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Effect of Buoyancy Force on the Flow Field in a Square Cavity with Heated from Below

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Abstract: In this paper, the effect of buoyancy force in a square enclosure is studied numerically. The coupled equations of continuity, momentum and energy are solved by a finite difference method. The SIMPLE algorithm is used to solve iteratively the pressure-velocities coupling. The numerical investigations in this analysis are made over a wide range of parameters, Rayleigh number and dimensionless heater lengths. Results are presented graphically in the form of streamlines, isotherms and also with a velocity profiles and average Nusselt numbers.

Keywords: Natural Convection, Nusselt Number, Square Cavity, Numerical Simulation, Finite Difference Method, Heat Source, Streamline, Isotherm, Average Nusselt Numbers

1. Introduction

The buoyancy-driven flow in a square cavity with differentially heated walls is one of the least pursuedareas in finite element methods, although it has been an extensively explored area in finite difference methods. Physics involved in the buoyancy-driven flow inside a square domain has relevance to a variety of practical problems such as nuclear reactor insulation, ventilation of rooms, solar energy collection, crystal growth in liquids and convective heat transfer associated with boilers and electronics etc. Buoyancy-driven flows have added complexity in form of coupling between transport properties of the flow and the thermal fields.

Due to their practical importance as reviewed by [1] and [2]. Nearly forty years ago as in [3] and [4] Studied the natural convection in enclosures with internal heat generation occurs in nuclear reactors and geothermal heat extraction processes. Based on technological applications of internal heat generation problems in nuclear reactors and geometrical applications, as in [5] and [6] Obtained several solutions. Besides regular geometries such as square or rectangle, many studies wavy-walled enclosures with or without internal heat generation. Natural convection in wavy enclosures with volumetric heat sources was investigated by [7]. They found that, both the function of wavy wall and the ratio of internal Rayleigh number Ra1 to external Rayleigh number RaE affect the heat transfer and fluid flow significantly.

On the other hand, the study of heat transfer in porous media has also got attention of many researchers. Neild and Bejan [8] and Ingham and Pop [9] contributed to an extensive overview of this important area of heat transfer in porous media. The convective heat transfer in the square enclosures has been studied extensively on natural convection in cavities can be found in Ostrich [10]. Natural convection in a square cavity heated from below and cooled from one side has been studied by Anderson and Lauriat [11]. Lattice Boltzmann method was employed for investigation the effect of the heater location on flow pattern, heat transfer and entropy generation in a cavity Delavar and Sedighi [12].

Natural convection heat transfer in a square air-filled enclosure with one discrete flush heater is examined numerically by Radhwan and Zaki [13]. The optimum location over the range of Rayleigh number is for the heater mounted at the center of the wall, a result confirmed by previous experiments. The phenomena of natural convection in an inclined square enclosure heated via corner heater have been studied numerically by Varol et al [14]. One wall of the enclosure is isothermal but its temperature is colder than that of heaters while the remaining walls are adiabatic. It is observed that heat transfer is maximum or minimum depending on the inclination angle and depending on the length of the corner heaters. The effect of Prandtl number on mean Nusselt number is more significant for Pr < 1.

The same case has been studied Che Sidik [15] using finite difference double-distribution function thermal lattice Boltzmann model. The results obtained demonstrate that this approach is very efficient procedure to study flow and heat transfer in a differentially heated cavity flow. Results show that higher heat transfer was observed from the cold walls when the heater located on vertical wall. On the other hand, heat transfer increases from the heater surface when it is located on the horizontal wall.

Dimensionless heat transfer correlating equations are proposed. In this context, the main aim of the present paper is to study the thermal behavior of tilted square enclosures that was locally one discrete heated from below and with different locations of the heating portion mounted symmetrically on the two vertical sides. The study is carried out numerically through a computational code based on the SIMPLE algorithm, which is used for the solution of the mass, momentum and energy transfer governing equations. Simulations are performed for different values of the Rayleigh number Ra in the range between 10^2 to 10^5, and of the different heater locations (0.2 to 0.8). Most of the enclosures commonly used in industries are cylindrical, rectangular, and square trapezoidal and triangular etc. In recent years, square enclosures have received a considerable attention because of its applicability in various fields. In a square cavity analysis it is found that at low Rayleigh numbers (Ra ≤10^4), the isotherms are almost parallel near the bottom portion of the square enclosure while at Ra = 10^5, the isotherms are more distorted.

This is because the heat transfer is primarily due to conduction for lower values of Rayleigh number. As Rayleigh number increases, there is a change from conduction dominant region to convection dominant region, and the critical Rayleigh number corresponding to on-set of convection is obtained.

The main objective of this paper is to study natural convection heat transfer in a square shape enclosure. From the above literatures, the aim of present investigation is to investigate the effect of buoyancy force in square shape enclosure. The results are presented in terms of streamlines, isotherms, velocity profiles, temperature profiles and local Nusselt number.

2. Mathematical Formulation and Numerical Computation

The configuration of interest for the present study is shown in Fig. 1, which is two dimensional square enclosure with a side of length L and adiabatic top wall. The non dimensional governing equations are obtained with the following assumptions: The enclosure is completely filled with porous materials, Darcy's law is assumed to be hold, the saturated porous medium is assumed to be isotropic in thermal conductivity, the bottom wall has a centrally located heat source which is assumed to be isothermally heated at constant temperature T_H, side walls are isothermally cooled at a constant temperature T_C, while the bottom wall except the heated part and the top of the wall are considered to insulated. The fluid is permeated by a uniform magnetic field B which is applied normal to the direction of the flow and the gravitational force.

(g) acts in the vertically downward direction.

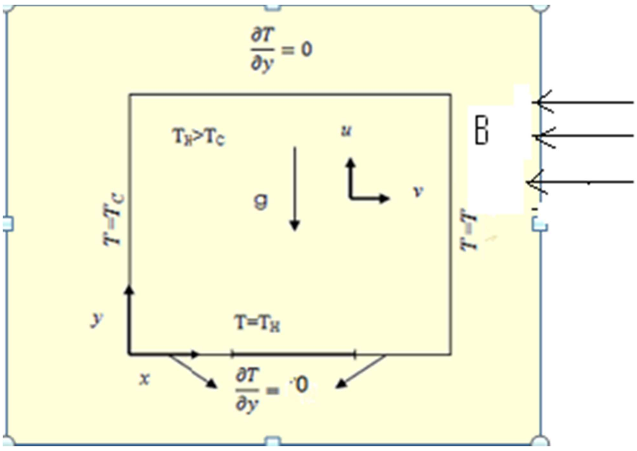

Figure 1. *Physical Model and coordinate system.*

The non dimensional governing equations in terms of the stream function ψ and the temperature θ are as follows.

$$\left(\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}\right\} = -\mathrm{Ra}\frac{\partial\theta}{\partial X} \tag{1}$$

$$\frac{\partial\theta}{\partial\tau}+\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y} = \left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right\} \tag{2}$$

Where the dimension variables are defined by

$$X=\frac{x}{L,}, Y=\frac{y}{L}, \tau=\frac{t}{\frac{L^2}{\alpha}}, U\frac{u}{\frac{\alpha}{L}} \tag{3}$$

$$V=\frac{v}{\frac{\alpha}{L}}, \varphi=\frac{\varphi}{\alpha}, \theta=\frac{T-T\_C}{T-T\_H}\ \mathrm{Ra}=\frac{g\beta\Delta TK}{\alpha v}$$

The non-dimensional stream function ψ, satisfies the following equations

$$U=\frac{\partial\Psi}{\partial Y}, V=-\frac{\partial\Psi}{\partial X} \tag{4}$$

Equation (1) and (2) are subject to the following boundary conditions

$$\Psi=\theta=0, \text{for } \tau=0$$

$$\Psi=\theta=0, \text{for } 0\le Y\le 1 \text{ at } X=0$$

$$\Psi=\theta=0, \text{for } 0\le Y\le 1 \text{ at } X=1$$

$$\Psi=\theta=1, \text{for } \frac{1-\epsilon}{2}\le X\le\frac{1+\epsilon}{2} \text{ at } Y=0$$

$$\Psi = 0, \frac{\partial \theta}{\partial Y} = 0 \text{ for}$$

$$0 < X < \frac{1-\epsilon}{2} \text{ and } \frac{1+\epsilon}{2} < X < 1 \; at \; Y = 0$$

$$\Psi = 0, \frac{\partial \theta}{\partial Y} = 0, \text{for } 0 \le X \le 1 \text{ at } Y = 0$$

Where ϵ is the non-dimensional heat source length.

Once we know the numerical values of the temperature function we may obtain the rate of heat tyransfer in terms of the local Nusselt number, Nu from the heated portion of the bottom wall using the following relation.

$$Nu = \left(\frac{\partial T}{\partial Y}\right)_{Y=0}$$

The average Nusselt number, Nu_{av} is given by

$$Nu_{av} = \int_{\frac{1-\epsilon}{2}}^{\frac{1+\epsilon}{2}} \left(\frac{\partial T}{\partial Y}\right)_{Y=0} dX$$

The governing equation (1)-(2) along with the boundary condition condition (5) are solved numerically, employing implicit finite difference method. the Poisson like momentum equation (1) and the Energy equation (2) are discretised using the central difference but time derivative is discretised using the three points backward difference formula to ensure the second order accuracy in both time and space, even though we have presented only steady state solution.

3. Numerical Setting

The streamlines and isotherms are shown in the below figure 1: for different values of Raleigh number $Ra = 10^2 \; to \; 10^5 \; for \; \epsilon = 0.4$. For higher Rayleigh number the streamlines as well as the isotherms are more dominant which are visualized in the contours.

The test cell is reproduced with real dimensions. The temperatures of the heated strips are assigned in order to obtain the same Rayleigh numbers as in the experimental analysis. The isothermal lines, streamlines and velocity maps from

$Ra = 10^2 \; to \; Ra = 10^5$. Analyzing this Figs. 2, it is observed that there is a good agreement between the experimental isothermal lines and the numerical pattern for all configurations under test. So in the configuration the heat transfer is higher than in other configurations for the same Rayleigh numbers. The maximum Values of stream lines 0.55, 2.82, 3.86 and 16.21.

For different heat source ϵ= 0.2 to 0.8 for $Ra = 10^4$ are shown in fig. 3: The fields are identical for different values of heat size. The maximum values of the stream function are 12.08, 16.21, 20.10 and 24.02. The isotherms are affected for increasing the heat source size. For varying the heat source keeping Ra constant, the flow fields are almost the same.

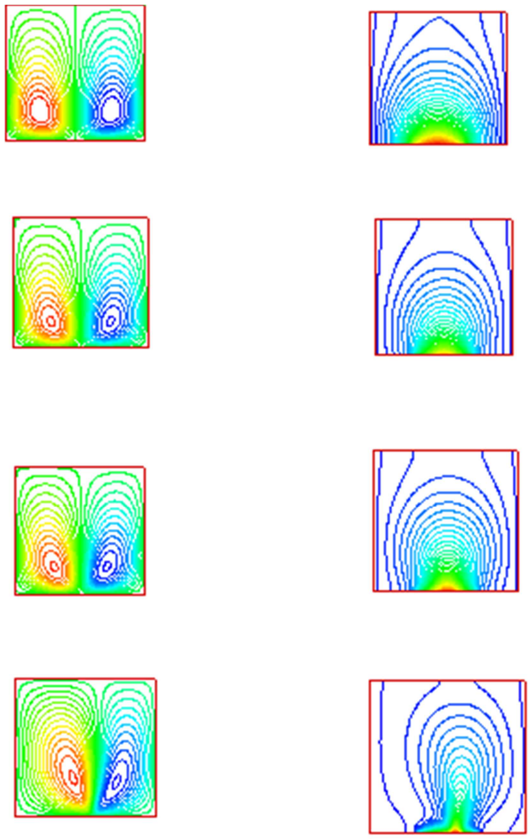

Figure 2. *Streamlines and isotherms for different Darcy's Rayleigh number Ra=10^1, Ra=5×10^2, Ra=10^3 and Ra =10^5 while the heat source length ϵ=0.4.*

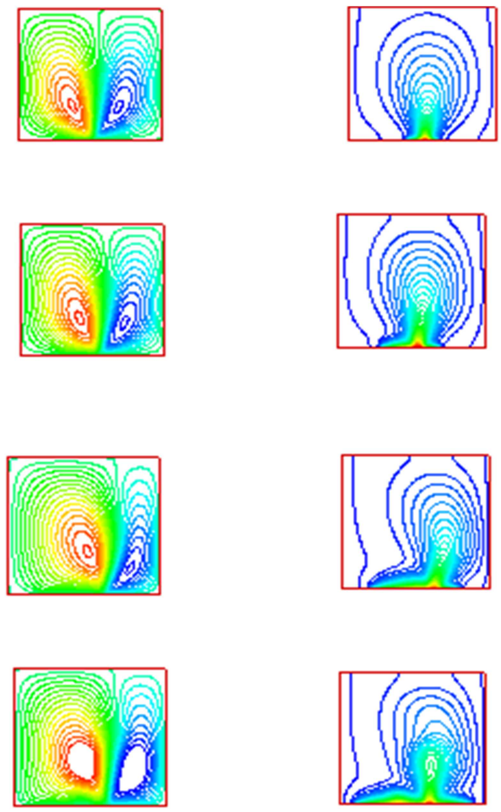

Figure 3. *Streamlines and isotherms for different heat source length ϵ=0.2 to 0.8 while Darcy's Rayleigh number Ra= 10^5.*

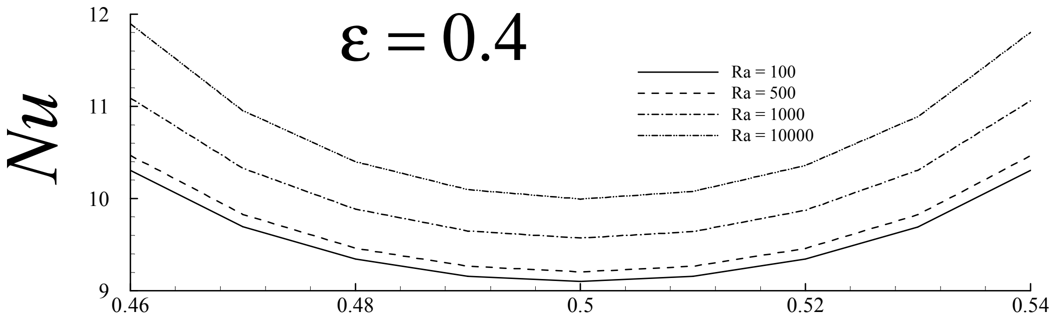

Figure 4. *The local nusselt numbers where ϵ=0.4 and Ra Varies from 10^2 to 10^4.*

Table 1. *For average Nusselt numbers.*

$\frac{Ra}{\epsilon}=$	10^2	5×10^2	10^3	10^4
0.2	4.13	6.11	8.58	25.01
0.4	4.27	6.05	8.21	21.18
0.6	4.12	5.98	4.87	18.39
0.8	4.12	5.90	8.31	17.40

4. Code Validation

The validation of the computations against suitable experimental data could not be performed. However, in order to validate the predictive capability and accuracy of the present code, three published works have been chosen.

Table 2. *Comparisons of present numerical values.*

Nu_av	$Ra = 10^2$	$Ra = 10^3$
Baytas and Pop	3.16	14.06
Moya et al	2.80	------
Mahmud and fraser	3.12	13.64
Present prediction	3.12	13.69

For validation purpose, a differentially heated square cavity has been considered. Average nusselt number has been calculated and shown in the table for different Darcy's Rayleigh number Ra $= 10^2, 10^3$ and compare with the earlier investigations. So the present numerical method and the presented results are very accurate.

5. Results and Discussions

The numerical results for the streamline and isothermals contours for various values of thermal Rayleigh number and the heater location are presented and discussed. In addition, the results for both average Nusselt, and velocity profiles, at various conditions are also discussed.

The symmetry in boundary conditions in the vertical walls, flow and heat transfer are controlled by the local heat source and the difference in temperature on the vertical walls.

It is observed that that the increase in temperature differences between the vertical walls (Ra number) affects the fluid dynamic behavior, increasing the intensity of the flow in the enclosure. As Ra increases 10^2 to 10^5, the buoyancy forces become strong and the heat transfer is dominated by convection for Pr=0.71. The stream lines and the isotherms are visualized for different value of Rayleigh number and different heat source size. The average nusselt number has also increased for increasing Rayleigh number which is shown in the table.

6. Conclusion

The main Parameter Rayleigh number (Ra) and discrete heat source size (ϵ) and the dependency on fluid and heat transfer have been discussed. The flow field and the isotherm are symmetric owing to the symmetric boundary condition for the parameters. The heat transfer enhances for the increases of Rayleigh number and discrete heat source size. Conduction is found dominant for low Rayleigh number and convection found for higher Rayleigh number.

Nomenclature

g: acceleration due to gravity
L: Enclosure height
K: permeability of the porous media
Nu: local Nusselt Number
Nu_{av}: Average nusselt number
Ra: Darcy-Rayleigh number
t: Dimensional time
T: fluid temperature
T_C: Temperature of the side walls
T_H: Temperature of localized heat source
x, y: Dimensional coordinates
X, Y: Dimensionless coordinates
u, v: Dimensional velocity components along x and y directions
U, V: Dimensional less velocity components along X and Ydirections

Greek Symbols

ψ: Dimensionless stream function
α: Thermal diffusivity of the fluid
β: Thermal expansion coefficient of the fluid
τ: Dimensionless time
ν: Kinematic viscosity of the fluid
ε: Dimensionless heat source length

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