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Floyd's Modified Computational Method Applied to Calculate Step Response of a Regilator from Frequency Response

Jose Flavio Feiteira, Jose Luiz Guarino, Rodrigo Guerra de Souza

Computational Modeling in Science and Technology, Universidade Federal Fluminense, Volta Redonda, Brazil

Email address:
galloguarino@gmail.com (J. L. Guarino), rodrigomcct@gmail.com (R. G. de Souza)

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Abstract: This work explains how to obtain the unit step time domain response by means of the frequency response of a regulator (gain and phase) using the Floyd's Modified Computational Method. The preliminary condition is that the gain of the system tends to zero as the frequency tends to infinite. Floyd's Method uses the Fourier's Inverse Transform to achieve the Impulse Unit response. The Modified Method calculates the integral. This work details the mathematical developnent of Floyd's Method. Authors introduce the integral of the Impulse Unit response to obtain the Step Unit response and also the linearization of the Method in order to approximate it and obtain an equation to do the computational calculation. We apply the modified method in a second order system, calculating its frequency response and its analytic step unit response by means of the MNatlab. Then we use the equation developed in this work by the linearization of Floyd's Modified Method applied in the frequency response of the system and compare with the step unit analytic response. The relative error is calculated and we can observe that Floyd's Modified Method generates a step unit response in the time domain that has some time little retard and with values a little inferior to the analytic response. This behavior is attributed to the linearization and to do not use the complete frequency band of the system. However the final values are very exact. T.

Keywords: Floyd's Method, Impulse Unit Response, Step Unit Response, Frequency Domain, Time Domain, Fourier's Inverse Transform

1. Introduction

Figure 1 below shows a canonical form of a regulator, where X(s) is the reference, input, and the output is Y(s), which we desire to control.

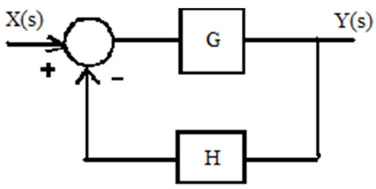

Figure 1. *Canonical System.*

Normally, there is no direct relation between the frequency response and the step unit response of regulators. As a rule of thumb we inject a 10% of the step input in the biased nominal input to determine the fulfillment of the system and to avoid saturations. [3-8]

This work, has the principle that having the response of the regulator in the frequency domain we wish to obtain the step unit response in the time domain i.e. the Floyd Method

Another objective of this work is to realize the Floyd's Modified Method with the Matlab.

We demonstrate the method mathematically, applying it to a second order system and analyze the errors. However this method is restrict to a regulator with closed loop gain G(w) following the equation below, where w is the frequency in rad/s:

$$\lim_{w\to\infty} G(w) = 0$$

2. Mathematical Discussion

2.1. Impulse Unit Response [2, 10-12]

Fourier's Inverse Transform is:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)e^{jwt}\, dw \quad (1)$$

Supposing an impulse unit step $\delta(t)$ to the system G, as shown in figure 2, the Fourier's Transform of the impulse unit is:

$$\Delta(w) = \int_{-\infty}^{\infty} \delta(t)e^{-jwt}\, dt$$

$$\Delta(w) = 1$$

where Δ(w) is the impulse unit response in the frequency domain.

This way in the frequency domain we have:

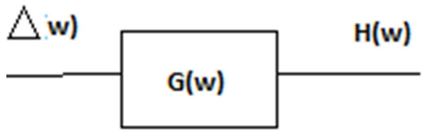

Figure 2. *Impulse response frequency domain.*

$$H(w) = \Delta(w).G(w)$$

$$H(w) = G(w)$$

We can say that the response of the system to the impulse unit is the system itself.

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(w)e^{jwt}\, dw$$

$$G(w) = G\, cos\theta + j\, G\, sin\theta$$

$$e^{jwt} = \cos(wt) + j\, sin(wt)$$

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [(G\, cos\theta + j\, G\, sin\theta).(\cos(wt) + j\, sin(wt))]\, dw$$

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [(G\, cos\theta \cos(wt) + j\, G\, cos\theta\, sin(wt) + j\, G\, sin\theta \cos(wt) - G\, sin\theta\, sin(wt)]\, dw$$

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [(G\, cos\theta \cos(wt) - \ G\, sin\theta\, sin(wt) + j\, G\, cos\theta \sin(wt) + j\, G\, sin\theta\, cos(wt)]dw$$

But, G cosθ is an even function and sin(wt) is an odd function so, the product is an odd function and the integral is null G sinθ is an odd function and cos(wt) is an even function; so the product is an odd function.

Then the integral is null.

G cosΘ cos(wt) is an even function as the same way G sinθ sin(wt).

So the equation is reduced to:

$$g(t) = \frac{1}{\pi}\int_{0}^{\infty} [(G\, cos\theta \cos(wt) - \ G\, sin\theta\, sin(wt)]dw \quad (2)$$

What is logical, because g (t) is a function in the domain of Real numbers. There is another simplification, from the Theory of Complex Numbers:

$$g(t) = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} [\, G(s)e^{st}\,]ds$$

To calculate this integral, with its distribution on the imaginary axis to close path, using a half circle with an infinite radius in the right side of the complex plan; the value of the integral is zero to all singularities that are to the left of imaginary axis. Then to (2), we have:

$$\int_0^{\infty}[G\, cos\theta\, cos(wt)]dw = \int_0^{\infty}[G\, sen\theta\, sin(wt)]dw; \quad to\, t\ < 0 \quad (3)$$

However by inspection, we verify that the left side of (3) is an even function, although the right side is an odd function, so we have for all $t > 0$:

$$\int_{0}^{\infty} [G\, cos\theta\, cos(wt)]dw = -\int_{0}^{\infty} [G\, sen\theta\, sen(wt)]dw$$

So we can rewrite (2),

$$g(t) = \frac{2}{\pi}\int_{0}^{\infty} [(G\, cos\theta\, cos(wt)]dw \quad (4)$$

This integral exists, since:

$$\lim_{w\to\infty} G(w) = 0$$

2.2. Step Unit Response [1-2]

Until now it was Floyd's Method. From now one we introduce some modifications to calculate the Step Unit Response. We know that the step unit function $r(t)$ is the integral of the impulse unit function, $h\,(t)$. So the step unit response will be the integral of the impulse unit response:

$$r(t) = \int_0^t g(t)\, dt$$

$$r(t) = \frac{2}{\pi}\int_0^t \int_0^\infty [G\, cos\theta \cos(w\lambda)]\, dw\ \, d\lambda$$

$$r(t) = \frac{2}{\pi}\int_0^\infty \int_0^t [G\, cos\theta \cos(w\lambda)]\, d\lambda\ \, dw$$

$$r(t) = \frac{2}{\pi}\int_0^\infty [G\, cos\theta\ \frac{sin(w\lambda)}{w}\Big|_0^t]\, dw$$

$$r(t) = \frac{2}{\pi}\int_0^\infty G\, cos\theta\ \frac{sin(wt)}{w}\ dw \tag{5}$$

2.3. Linearization of the Integral

Imposing that $G\, cos\theta$ were linearized by sectors in the frequency band B, we have that equation (5) becomes the following equation:

$$r(t) = \frac{2}{\pi}\sum_{w=0}^{B}\int_{w_i}^{w_j} \frac{G\, cos\theta\ sin(wt)}{w}\, dw$$

We can consider G cosθ sin the linear form aw+b. Then:

$$r(t) = \frac{2}{\pi}\sum_{w=0}^{B}\int_{w_i}^{w_j} \frac{(a_i w + b_i)\ sin(wt)}{w}\, dw$$

$$r(t) = \frac{2}{\pi}\sum_{w=0}^{B}\left[a_i\int_{w_i}^{w_j} sin(wt) dw + b_i\int_{w_i}^{w_j}\frac{sin(wt)}{w}\, dw\right]$$

$$r(t) = -\frac{2}{\pi}\sum_{w=0}^{B}\frac{a_i\ cos(wt)}{t}\Big|_{w_i}^{w_j} + \frac{2}{\pi}\sum_{w=0}^{B} b_i\int_{w_i}^{w_j}\frac{sin(wt)}{w}\, dw$$

Calculating this integral,

$$\int_{w_i}^{w_j}\frac{sin(wt)}{w}\, dw;\ doing\ wt = x\ and\ dw = \frac{dx}{t}$$

$$\int_{w_i}^{w_j}\frac{sin(wt)}{w}\, dw = \int_{w_it}^{w_jt}\frac{sin(x)}{\frac{x}{t}}\frac{dx}{t} = \int_{w_it}^{w_jt}\frac{sin(x)}{x}\, dx$$

Then

$$\int_{w_it}^{w_jt}\frac{sin(x)}{x}\, dx = \int_0^{w_jt}\frac{sin(x)}{x}\, dx - \int_0^{w_it}\frac{sin(x)}{x}\, dx$$

$$The\ function \int_{w_it}^{w_jt}\frac{sin(x)}{x}\, dx\ is\ known\ as\ Integral - sinus, or\ Si(x)$$

Then

$$r(t) = -\frac{2}{\pi}\sum_{w=0}^{B}\left\{\frac{a_i[\cos(w_i t - \cos(w_j t))]}{t}\right\} + \frac{2}{\pi}\sum_{w=0}^{B}\{b_i[\mathrm{Si}(w_j t) - \mathrm{Si}(w_i t)]\} \tag{6}$$

2.4. Determination of the Angular and Linear Coefficients of the Linearization

In figure 3, we have:

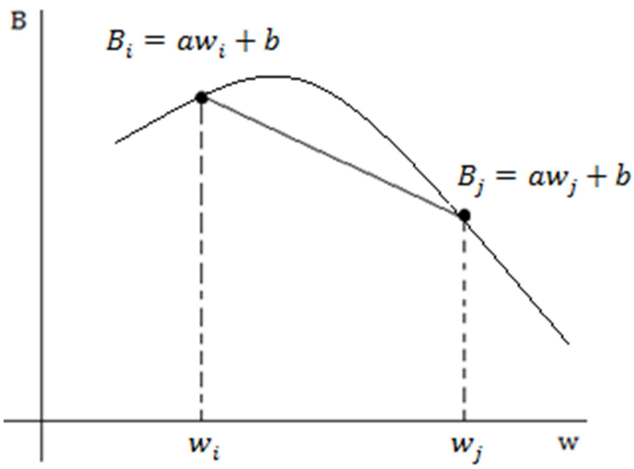

Figure 3. *GcosΘ graphic.*

$$B_i = G_i\cos(\theta_i) = aw_i + b \tag{7}$$

$$B_j = G_j\cos(\theta_j) = aw_j + b \tag{8}$$

If we do (7) – (8), we have:

$$B_i - B_j = a(w_i - w_j)$$

$$a = \frac{B_i - B_j}{w_i - w_j} \tag{9}$$

$$Putting\ (9)\ em\ (7), we\ determine\ the\ coefficients$$

$$B_i = \left(\frac{B_i - B_j}{w_i - w_j}\right) w_i + b$$

$$B_i w_i - B_i w_j = B_i w_i - B_j w_i + b(w_i - w_j)$$

$$So, b = \frac{B_j w_i - B_i w_j}{w_i - w_j}.$$

Then the equation of the straight right is:

$$B = \left(\frac{B_i - B_j}{w_i - w_j}\right) w + \left(\frac{B_j w_i - B_i w_j}{w_i - w_j}\right) \quad (10)$$

Putting (7) and (8) in (10) we have the final equation:

$$r(t) = \frac{2}{\pi} \sum_{w=0}^{B} \left[\left(\frac{G_j \cos(\theta_j) - G_i \cos(\theta_i)}{w_i - w_j} \right) . \left(\frac{\cos(w_j t) - \cos(w_i t)}{t} \right) + \left(\frac{w_i G_j \cos(\theta_j) - w_j G_i \cos(\theta_i)}{w_i - w_j} \right) . \left(\mathrm{Si}(w_j t) - \mathrm{Si}(w_i t) \right) \right] \quad (11)$$

2.5. Realization [6, 9, 13-17]

We have the second order system whose expression in the frequency domain (Laplace Transform) is:

$$G(s) = \frac{w_n{}^2}{s^2 + 2\zeta w_n s + w_n{}^2} \quad (12)$$

We make $\zeta = 0{,}2$ and $w_n = 1$ rad/s. Making $s = jw$.
It follows the graphic of Gain and Phase of the above system given using Matlab:

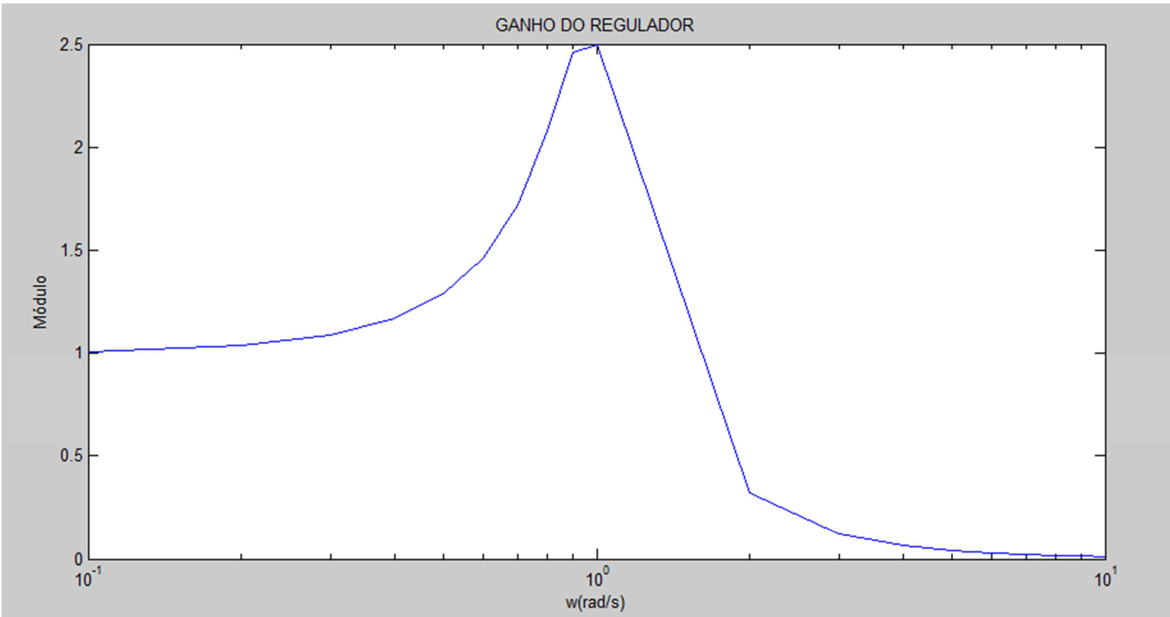

Figure 4. *Gain of the second order system.*

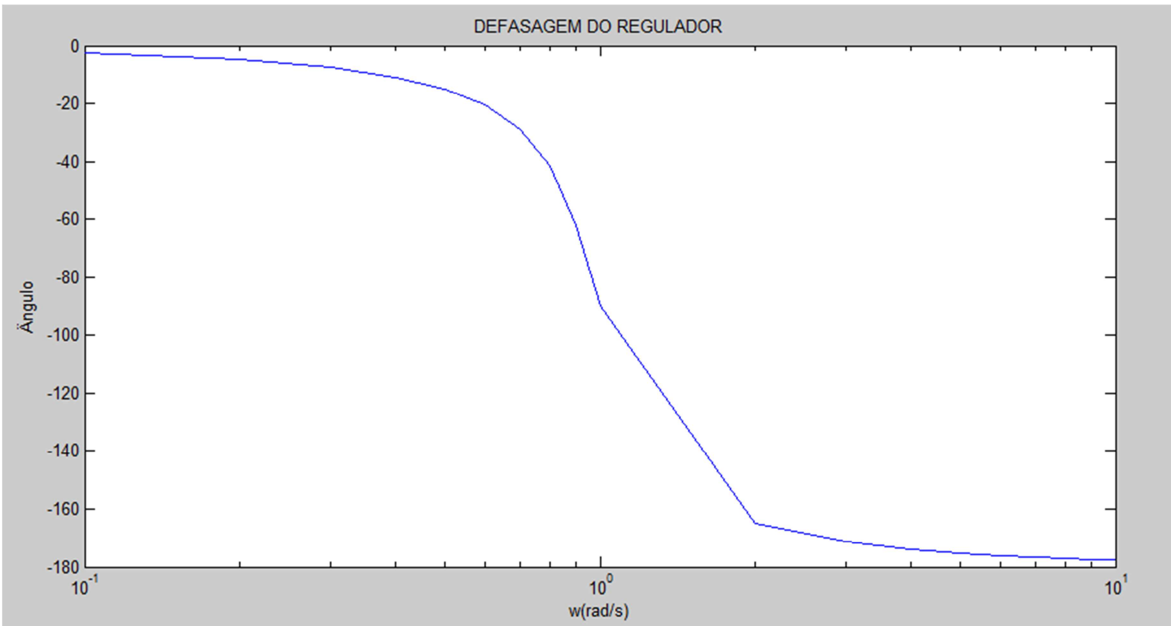

Figure 5. *Phase of the second order system.*

In the case we have the frequency, f in Hertz (Hz), we should convert:

$$w = 2\pi f\ (rad/s)$$

In the case we have the gain (g) in Decibell (dB), we should convert, $G = 10^{\frac{g}{20}}$.

Using Matlab code we calculate the step unit response $r(t)$ for each t, varying from 1 to 20 by Floyd's Modified Method employing (11) (the cross).

```
% Escopo
clear all; clc; close all;
%
w=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 2 3 4 5 6 7 8 9 10];
[G,mG,aG]=respfreq(w);
t=1:20;
[lims]=geramatriz(w(t));
for m=1:20
    for k=1:20
        senointegral(m,k)=quad('senodivx',0,lims(m,k));
    end
end
for k=1:20
    senointegral(k,1)=0;
end
for t=1:20
    respostafinal(t)=0;
    for k=1:19
        degrau=floyd(t,w,k,senointegral,mG,aG);
        respostafinal(t)=respostafinal(t)+degrau;
    end
end
t=1:20;
plot(t,respostafinal(t));
title('RESPOSTA AO DEGRAU UNITARIO NO DOMINIO DO TEMPO');
xlabel('tempo(s)'); ylabel('r(t)')
function [G,mG,aG]=respfreq(w)

G = 1./(-w.^2+1+j.*0.4.*w);
mG = abs(G);
aG = angle(G);

function [lims] = geramatriz(w)

for t = 1:20;
    for m = 1:20;
        lims(t,m)=t.*w(m);
    end
end
function [resposta] = floyd(t,w,k,senointegral,mG,aG)

auxa=((mG(k).*cos(aG(k))-mG(k+1).*cos(aG(k+1)))./(w(k)-w(k+1))).*(cos(w(k).*t)-cos((w(k+1).*t)))./t;
%
auxb=(w(k).*mG(k+1).*cos(aG(k+1))-w(k+1).*mG(k).*cos(aG(k)))./(w(k)-w(k+1)).*(senointegral(t,k+1)-senointegral(t,k));
%
resposta=(auxa+auxb).*2./pi;
function [razao] = senodivx(x)

razao = (sin(x))./x;
```

Note: is of fundamental importance to start with gain and phase with w=0, otherwise we find incoherent results.

The analytic step unit response of the system (12) is:

$$r(t) = 1 - \frac{1}{\beta}e^{-\zeta w_n t}sin(w_n\beta t\ +\ \theta) \qquad (13)$$

where,

$$\beta = \sqrt{1-\zeta^2}\ and\ \theta = arc\ tan\frac{\beta}{\zeta}$$

And we compare the computational result of equation (11) with (13) (the continuous line) and plot them in the figure 6 bellow.

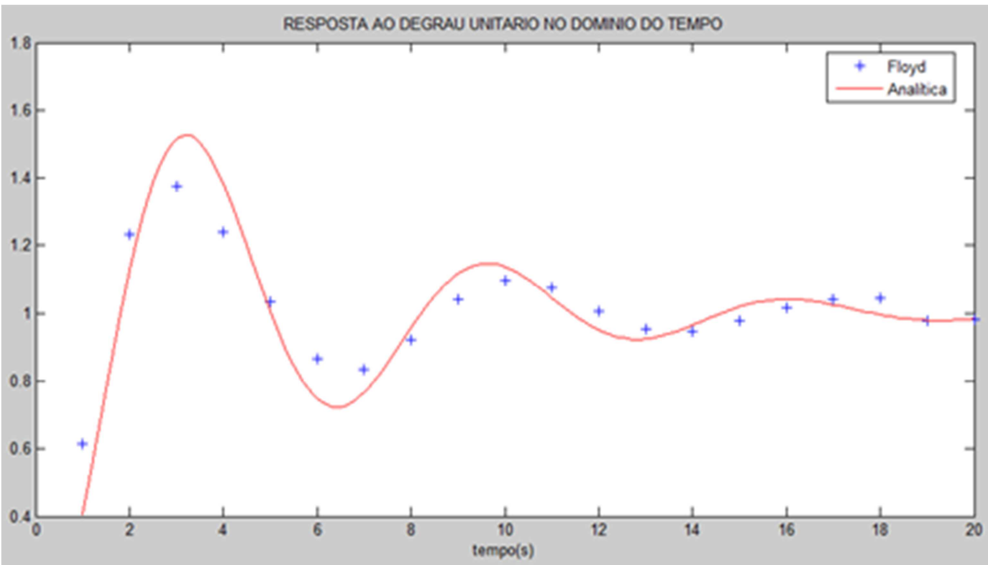

Figure 6. *Step Response.*

2.6. Error Analysis

The table 1 bellow shows the relative error between the analytic response and linearization of Floyd's Modified Method.

Table 1. *Relative Error-%.*

Time (s)	error (%)
1	51,7522
2	9,3152
3	-9,2008
4	-10,5220
5	2,7339
6	15,9879
7	8,9201
8	-3,4275
9	-6,9685
10	-3,6313
11	2,7998
12	5,8672
13	3,0082
14	-1,9419
15	-3,9843
16	-2,2213
17	1,4295
18	4,7976
19	-0,1563
20	-0,3408

3. Conclusion

With the results attained, we observe that with the linearization of the Floyd's Modified Method we have some little retard of the time, and inferior values, since we do not consider the whole curve of frequency response and we linearize i too. But to final values the Floyd's Method is good.

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