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Thermal Instability in a Horizontal Layer of Ferrofluid Confined Within Hele-Shaw Cell

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Abstract: Linear thermal instability analysis of a ferrofluid layer confined between in Hele-Shaw cell is investigated. The stability theory is based upon perturbation method and normal mode technique and the resulting equations are solved by using Galerkin weighted residuals method to find expressions for Rayleigh number and critical Rayleigh number. 'Principle of Exchange of Stabilities' hold and the oscillatory modes are not allowed in the problem. It is found that Hele-Shaw number delays the onset of convection while magnetization parameter and buoyancy magnetization parameter hasten the onset of convection.

Keywords: Ferrofluid, Perturbation Method, Galerkin Method, Hele-Shaw Number, Magnetization Parameter

1. Introduction

Ferrodynamics deals with the interaction of the magnetic fields on conducting, as well as non-conducting ferromagnetic fluids. Magnetic fluids also called 'ferromagnetic fluids' are electrically non-conducting colloidal suspensions of solid ferromagnetic magnetite particles in a non-electrically conducting carrier fluid like water, kerosene, hydrocarbon etc. A typical ferromagnetic fluid contains 10^{23} particles per cubic meter. These fluids are not found in nature but are artificially synthesized. The investigations on ferrofluids attracted researchers because of its applications in area such as instrumentation, lubrication, vacuum technology, metals recovery, bio- medical applications, acoustics etc. These fluids are widely used in sealing of hard disc drives rotating X-ray tubes under engineering applications. One of the major applications of ferrofluid is its use in medical fields such as the transport of drugs to an injured site and the removal of tumors from the body. Convection of ferromagnetic is gaining much importance due to their astounding physical properties and one such property is viscosity of ferrofluid. A detailed introduction to this subject has been given in the celebrated monograph by Rosensweig [1]. This monograph reviews several applications of heat transfer through ferrofluids. Since magnetization depends on the magnetic field, temperature and density, any variation of the above causes a change in the body force of the fluid and gives rise ferro convection which is similar to Bénard convection given by Chandrasekhar [2]. Finlayson [3] has studied the convective instability of ferromagnetic fluids, whereas thermo convective stability of ferrofluids without considering buoyancy effects has been investigated by Lalas and Carmi [4]. Thermo convective stability of ferromagnetic fluids was continued by Blennerhassett et al. [5]. Thermosolutal convection in ferromagnetic fluid was studied by Sunil et al. [6]. Mahajan [7] studied the linear and nonlinear convective instability of a ferromagnetic fluid for a fluid layer heated from below under various assumptions. Recently Chand and Bala [8-9], Bala and Chand [10-12] studied the problem related to the thermal instability in a horizontal layer of ferrofluid by using Galerkin weighted residuals method. But in the present study, we studied the stability of ferrofluid layer confined within Hel-Shaw cell heated from below. Hele-Shaw cell is device, whose essential features are two parallel plates separated by an infinitesimally small gap containing a thin layer of fluid. Hele-Shaw flow finds its applications in various fields of sciences and engineering particular in matter physics and material science. The governing equations of the fluid flow in Hele-Shaw cell are similar to those governing the fluid flow in porous medium. Hele-Shaw [13] was first showed the analogy between flow in porous medium and Hele-Shaw cell by defining an equivalent permeability of $b^2/12$ for the Hele-Shaw cell, where b is the width of fluid layer gap. Wooding [14] used free convection in Hele-Shaw cells to simulate thermal convection in porous medium.The objective of the present investigation is to study the onset of the convection ofa ferromagnetic fluid confined within a Hele-Shaw cell using Brinkman model.

2. Mathematical Formulation of the Problem

Consider a horizontal layer of an electrically non-conducting incompressible ferromagnetic fluid of height 'd', vertically confined between two parallel boundaries at z = 0 and z = d. Fluid layer shall be infinitely extended in the x-direction, but confined in the y-direction by vertical impermeable boundaries (side walls) at y = 0 and y = b (<<d). Fluid layer is heated from below in a porous medium of porosity is ε and medium permeability is k(= b2/12)in such a way that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained, where T denote the temperature. The temperature T at z = 0 taken to be T0 and T1 at z = d, (T0 > T1). Let the system is acted upon by gravity force g(0,0,-g) and a uniform magnetic field $\mathbf{H}=H_0^{ext}\hat{k}$ also acts outside the fluidlayer.

2.1. Assumptions

The mathematical equations describing the physical model are based upon the following assumptions

ii. Thermo physical properties of fluid expect for density in the buoyancy force (Boussinesq hypothesis) are constant for the purpose of characterization and estimates of the various effects on the order of magnitude.
iii. Dilute mixture.
iv. No chemical reactions take place in fluid layer.
v. Negligible viscous dissipation.
vi. Negligible radiative heat transfer.
vii. Fluid is incompressibleand laminar flow.

2.2. Governing Equations

When the Hele-shaw cell gap width is not sufficiently small with regard to appearing wavelength of the instability, the correction to Darcy's law is needed. Therefore on employing the Brinkman model, the governing equations under Hele-Shaw approximation for ferrofluid (Chandrasekhar [2], Resenweig [1] and Finlayson [3]) are

$$\nabla\cdot\mathbf{q}=0\,, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{d\mathbf{q}}{dt}=-\nabla p+\rho_0\mathbf{g}-\frac{\mu}{k}\mathbf{q}+\mu\nabla^2\mathbf{q}+\mu_0\left(\mathbf{M}\cdot\nabla\right)\mathbf{H}\,, \tag{2}$$

where $\frac{\mathrm{d}}{\mathrm{dt}}=\frac{\partial}{\partial \mathrm{t}}+\frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)$ stands for convection derivative, **q**(u, v, w) is the velocity vector, ρ0 is reference density, p is the hydrostatic pressure, k(= b2/12) is medium permeability of fluid, μ is viscosity of the fluid,μ0 is magnetic permeability, H magnetic field, M is magnetization.

$$\left(\rho_\circ C_0\right)_m\frac{\partial T}{\partial t}+\left(\rho_\circ C_0\right)_f q\cdot\nabla T=k_m\nabla^2 T\,, \tag{3}$$

where $(\rho_0C_0)_m$ is heat capacity offluid in porous medium, $(\rho_0C_0)_f$ is heat capacity offluid andk_m is thermal conductivity.

Maxwell's equations, in magnetostatic limit:

$$\nabla\cdot\mathbf{B}=0\,,\ \nabla\times\mathbf{H}=0\,, B=\mu_0\left(\mathbf{H}+\mathbf{M}\right).. \tag{4}$$

The magnetization has the relationship

$$\mathbf{M}=\frac{\mathbf{H}}{H}\left[M_0+\chi\left(H-H_0\right)-K_1\left(T-T_1\right)\right] \tag{5}$$

Where B is magnetic induction, K1 is thermal conductivity, $H=|\mathbf{H}|$, $M=|\mathbf{M}|$ and $M_0=M\left(H_0,T_a\right)$.

The magnetic susceptibility and pyomagnetic coefficient are defined by $\chi=\left(\frac{\partial M}{\partial H}\right)_{H\circ,Ta}$ and $K_1=\left(\frac{\partial M}{\partial T}\right)_{H\circ,Ta}$ respectively.

The density equation of state is taken as

$$\rho=\rho_\circ\left[1-\alpha\left(T-T_a\right)\right], \tag{6}$$

Where T_a is the average temperature given by $T_a=\left(\frac{T_0+T_1}{2}\right)$.

The boundary conditions (Chandrasekhar [2]) are

$$\mathrm{w}=0,\quad \mathrm{T}=\mathrm{T}_{0,}\ \mathrm{H}=0\ \text{ at }\ \mathrm{z}=0 \text{ and}$$
$$\mathrm{w}=0,\quad \mathrm{T}=\mathrm{T}_{1,}\ \mathrm{H}=0\ \text{ at }\ \mathrm{z}=\mathrm{d}. \tag{7}$$

2.3. Basic Solutions

The basic state is assumed to be a quiescent state and is given by

$$q\left(u,v,w\right)=q_b\left(u,v,w\right)=0\,, p=p_b\left(z\right),$$
$$T=T_b\left(z\right)=-\beta z+T_a\,, H_b=\left[H_\circ+\frac{K_1\left(T_b-T_a\right)}{1+\chi}\right]\hat{k}\,,$$
$$M_b=\left[M_\circ-\frac{K_2\left(T_b-T_a\right)}{1+\chi}\right]\hat{k}\,,\ H_\circ+M_\circ=H_\circ ext\,. \tag{8}$$

2.4. The Perturbation Equations

We shall analyze the stability of the basic state by introducing the following perturbations:

$$q = q_b + q',\ p = p_b(z) + \delta p,\ T = T_b(z) + \theta,\ H = H_b(z) + H'$$
$$M = M_b(z) + M', \tag{9}$$

where q′(u,v,w), δp, θ, H′(H′$_1$,H′$_2$,H′$_3$) and M′(M′$_1$,M′$_2$,M′$_3$)are perturbations in velocity, pressure, temperature, magnetic field and magnetization. These perturbations are assumed to be small. Then the linearized perturbation equations are

$$\nabla\cdot\mathbf{q}' = 0, \tag{10}$$

$$\frac{\rho_\circ}{\varepsilon}\frac{\partial\mathbf{q}'}{\partial t'} = -\nabla\delta p - \frac{\mu}{k}\mathbf{q}' + \mu\nabla^2\mathbf{q}' + \rho_0\alpha g\theta\hat{k} - \frac{\mu_\circ K_1\beta}{1+\chi}\left((1+\chi)\frac{\partial\phi_1'}{\partial z'}\hat{k} - K_1\theta\hat{k}\right), \tag{11}$$

$$\sigma\frac{\partial\theta}{\partial t'} = \kappa\nabla^2\theta + \beta w', \tag{12}$$

$$\left(1+\frac{M_0}{H_0}\right)\nabla^2\phi_1' - \left(\frac{M_0}{H_0} - \chi\right)\frac{\partial^2\phi_1'}{\partial z'^2} = K_1\frac{\partial\theta}{\partial z'}, \tag{13}$$

where $\mathbf{H}' = \nabla\phi_1'$ and ϕ_1' is the perturbed magnetic potential, $\sigma = \frac{(\rho_0 c_0)_m}{(\rho_0 c_0)_f}$ and $\kappa = \frac{k}{(\rho_0 c_0)_f}$ is thermal diffusivity of the fluid.

The dimensionless boundary conditions are

$$w' = 0,\quad \theta = 1,\ D\phi_1' = 0 \quad \text{at} \quad z = 0 \text{ and}$$
$$w' = 0,\quad \theta = 0,\ D\phi_1' = 0 \quad \text{at} \quad z = d. \tag{14}$$

Introducing non-dimensional variables as

$$(x'',y'',z'') = \left(\frac{x',y',z'}{d}\right),\ \mathbf{q}'' = \mathbf{q}'\frac{d}{\kappa},\ t' = \frac{\kappa}{d^2}t,\ \delta p' = \frac{k}{\mu\kappa}\delta p,$$
$$\theta' = \frac{\theta}{\beta d},\ \phi_1'' = \frac{(1+\chi)}{K_1\beta d^2}\phi_1',$$

Where $\kappa = \frac{k_m}{(\rho_0 c_0)_f}$ is thermal diffusivity of the fluid.

Equations (10)-(14), in non-dimensional form can be written as

$$\nabla\cdot\mathbf{q} = 0, \tag{15}$$

$$\frac{Hs}{\Pr}\frac{\partial\mathbf{q}}{\partial t} = -\nabla\delta p - \mathbf{q} + Hs\nabla^2\mathbf{q} + R(1+M_1)\theta\hat{k} - RM_1\frac{\partial\phi_1}{\partial z}\hat{k}, \tag{16}$$

$$\sigma\frac{\partial\theta}{\partial t} = \nabla^2\theta + w, \tag{17}$$

$$M_3\nabla^2\varphi_1 - (M_3 - 1)\frac{\partial^2\varphi_1}{\partial z^2} = \frac{\partial\theta}{\partial z}. \tag{18}$$

[Dashes (") have beendroppedfor simplicity]

Here non-dimensional parameters are given as

$\Pr = \frac{\mu}{\rho_0\kappa}$ is the Prandtl number;

$Hs = \frac{k_1}{d^2}$ is the Hele-Shaw number;

$R = \frac{\rho_0 g\alpha\beta d^2 k}{\mu\kappa}$ is the Rayleigh number;

$M_1 = \frac{\mu_0 K_1^2\beta}{\alpha\rho_0 g(1+\chi)}$ measure the ratio of magnetic to gravitational forces; $N = RM_1 = \frac{\mu_0 K_1^2\beta^2 d^4}{\mu\kappa(1+\chi)}$ is the magnetic thermal Rayleigh number;

$M_3 = \frac{\left(1+\frac{M_0}{H_0}\right)}{(1+\chi)}$ measure the departure of linearity in the magnetic equation of state and values from one $(M_0 = \chi H_0)$ higher values are possible for the usual equation of state.

Non- dimensional boundary conditions are given by

$$w = 0,\ \theta = 0,\ D\phi_1 = 0 \quad \text{at} \quad z = 0 \text{ and}$$
$$w = 0,\ \theta = 0,\ D\phi_1 = 0 \quad \text{at} \quad z = 1. \tag{19}$$

On eliminating δp from equation (16), we have

$$\left(Hs\nabla^2 - 1 - \frac{Hs}{\Pr}\frac{\partial}{\partial t}\right)\nabla^2 w + R(1+M_1)\nabla_H^2\theta - RM_1\nabla_H^2 D\phi_1 = 0, \tag{20}$$

where $\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is two-dimensional Laplacian operator on horizontal plane.

3. Normal Mode Analysis

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w,\theta,\varphi_1] = [W(z),\Theta(z),\Phi(z)]\exp(ik_x x + ik_y y + nt), \tag{21}$$

Where k_x and k_y are wave numbers in x and y directions respectively, while n is the growth rate of disturbances.

Using equation (21), equations (20) and (17) - (18) become

$$\left(Hs(D^2 - a^2) - 1 - \frac{n\,Hs}{\Pr}\right)(D^2 - a^2)W - a^2R(1+M_1)\Theta + a^2RM_1D\Phi = 0, \tag{22}$$

$$W + \left(D^2 - a^2 - \frac{n}{\sigma}\right)\Theta = 0, \tag{23}$$

$$D\Theta - (D^2 - a^2M_3)\Phi = 0, \tag{24}$$

where $D \equiv \frac{d}{dz}$ and $a = \sqrt{k_x^2 + k_y^2}$ is the dimensionless the resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are

$$W = 0,\ D^2W = 0,\ \Theta = 0,\quad D\Phi = 0 \quad \text{at} \quad z = 0,1. \tag{25}$$

4. Method of Solution

The Galerkin weighted residuals method is used to obtain an approximate solute on to the system of equations (22) – (24) with the corresponding boundary conditions (25). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W = \sum_{p=1}^{N} A_p W_p,\ \Theta = \sum_{p=1}^{N} B_p \Theta_p, D\Phi = \sum_{p=1}^{N} C_p D\Phi_p, \tag{26}$$

where A_p, B_p and C_p are unknown coefficients, p =1, 2, 3,..., N and the base functions W_p, Θ_p and $D\Phi_p$ are assumed in the following form for free-free boundary conditions are:

$$W_p = Cos p\pi\ z,\ \Theta_p = Cos p\ \pi\ z,\ D\Phi_p = Cos p\pi\ z, \tag{27}$$

such that W_p, Θ_p and Φ_p satisfying the corresponding boundary conditions. Using expression for W, Θ and DΦ in equations (22) – (24)and multiplying first equation by W_p second equation by Θ_p and third by $D\Phi_p$ and integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations in 3N unknown A_p, B_p and C_p;p =1,2,3,..., N. For existing of non trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number R.

5. Linear Stability Analysis

We confined our analysis to the one term Galerkin approximation; for one term Galerkin approximation, we take N=1, the appropriate trial function are given as

$$W_p = \cos\pi\ z,\ \Theta_p = \cos\pi\ z,\ D\Phi_p = \cos\pi\ z, \tag{28}$$

which satisfies boundary conditions

$$W = 0, D^2W = 0,\ \Theta = 0,\ D\Phi = 0 \ \text{ at } \ z = 0 \text{ and}$$
$$W = 0, D^2W = 0,\ \Theta = 0,\ D\ \Phi = 0 \ \text{ at } \ z = 1. \tag{29}$$

Substituting solution (28) into equations (22)-(24), integrating each equation from z = 0 to z = 1, by parts and using boundary conditions (29), we obtain following matrix equation

$$\begin{bmatrix} J\left(HsJ + 1 + \frac{nHs}{Pr}\right) & -a^2R\left(1+M_1\right) & a^2RM_1 \\ 1 & -\left(J + \frac{n}{\sigma}\right) & 0 \\ 0 & -\pi & \pi + \frac{a^2M_3}{\pi} \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ D\Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $J = \pi^2 + a^2$.

The non-trivial solution of the above matrix requires that

$$a^2R\left(\left(1+M_1\right)\left(\pi^2 + a^2M_3\right) - M_1\pi\right) = \left(HsJ^2 + J + \frac{nHs}{Pr}\right)\left(J + \frac{n}{\sigma}\right)\left(\pi^2 + a^2M_3\right). \tag{30}$$

For neutral stability, the real part of n is zero. Hence on putting n = iω, (where ω is real and is dimensionless frequency) in equation (30), we have

$$R = \Delta_1 + i\omega\Delta_2, \tag{31}$$

Where

$$\Delta_1 = \frac{\left(HsJ^3 + J^2 - \frac{\omega^2 Hs}{Pr}\right)\left(\pi^2 + a^2M_3\right)}{a^2\left(\left(1+M_1\right)\left(\pi^2 + a^2M_3\right) - M_1\pi^2\right)}, \tag{32}$$

and

$$\Delta_2 = \frac{\left(\frac{HsJ^2 + J}{\sigma} - \frac{JHs}{Pr}\right)\left(\pi^2 + a^2M_3\right)}{a^2\left(\left(1+M_1\right)\left(\pi^2 + a^2M_3\right) - M_1\pi^2\right)}. \tag{33}$$

Since Ris a physical quantity, so it must be real. Hence, it follow from the equation (31) that eitherω= 0 (exchange of stability, steady state) or Δ_2 = 0 (ω ≠ 0 overstability or oscillatory onset).

But $\Delta_2 \neq 0$, we must have ω = 0, which means that oscillatory modes are not allowed and the ‘Principle of Exchange of Stabilities’ is satisfied. This is the good agreement of the result as obtained by Finlayson [3].

Consider the case of stationary convection (n=ω = 0), from equation (30), we have

$$R = \frac{\left(Hs\left(\pi^2 + a^2\right)^3 + \left(\pi^2 + a^2\right)^2\right)\left(\pi^2 + a^2M_3\right)}{a^2\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)} \tag{34}$$

In the absence of Hele-Shaw number i.e. Hs = 0, then equation (34) gives

$$R = \frac{\left(\pi^2 + a^2\right)^2\left(\pi^2 + a^2M_3\right)}{a^2\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)}.$$

This is the good agreement of the result as obtained by Finlayson [3].

In the absence of magnetic parameters ($M_1 = M_3 = 0$) and Hele-Shaw number (Hs = 0), the Rayleigh number R for steady onset is given by

$$R = \frac{\left(\pi^2 + a^2\right)^2}{a^2} \tag{35}$$

Consequently critical Rayleigh number is given by $\mathrm{Rc} = 4\pi^2$.

This is good agreement of the classical result of Rayleigh-Bénard problem for Newtonian fluid.

6. Result and Discussions

In order to investigate the effects of magnetization parameter M_3buoyancy magnetization M_1 and Hs on the stationary convection, we examine the behavior of $\frac{\partial R}{\partial M_3}, \frac{\partial R}{\partial M_1}$ and $\frac{\partial R}{\partial Hs}$ analytically.

Equation (37), we have

$\frac{\partial R}{\partial M_3} < 0$, which mean magnetization parameter M_3 has destabilizing effect on fluid layer. This is the good agreement of the result obtained by Bala and Chand [10-12].

$\frac{\partial R}{\partial M_1} < 0$, which imply buoyancy magnetization M_1 destabilize the fluid layer. This is the good agreement of the result obtained by Bala and Chand [10-12].

$\frac{\partial R}{\partial Hs} > 0$ which mean Hele-Shaw parameter Hs has stabilizing effect on fluid layer.

Thus Hele-Shaw parameter Hs has stabilizing effect while magnetization parameters M_3 and buoyancy magnetization M_1 have destabilizing effect on stationary convection of the fluid layer.

7. Conclusions

Thermal instability in a vertically oriented Hele-Shaw cell was investigated using linearly stability theory. Anexpression for Rayleigh number for the stationary convection is obtained by using Galerkin residual weighted method.

The main conclusions are as follows:

(i) The magnetization parameters M_3 and buoyancy magnetization M_1have destabilizing effect while Hele-Shaw parameter Hs has stabilizing effect on stationary convection of the fluid layer.

(ii) In the absence of magnetic parameters and Hele-Shaw parameter the obtained result is same as the result obtained by Chandrasekhar in the classical Bénard problem.

(iii) The oscillatory modes are does not exist for the problem.

(iv) The ‘Principle of Exchange of Stabilities’ is valid for the problem.

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