity $\dot{R}_0 = 5 \times 10^{-5}$m/s.

Table 1. *Parameters' values used in the present problem.*

Parameter	Value	Unite	Parameter	Value	Unite
ρ_v	0.597	kg/m^3	k	0.6857	W/mk^0
ρ_l	958.3	kg/m^3	σ	0.0535	kg/s^2
ρ_g	1.37	kg/m^3	$\Delta\theta_0$	2.5	k^0
R_0	1×10^{-5}	m	T_0	273.15	k^0
c_p	4240	$J(kgk^0)$			
L	533000	Jkg			
$\dot{R}_0$	$R_0\varphi_0^{\frac{-1}{3}}$	m/s			
R_m	10^{-4}	m			

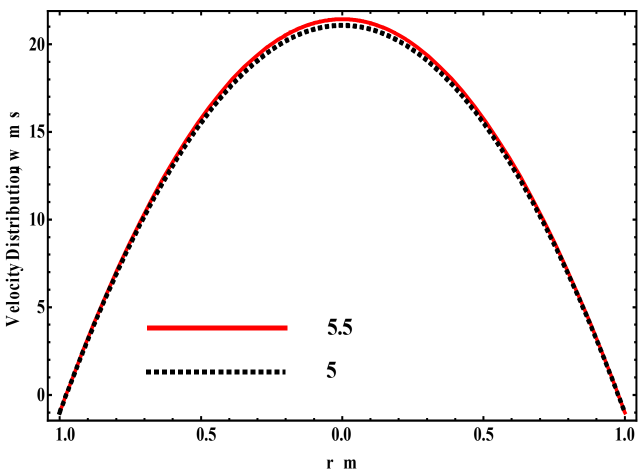

Fig. 2a. *The velocity distribution is plotted as a function of r with thedifferent values of heat source parameter β (β =5.5, 5).*

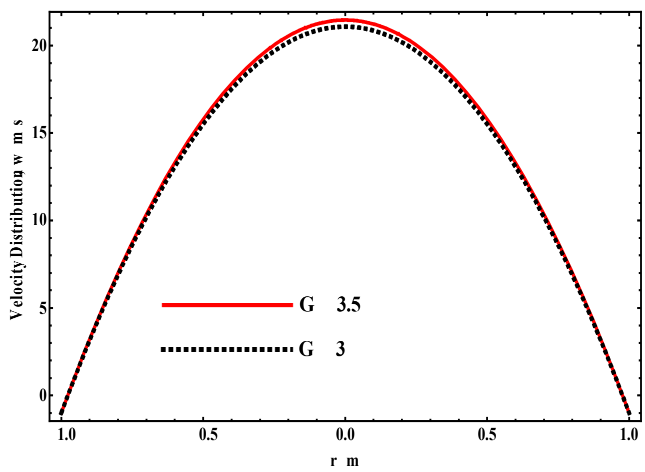

Fig. 2b. *The velocity distribution is plotted as a function of r with the different values of Grashof number G (G=3, 3.5).*

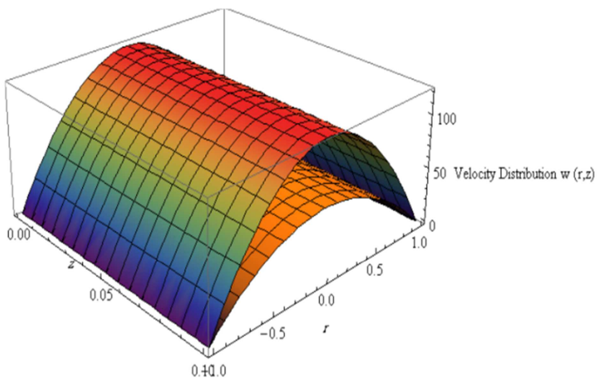

Fig. 3. *The velocity distribution is plotted in 3D as a function of* r, z *with thedifferent values of volume flow rate* q.

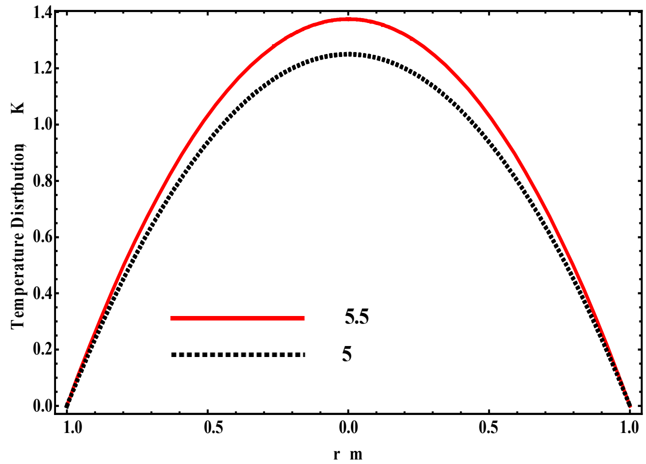

Fig. 4. *Temperature distribution* $\theta(r,z)$ *is plotted as a function of* r *with thedifferent Values of heat source parameter* β *(*β *=5, 5.5).*

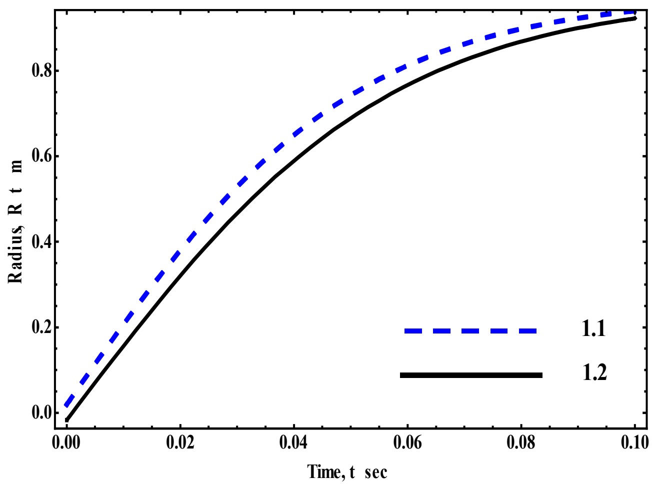

Fig. 5. *The radius of vapour bubble is plotted as a function of time t for two different values of parameter* ε ($\varepsilon = 1.1, 1.2$).

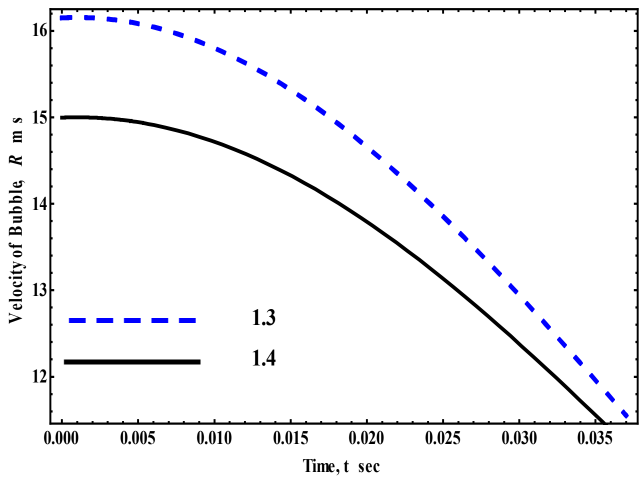

Fig. 6. *The Velocity of bubble radius is plotted as a function of time twith the different values of*ε ($\varepsilon = 1.3, 1.4$).

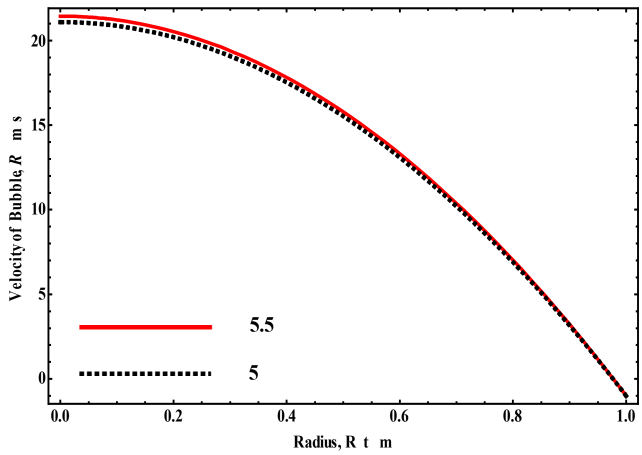

Fig. 7. *The Velocity of bubble radius is plotted as a function of bubble radius R with the different Values of heat source parameter*β *(*β *=5, 5.5).*

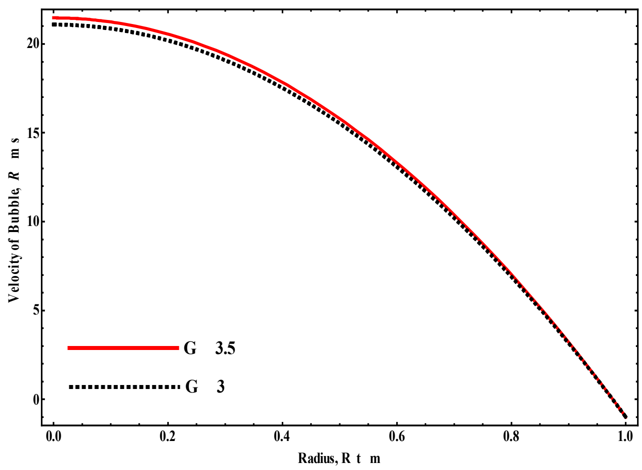

Fig. 8. *The Velocity of bubble radius is plotted as a function of bubble radius R with the different Values of Grashof number G (G=3.5, 3).*

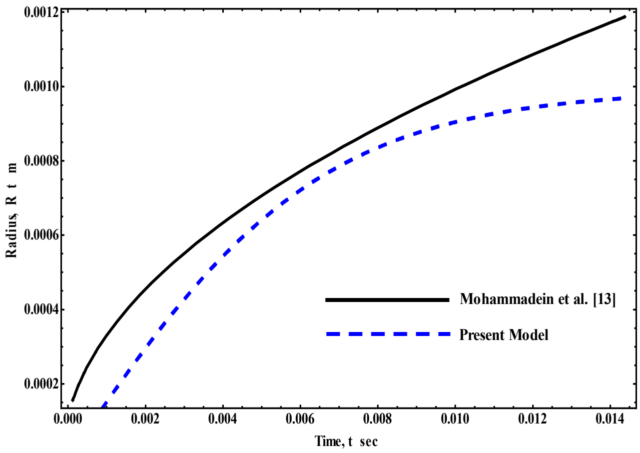

Fig. 9. *The comparison of present model with Mohammadein et al. [13].*

2. Conclusion

The flow of a viscous incompressible Newtonian flow in a vertical tube with observing of vapour bubbles is formulated. The flow is generated by sinusoidal wave trains propagating with constant speed c along the wall of the outer tube. Temperature distribution in the mixture is proportional with different values of amplitude ratio e . The velocity distribution in the mixture is proportional with different values of amplitude ratio e, Grashof number G, heat source parameter β. The behavior of vapour bubble radius in the mixture increasing with the increasing values of physical parameter ε. The temperature, velocity distributions, and behavior of bubble radius in vertical cylindrical tube are obtained as special case when the amplitude ratio tends to zero.

3. Nomenclature

a	Radius of the tube
b	Amplitude of the wave
λ	Wavelength
c_p	Specific heat of liquid at constant pressure $(\mathrm{Kg}^{-1}\mathrm{J/k})$
ρ_v, ρ_l	Density of vapour and liquid $(\mathrm{Kg\ m}^{-3})$
Q_0	Constant heat addition/absorption
k	Thermal conductivity
r	Radial coordinate
z	Coordinate z
R	Instantaneous bubble radius (m)
$\dot{R}$	Instantaneous radial velocity of bubble boundary
T	Time of bubble growth (s)
T	Temperature of liquid(K^0)
T_0	Initial temperature of liquid (K^0)
w	Liquid velocity
θ	Temperature distribution
e	Amplitude Factor
ε	Constant defined by Eq. 25
G	Grashof number
Pr	Prandtl number
β	Non-dimensional heat source parameter
q	Dimensionless volume flow rat
R_e	Reynolds number
δ	Wave number
Subscript l	Liquid
v	Vapour
P	Pressure

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