

# Probabilistic Voltage Stability Analysis Based on Unscented Transformation and Maximum Entropy Principle

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**Abstract:** Considering the uncertainty of load and the random variation of wind farm output power in power system, a probabilistic voltage stability analysis method is proposed based on unscented transformation technique. According to the statistical characteristics of random variables in power system, the statistical characteristics of voltage stability margins, such as mean, standard deviation and moments, can be calculated by using a small number of samples and the conventional method. The maximum entropy method is applied to determine the probability distribution of voltage stability margin. In compared with Monte Carlo method, the effectiveness of the proposed method is verified on 39-bus and IEEE 57-bus system. The results show that the proposed method can accurately compute the statistical characteristics and the probability distribution of the voltage stability margin, and the computational efficiency is improved.

**Keywords:** Voltage Stability, Probabilistic Voltage Stability Margin, Unscented Transformation Technique, Sigma Points, Maximum Entropy Principle

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## 1. Introduction

The security and stability of power system is the basic requirement, and voltage stability is an important content in the research of power system stability. The reformation of the power market in the world makes the network operator make full use of the transmission ability of the existing transmission system, and makes the system run on the operating point closer to the critical point of voltage stability. In order to prevent the voltage collapse accident, power system dispatchers are very concerned about voltage stability and stability margins [1, 2] of power system under the current operating conditions. Voltage stability margins are related to the current operating point and the increase direction of the loads and the generator power. As the loads in the system vary randomly and there are random errors in the process of measurement, estimation and prediction, the voltage stability

margin of power system is also uncertain.

To address the issue of the stable operation of power systems, Dobson and Alvarado presented the concepts of the closest voltage stability critical point, the load increase worst scenario, the minimum load margin, and the method to calculate the minimum load margin [3, 4]. A fast calculation method for the minimum voltage stability margin was proposed in [5]. Since the closest critical voltage stability point is the one that has the shortest distance from the initial operating point, the voltage stability margin is the smallest and the most conservative. In the practical system the load may not increase in the worst scenario, which will affect the critical point. To address this problem, the super-cone and super-pyramid load model which simulate the uncertainty of load increase direction were presented [6-8], and the closest critical point are calculated with loads increased among the super-cone and super-pyramid. However, to the best of our

knowledge, the probability of the closest critical point occurrence and the probability distribution of the voltage stability margin are not provided.

Loads in the power systems are timely variable. With the development of electric vehicles, high-power charging enhances the load fluctuation [9]. Many renewable energy sources (including wind power, solar power) and other distributed power generation are integrated into the grid. These factors are stochastic and will certainly affect the power system voltage stability. In order to consider the influence of these new uncertainties on voltage stability, researchers proposed some voltage stability analysis methods based on probability theory. The probability distribution of power system voltage stability margin is calculated by the probability distribution or statistic characteristics of the stochastic factors in the power system, so that the power system voltage stability probability at a certain load level can be determined. At present, Monte Carlo method, analytic method [12, 14, 17, 19], point estimation method [11, 16, 20] are the common techniques for probabilistic voltage stability analysis [10-20]. Monte Carlo method is used to simulate various uncertainties, such as the varieties of loads and the outages of equipment in power systems. Since Monte Carlo method is time-consuming, its application is limited in the practical system. However, it is usually treated as a reference to evaluate the accuracy of other probabilistic methods [14-16, 19, 20]. The analytic method exploits the linearized relationship between the input and output random variables at the critical point by using mathematical assumptions. According to the probabilistic distribution or statistic characteristics of the input random variables, the cumulants of other variables such as load margin can be calculated, and then the Gram-Charlier series or Edgeworth series are used to determine the cumulative probability or probability density function of the load margin [12, 14, 17]. This method is more efficient, but cumulants computation needs to assume that the input variables are independent of each other. The point estimation method applies the statistical information of the input random variables to extract the sampling points and to determine the probability of the sampling points. The statistical information of voltage stability margin can be calculated in terms of the sampling points, and the probability distribution of the system voltage stability margin is estimated by Cornish-Fisher series [11, 16]. Point estimation method is convenient for independent input random variables. However, for the correlative random variables, additional transformation is required to make these variables irrelative [21]. The stochastic response surface method is also used to determine the probability distributions of the power system voltage stability margin [13, 18]. This method treats the voltage stability margin as a Hermite polynomial of standard normal random variables and the coefficients of the Hermite polynomial are determined according to the samples. Therefore the distribution of input random variable should be unambiguous and can be transformed into a standard normal distribution.

In this paper, a probabilistic voltage stability analysis method based on unscented transformation (UT) [22] is

proposed, and this method is combined with the maximum entropy method [19, 23] to determine the probability distribution of system voltage stability margin. The unscented transformation determines a small number of sampling points and their distribution probability. These sampling points represent different loads and output power of the wind farm, and are named as sigma points. The sampling voltage stability margins for these sigma points are calculated by the direct method and continuous power flow method. With these sampling values, the mean and variance for the population of voltage stability margin can be estimated. And the probability distribution for this population is determined by the maximum entropy method.

The rest of this paper is organized as follows. Section II gives the power flow equations at the critical point and the probabilistic models of different random variables in the power system. Section III outlines the process for the determination of the voltage stability margin probability distribution. The validation of the proposed method is examined on 39 bus and IEEE 57 bus system in section IV. Conclusion are given in section V.

## 2. Probabilistic Voltage Stability Analysis Model

In the static voltage stability analysis, the voltage stability critical point is the point where the Jacobian matrix of power flow equation is singular. The power flow equation at the critical point is

$$F(X) - (S_G - S_L) = 0 \quad (1)$$

where  $X$  is the bus voltage vector,  $S_G$  and  $S_L$  represent respectively the generator and the load vectors at the critical point. The main methods for finding the critical points are continuous power flow method, direct method and optimization method. In this paper, we used the continuous power flow and direct method to calculate the critical point.

The load increment from the current operating point to the critical point is named as the voltage stability margin, and it is a measure of the voltage stability of the power system. The voltage stability margin is related to the current operating point and increase pattern of load and generator power. That means the voltage stability margin depends on the initial operating point and the increase direction of loads and generator outputs. Deterministic voltage stability margin is calculated under deterministic operating condition and the predefined load increase direction [1, 2]. The minimum voltage stability is obtained under the conditions of the worst load increase scenario and the determined operation condition [3, 4]. The probabilistic voltage stability margin is obtained considering the random variation of the initial operating point [10, 11, 13, 14, 16-19].

With the stochastic variance of loads and the output uncertainties of wind farms led by wind speed, the probability characteristics and probability distribution of voltage stability margin can be determined. Assuming that the loads at the

initial operating point are subject to normal distribution, all loads are increased following the conditions that the increased gains are same and the power factors maintain constant. The generators in the system consist of conventional synchronous generators and wind generators. The outputs of the conventional synchronous generators can be adjusted and their power increases in proportion to the power at the initial operating point. The outputs of the wind turbines affected by wind speed are random variables, and they do not increase with the load. The probability distribution of wind speed is assumed as Weibull distribution. The random distribution of the wind turbine output power is acquired in terms of the approximate relationship between the output power and wind speed of the wind turbine [16]. As the load and the wind turbine outputs are random variables, the corresponding voltage stability margin is also a random variable. According to the statistical characteristics of the voltage stability margin, the maximum entropy method is exploited to determine the probability distribution and cumulative probability of voltage stability margin.

### 3. Probability Voltage Stability Margin Calculation

#### 3.1. Unscented Transformation Sampling Method

The probability characteristic of the voltage stability margin is calculated by the unscented transformation technique in this paper. The sigma points and their probability are obtained according to the statistical characteristics of input random variables. Here the input variables are the loads and the wind turbine outputs. Related to these sigma points of input variables, the sigma points of the output variable (i.e. sampling voltage stability margin) are determined by the conventional method. And from the output sigma points, the statistical characteristics for the voltage stability margin  $\lambda_p$  of the power system (i.e. the population) can be calculated [22]. The distribution of wind turbine output is determined according to the distribution of the wind speed, and then the mean  $\bar{P}$  and covariance  $C_p$  for wind turbine output and load can be estimated. After that the sigma point set  $\{P_1, P_2, \dots, P_l, \dots, P_N\}$  is obtained by using the symmetric sampling method as follows,

$$P_0 = \bar{P} \quad (2)$$

$$P_i = \bar{P} + \sqrt{\frac{n}{1-w_0}} S_{Pi} \quad i = 1, 2, \dots, n \quad (3)$$

$$P_{i+n} = \bar{P} - \sqrt{\frac{n}{1-w_0}} S_{Pi} \quad i = 1, 2, \dots, n \quad (4)$$

where  $n$  is the number of input random variables.  $S_{Pi}$  is the  $i$ th  $n$ -dimensional column vector of matrix  $S_p$  obtained by Cholesky decomposition, and  $S_p$  satisfies  $C_p = S_p S_p^T$ .

Each sample point  $P_l, l = 1, 2, \dots, N$  is also a column vector. The weights corresponding to each sample point are:

$$w_0 = w_0 \quad (5)$$

$$w_i = \frac{1-w_0}{2n} \quad i = 1, 2, \dots, n \quad (6)$$

$$w_{i+n} = \frac{1-w_0}{2n} \quad i = 1, 2, \dots, n \quad (7)$$

The mean, standard deviation and probability of the sigma sample points satisfy the following expression

$$\sum_{l=1}^N w_l = 1 \quad (8)$$

$$\sum_{l=1}^N w_l P_l = \bar{P} \quad (9)$$

$$\sum_{l=1}^N w_l (P_{il} - \bar{P}_i)(P_{jl} - \bar{P}_j) = C_{Pij} \quad (10)$$

where  $P_{il}$  and  $P_{jl}$  represents respectively the  $i$ -th and  $j$ -th elements of the  $l$ -th sigma point.

From (2)-(10), it can be seen that the covariance matrix  $C_p$  is used to determine the sample points. That means the correlation between random variables has been taken into account. No additional transformation is required to deal with the correlation between random variables, and therefore the method is convenient to consider the correlation between random variables.

#### 3.2. Voltage Stability Margin Probability Characteristics Calculation

In this paper, with the weight of center sample  $w_0 = 0.5$ , the probabilistic characteristics of the voltage stability margin is calculated using the unscented transformation method. The steps are as follows:

- (1) According to the distribution of loads and the distribution of wind turbine outputs, the mean  $\bar{P}$  of load and wind turbine output and their covariance matrix  $C_p$  can be determined.
- (2) The symmetric sampling method is used to determine sigma sampling points  $P_l$  and its weight  $w_l, l = 1, 2, \dots, N$ .
- (3) For each sampling point  $P_l$ , the voltage stability margin  $\lambda_{Pl}$  can be calculated by deterministic continuation power flow and direct method.
- (4) By combining the voltage stability margin calculated for each sampling point and the weight  $w_l$  of each sampling point, the mean and standard deviation of voltage stability margin and other probability statistical information such as moments are obtained by weighted calculation.

### 3.3. Distribution Determination of Voltage Stability Margin

As is stated in reference [24], the random variable entropy is defined as  $H = -\int p(x) \ln p(x) dx$  which represents the measure of uncertainty, where  $p(x)$  is the probability density function of the random variable. The maximum entropy principle can be used to determine the probability distribution of the random variable with the expectation and other statistical characteristics of random variables. The general form of the maximum entropy principle is described as follows [25]:

$$\max H = -\int p(x) \ln p(x) dx \quad (11)$$

$$s.t. \quad E\{\phi_n(x)\} = \int \phi_n(x) p(x) dx = \mu_n, n = 0, \dots, M \quad (12)$$

where  $\phi_n(x)$  refers to known function, and in this paper  $\phi_n(x) = x^n, n = 1, 2, \dots, M$ .  $\mu_n$  is the  $n$ -th moment of random variable  $x$ , and when  $\mu_0 = 1, \phi_0 = 1$ .

The classical solution of this optimal problem is given by [25]

$$p(x) = \exp\left[-\sum_{n=0}^M \lambda_n \phi_n(x)\right] \quad (13)$$

where the  $M+1$  Lagrange parameters  $\lambda = [\lambda_0, \lambda_1, \dots, \lambda_M]$  are obtained by solving the following  $M+1$  nonlinear equations,

$$\int \phi_n(x) \exp\left[-\sum_{n=0}^M \lambda_n \phi_n(x)\right] dx = \mu_n, n = 0, \dots, M \quad (14)$$

Substituting the Lagrange parameters solved from equation (14) into equation (13), we can find the probability density function  $p(x)$  and the cumulative probability of stability margin.

## 4. Case Study and Result Analysis

### 4.1. Test System Information

In order to examine effectiveness of the proposed method, the probabilistic voltage stability analysis are performed on England 39-bus system [26] and IEEE 57-bus system, and the results will be compared with the ones obtained by Monte Carlo method. Real power loads of bus 12 and 20 are respectively modified to be 8.5 MW and 680 MW. The rate outputs of wind farm integrated at buses 30 and 35 are 250 MW and 350 MW in England 39-bus system. In the IEEE 57-bus system, the wind farms are attached to buses 9, 13 and 14 respectively. Wind turbines are equipped with DFIG, which could adjust reactive power by itself. Therefore, they are regarded as PQ nodes with constant power factor  $\cos\varphi=0.98$ , absorbing reactive power. The parameters of wind farms are listed in Table 1. Outages of power components, economic dispatch and generator power limit are not considered. The speeds of wind farms are assumed to follow two-parameter Weibull distribution. The loads are assumed to be normal distribution and independent of each other. All loads in the system increase in the same proportion and with the power factor kept.

Table 1. Parameters of wind farms.

	Wind farm	Rated power (MW)	Cut-in speed (m.s <sup>-1</sup> )	Cut-out speed (m.s <sup>-1</sup> )	Rated wind speed (m.s <sup>-1</sup> )	$k$	$c$
39-bus system	1	250	3.0	25	15	2.0	8.5
	2	350	3.0	25	15	2.0	8.5
	1	45	4.0	25	15	1.4	7.0
IEEE57-bus system	2	45	4.0	30	14	1.8	6.0
	3	10	3.0	25	14	1.6	6.5

### 4.2. Result and Analysis

The voltage stability margins of the two test systems are respectively obtained by the proposed unscented transformation technique and Monte Carlo with 10000 sampling sizes. In this paper, voltage stability margin is defined as the total active power increments of all loads (without the loads at slack bus) from the initial operating point to the critical point. The means and standard deviations of voltage stability margins are listed in Tables 2, 3 and 4.

Table 2 and 3 show the means and standard deviations of the

voltage stability margin for the 39-bus system and their relative errors compared with Monte Carlo results. In order to investigate the impact of standard deviations of loads on voltage stability margin, the variance of each load is selected such that 95%, 90%, 85% and 80% confidence interval are corresponding to the interval  $[\mu-10\%\mu, \mu-10\%\mu]$ .  $\mu$  is the mean value of random load. In other words, the standard deviations of loads are respectively  $0.0510\mu, 0.0606\mu, 0.0694\mu$  and  $0.0775\mu$ . The wind speeds of two wind farms are assumed to be uncorrelated for Table 2. The correlation coefficient of wind speed of two wind farms is assumed to be 0.2 for Table 3.

Table 2. Mean and standard deviation of voltage stability margin for uncorrelated wind speed.

$\sigma_L$		Proposed method		Monte Carlo		Relative error	
		Mean (MW)	Standard deviation (MW)	Mean (MW)	Standard deviation (MW)	Mean	Standard deviation
(a)	$0.0510\mu_i$	3665.69	449.31	3663.99	453.56	0.05%	0.94%
(b)	$0.0606\mu_i$	3665.18	497.99	3663.90	505.51	0.03%	1.49%
(c)	$0.0694\mu_i$	3664.44	543.85	3663.75	555.60	0.02%	2.11%
(d)	$0.0775\mu_i$	3663.47	585.88	3663.53	602.61	0.06%	2.78%

**Table 3.** Mean and standard deviation of voltage stability margin for correlated of wind speed.

$\sigma_L$		Proposed		Monte Carlo		Relative error	
		Mean (MW)	Standard deviation (MW)	Mean (MW)	Standard deviation (MW)	Mean	Standard deviation
(a)	0.0510 $\mu_i$	3743.86	469.08	3742.28	474.51	0.04%	1.15%
(b)	0.0606 $\mu_i$	3743.23	515.81	3742.07	524.41	0.03%	1.64%
(c)	0.0694 $\mu_i$	3742.35	560.10	3741.78	572.82	0.02%	2.22%
(d)	0.0775 $\mu_i$	3741.24	600.88	3741.41	618.47	0.01%	2.84%

It can be seen from Table 2 and 3 that the voltage stability margin obtained by using the unscented transformation method is very close to the ones of Monte Carlo method. The maximum relative errors for means of voltage stability margin are 0.06% and 0.04% respectively in table 2 and 3. The relative errors for standard deviation is larger than the ones for means of voltage stability margin, and the maximum errors are 2.78% and 2.84% respectively in table 2 and 3. The error is small and the accuracy of the proposed method is acceptable. Furthermore, in Tables 2 and 3 the results obtained by both the proposed and Monte Carlo methods show that when the standard deviations of the loads increase the means of the stability margins for probabilistic voltage stability decrease slightly, and the standard deviations for the probabilistic voltage stability margin increase. In addition, for the same standard deviation of the loads, the mean and standard deviation of the probability voltage stability margins in Table

3 are bigger than those in Table 2. For example, for  $\sigma_L = 0.0775\mu$ , the mean and standard deviation of stability margins are 3663.47MW and 585.88MW, respectively. The mean and standard deviation of stability margin are 3741.24MW and 600.88 MW respectively when the wind speeds are correlated. They were increased by 2.12% and 2.56% respectively.

Table 4 shows the results of the IEEE 57-bus system by the two methods. It is assumed that the partial loads (loads on buses 3, 6, 8, 9 and 12) are random variables and their standard deviations are 0.0775 $\mu$  ( $\mu$  is their means). Assuming the wind speed correlation coefficient matrix  $R$  is:

$$R = \begin{bmatrix} 1.0 & 0.28 & 0.22 \\ 0.28 & 1.0 & 0.18 \\ 0.22 & 0.18 & 1.0 \end{bmatrix}$$

**Table 4.** Mean and standard deviation of voltage stability margin of IEEE 57-bus system.

	Proposed method		Monte Carlo		Relative error	
	Mean (MW)	Standard deviation (MW)	Mean (MW)	Standard deviation (MW)	Mean	Standard deviation
Uncorrelated wind speed	891.65	19.76	891.44	19.85	0.02%	0.45%
Correlated wind speed	890.99	19.84	890.78	19.95	0.02%	0.55%

From Table 4, it also can be seen that the probability stability margin calculated by two methods is very close. The relative errors for the mean value are both 0.02% and the relative errors for standard deviation are 0.45% and 0.55% respectively, and these results are in the acceptable range for the practical applications. Table 4 also shows that the unscented transformation method can calculate the probability voltage stability margin accurately. In addition, when wind speed is correlated, the standard deviation is slightly larger than that when the wind speed is uncorrelated, however the mean value is slightly reduced. This is different from the result of 39-bus system. Therefore, if the wind speed correlation of the wind farm is neglected, especially when the correlation is strong, the analysis result may have a large deviation from the actual situation.

In order to illustrate the efficiency of the proposed method, the computational time and the number of sampling points are listed in Table 5. The standard deviation of the load is 0.0775 $\mu$  and the wind speeds are correlated. For 39-bus system, the direct method combined with the continuous power flow method is adopted and for IEEE 57 bus system continuous power flow method is applied. All computation procedures are performed in Matlab on the desktop computer. The CPU frequency is 3.1GHz, and the memory is 8G.

As is seen from Table 5, since 20 random variables are considered in the 39-bus system and 8 random variable are

considered in IEEE 57-bus system, the numbers of sampling points are 41 and 17 respectively. Compared with the Monte Carlo method, the number of samples is reduced significantly. Therefore the unscented transformation method requires less computational time which is only 0.81% and 0.26% of the ones of Monte Carlo method for two test systems. With the increase of the number of random variables in a large-scale system, the required sampling point will increase, and the computational performance of the proposed method will be reduced.

**Table 5.** Number of samples and calculation time.

		Proposed method	Monte Carlo
39-bus system	Simple number	41	10000
	Time	4.7850s	589.3729s
IEEE57-bus system	Simple number	17	10000
	Time	6.8219s	2587.4660

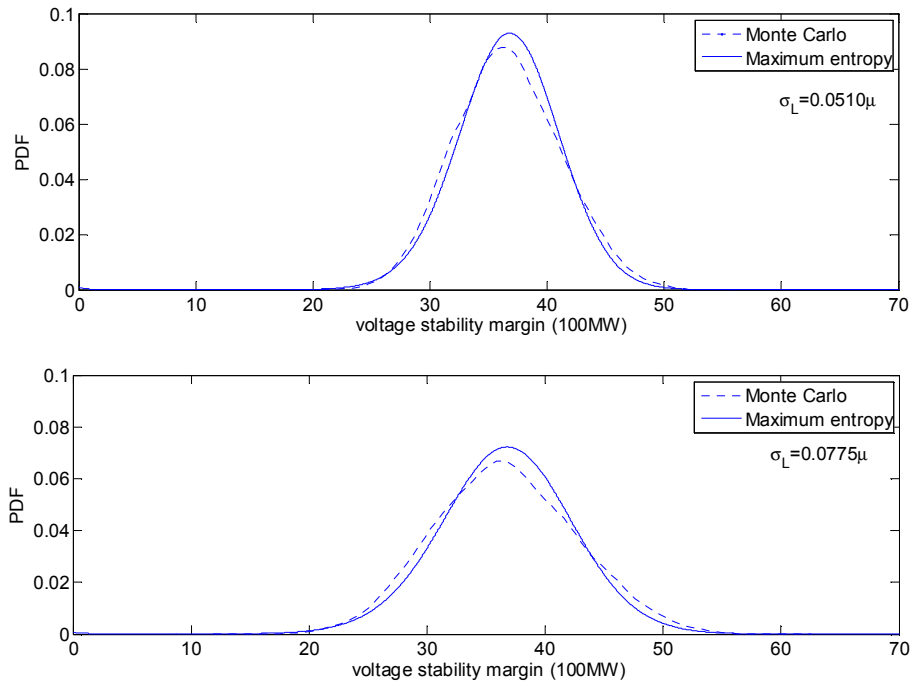
Besides the mean and standard deviation obtained by the unscented transformation method, the moments of the probability voltage stability margin can also be approximated by using the weights of sampling points and the voltage stability margins corresponding to the sampling points. Substituting the first fourth order moments of voltage stability margin into equation (14),  $\lambda = [\lambda_0, \lambda_1, \dots, \lambda_4]$  can be solved shown in table 6. The Probability Density Function (PDF) of

probabilistic stability margin is obtained in terms of equation (13). The PDF curve of probabilistic stability margin in terms of Monte Carlo results is also estimated to examine the

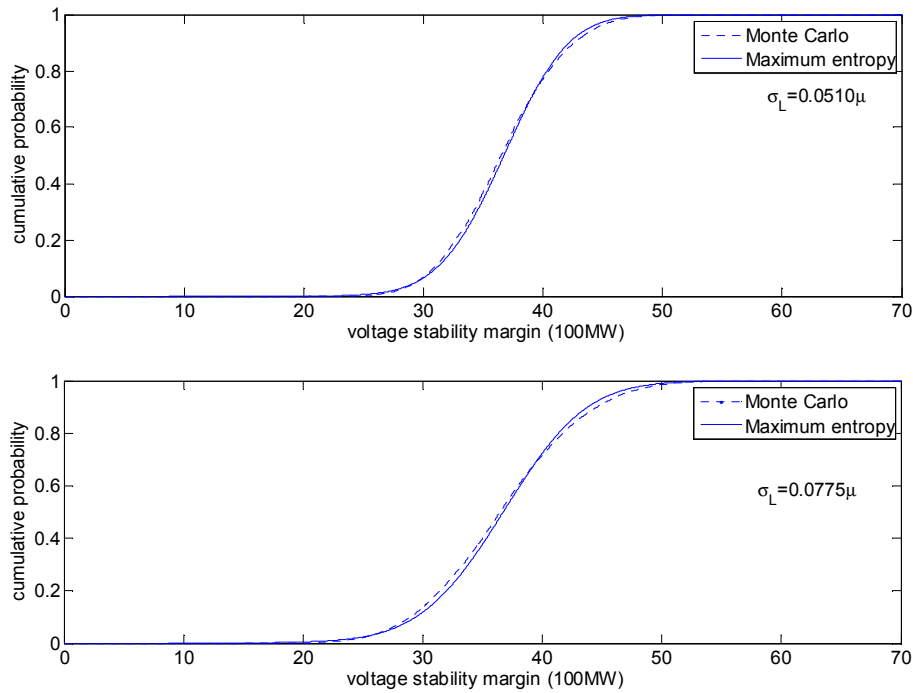
validation of the maximum entropy method. The PDF curves and the cumulative probability curves of 39 bus system and IEEE 57 bus system are shown in Fig. 1-4 and Fig. 5-6.

**Table 6.** Lagrange parameters for 39-bus system.

	$\sigma_L$	$\lambda_0$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
Uncorrelated	$0.0510\mu$	6.9407	1.4201	-0.1077	$0.2343 \times 10^{-2}$	$-0.1513 \times 10^{-4}$
	$0.0775\mu$	7.2047	0.7297	$-0.6229 \times 10^{-1}$	$0.1402 \times 10^{-2}$	$-0.9248 \times 10^{-5}$
Correlated	$0.0510\mu$	6.8301	1.3064	$-0.9681 \times 10^{-1}$	$0.2047 \times 10^{-2}$	$-0.1274 \times 10^{-4}$
	$0.0775\mu$	7.2605	0.6756	$-0.5679 \times 10^{-1}$	$0.1242 \times 10^{-2}$	$-0.7886 \times 10^{-5}$



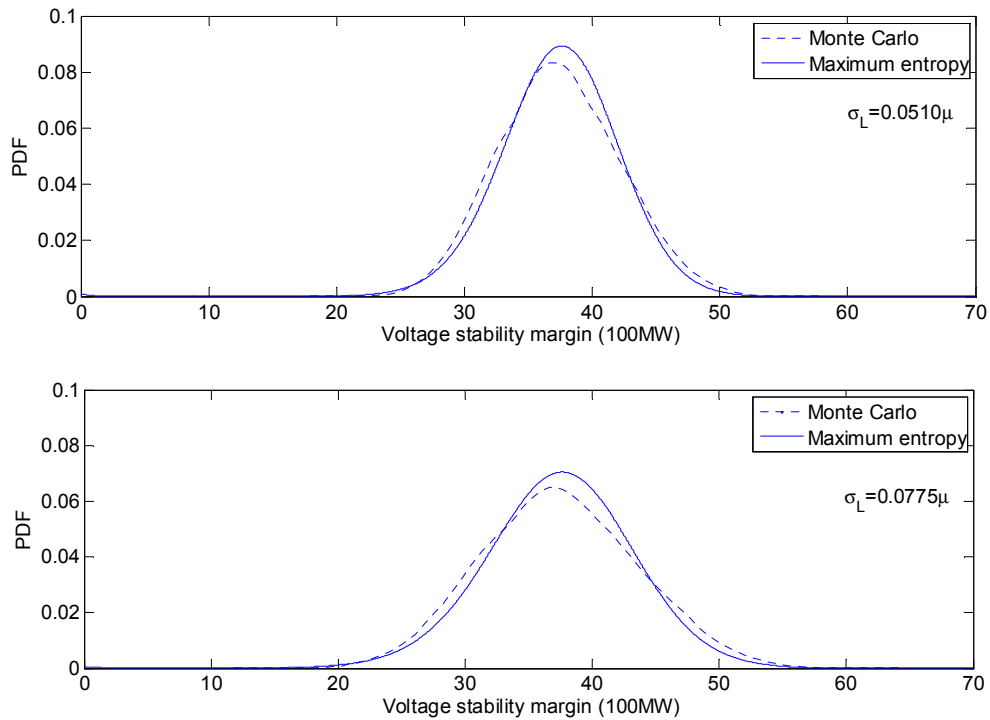
**Figure 1.** Probability density function of voltage stability margin of 39-bus system for uncorrelated wind speeds.



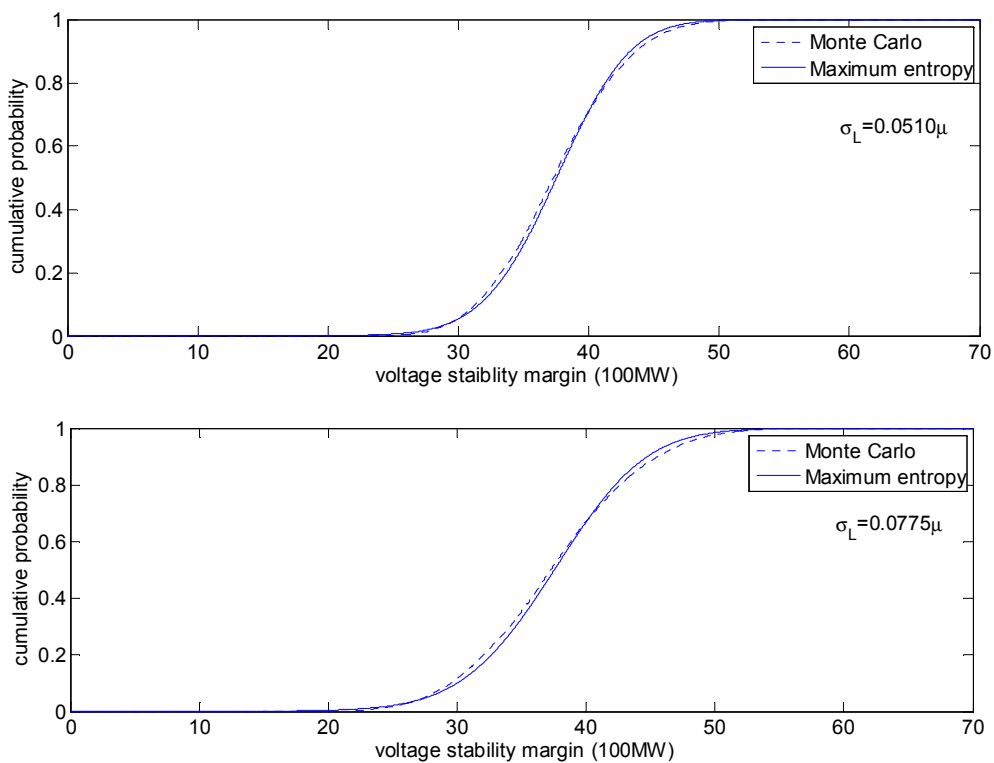
**Figure 2.** Cumulative probability of voltage stability margin of 39-bus system for uncorrelated wind speeds.

From Figures 1 to 6, we can see that PDF curves and cumulative probability curves of voltage stability margin by the two methods are very close, which shows that the maximum entropy method is effective and accurate to determine the

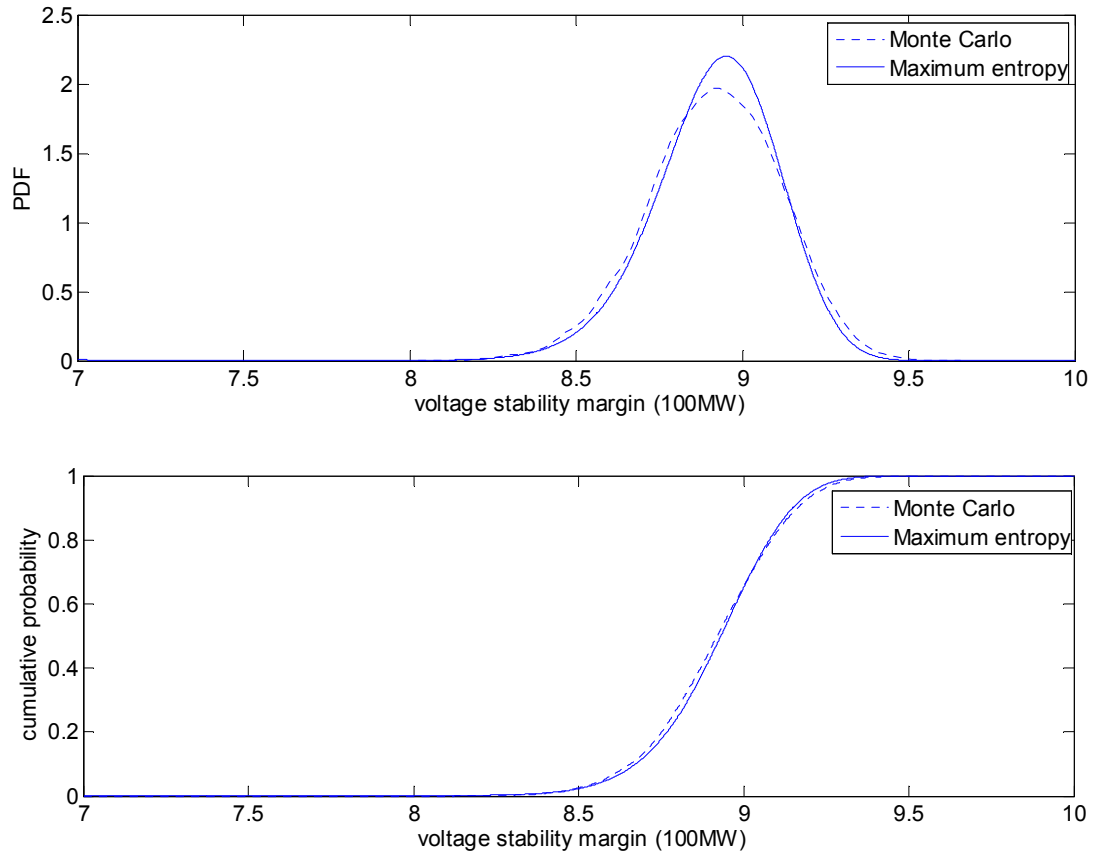
probabilistic distribution of voltage stability margin. In addition, it can be seen from Fig. 1-4 that when the standard deviation of the random variable increases, the distribution range of the voltage stability margin becomes wider.



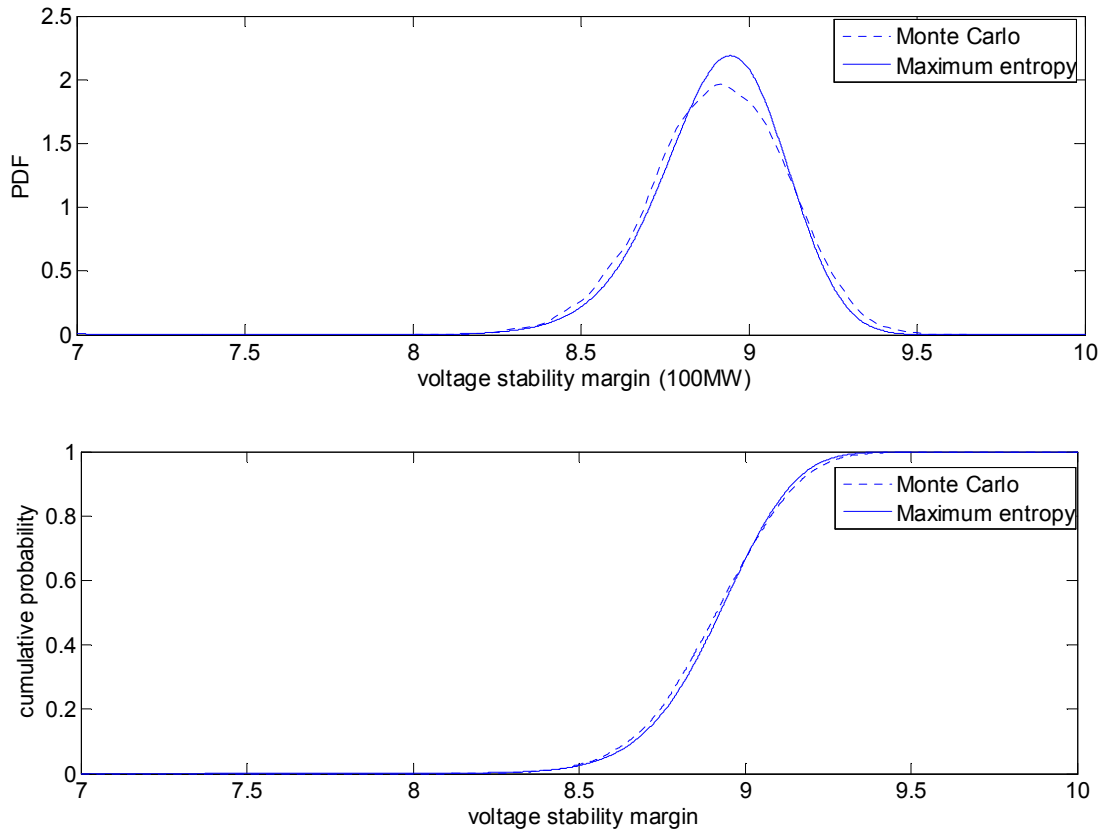
**Figure 3.** Probability density function of voltage stability margin of 39-bus system for correlated wind speeds.



**Figure 4.** Cumulative probability of voltage stability margin of 39-bus system for correlated wind speeds.



**Figure 5.** PDF and cumulative probability of voltage stability margin of IEEE 57-bus system for uncorrelated wind speeds.



**Figure 6.** PDF and cumulative probability of voltage stability margin of IEEE 57-bus system for correlated wind speeds.



## 5. Conclusion

In this paper, a method to calculate the probabilistic voltage stability margin of power system integrated with wind farm is proposed based on unscented transformation technology and maximum entropy principle. Unscented transformation technology is used to determine the statistical characteristics of probabilistic voltage stability margin, and the maximum entropy principle is used to determine the distribution of voltage stability margin. The uncertainties of wind farm output and random loads are considered. The effectiveness and accuracy of the proposed method are validated through comparing with the Monte Carlo method. The effect of standard deviation and correlation of random variables on voltage stability margin are discussed. Compared with Monte Carlo method, the computational time is reduced significantly, but with the increase of the number of random variables, the computational performance of this method will be reduced.

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