



Paradoxes the Model of an RLC Circuit

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Abstract: The special point is found in frequency characteristics of one model of an oscillatory contour about which it wasn't reported earlier in the scientific literature. The point settles down at a frequency, the smaller resonant frequency of a contour. The module of an impedance doesn't depend on the size of active resistance at this frequency. Physical interpretation of the phenomenon doesn't exist at the moment and it is main question. The aim of this article is the complete description of the properties system with the observed effect - results of calculations of existence region of the module are for the first time given in it. For comparison two models of an oscillatory contour are considered. Existence region of the impedance is essentially different for the considered models. Phase charts of an impedance are calculated with same aim. Results of the research can be used in the electrical and the electronic engineering and textbooks for profound studying of electrical equipment.

Keywords: RLC Circuit, Phase Chart, Impedance, Resonance of Current

1. Introduction

One of the basic elements of the radio engineering devices is the oscillatory contour. Two models of a parallel circuit normally covered in numerous textbooks and reference books on electrical engineering (Figure 1). Model (type A) is more popular than (type B), despite the fact that it is worse (than B) corresponds to the actual oscillation circuit, because the real coil is replaced by a series connected active and reactive resistance. The resonant frequency (type A) is the same for any value of active resistance.

The analysis of contour (type B) is usually limited to the statement that its resonant frequency decreases with the growth of size of active resistance [1-7].

Results of research of contour frequency characteristics (type B) were presented [8]. The special point or a point of stability is found in frequency characteristics of the module of complex resistance of this chain. Special point settles down at a frequency lower, then the frequency of resonance. The module of resistance doesn't depend on the size of active resistance. The author have studied a great number of textbooks on physics and electrical engineering and Internet search engines, but didn't find any source that mentions this the effect. As a result the article has no references to previous

work, because all the textbooks in electrical engineering and physics equal to each other in the part where there is no information about the detected effect.

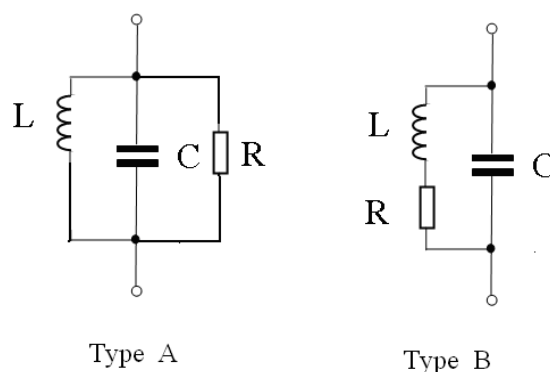


Figure 1. Two models of an oscillatory contour.

On the basis of the effect the mechanical model [9-10] was offered. The module of mechanical resistance of the model also didn't depend on the value of coefficient of viscosity at a certain frequency.

Since the question of the physical nature of the effect remains unresolved, the author of this article do attempt the most detailed descriptions of the properties of a this system.

Both types of circuit reviewed to identify major difference between the two models.

Results of calculations are given existence region of the relative module impedance. Circular charts various models are given in the wide range of frequencies. Formulas are given for the dependence of shift and radius of a phase semi-circle.

2. Analysis of Model A

The model of a contour (Figure 1A) is formed from in parallel the connected elements. The impedance (in a dimensionless form) of a chain can be presented in the following form:

$$\bar{Z} = \frac{j \bar{\omega} \bar{r}}{(1 - \bar{\omega}^2) \bar{r} + j \bar{\omega}} \quad (1)$$

where \bar{Z} - relation of the impedance to the characteristic resistance of a contour, equal $\sqrt{L/C}$,
 $\bar{\omega}$ - the frequency relation to resonant frequency,
 \bar{r} - the relation of active resistance to the contour characteristic resistance contour.

Existence region of the impedance module (type A) is presented on Figure 2.

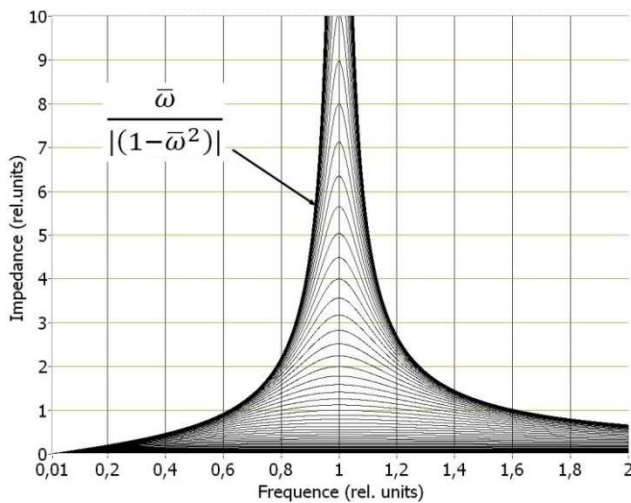


Figure 2. Existence region of the impedance module (Type A).

The existence region of the impedance module is limited from below by abscissa axis (at $\bar{r} = 0$), and from above by the asymptote $\frac{\bar{\omega}}{|(1 - \bar{\omega}^2)|}$, corresponding $\bar{r} \rightarrow \infty$, noted by the dashed line on the picture. Curves of an impedance come nearer to the second asymptote, differing from them in the most part in the area of a maximum.

The phase surface of model A is presented on Figure 3. It formed by the semi-circular charts of an impedance changing dependences on relative frequency. According to Figure 3 semi-circular charts include the point corresponding zero an impedance.

At each frequency the phase chart represents a semi-circle the radius $\frac{\bar{\omega}}{2|(1 - \bar{\omega}^2)|}$, which center is displaced at a radius size in a positive or negative side depending on a difference sign

between this and resonant frequencies.

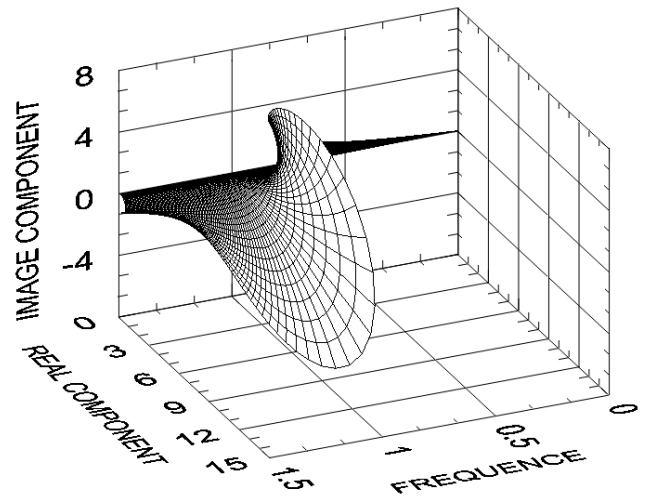


Figure 3. Phase charts of a parallel contour (type A).

3. Analysis of Model B

3.1. Module of Impedance

The existence region of the impedance module (Type B) is presented on Figure 3. This contour consists of the condenser and the inductance coil, modelled by the consistently connected active resistance and reactance. The impedance of a such contour can be presented in the form:

$$\bar{Z} = \frac{j \bar{\omega} + \bar{r}}{(1 - \bar{\omega}^2) + j \bar{\omega} \bar{r}} \quad (2)$$

where \bar{Z} , $\bar{\omega}$ and \bar{r} - such dimensionless sizes of an impedance, frequency and active resistance, as for a contour of type 1A.

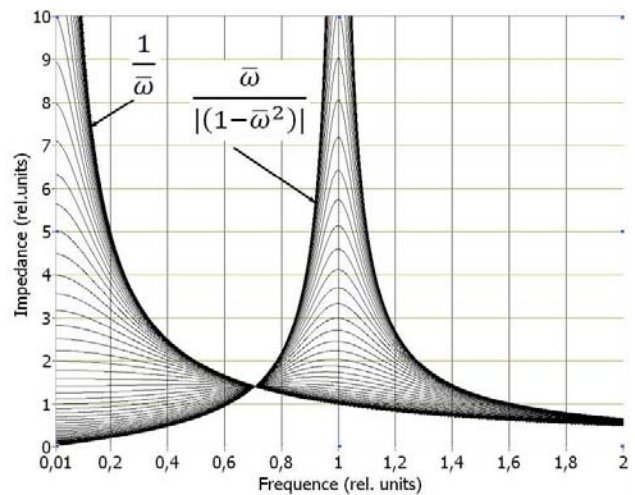


Figure 4. Existence region of the impedance module 1B.

The frequency characteristic of an impedance, significantly differs from the dependence on the Figure 2. The existence region of the impedance module is limited by two asymptotes noted on the figure by dashed lines:

1. asymptote $\frac{\bar{\omega}}{|(1-\bar{\omega}^2)|}$ ($\bar{r}=0$),
2. asymptote $\frac{1}{\bar{\omega}}$ ($\bar{r} \rightarrow \infty$).

All the curves of an impedance module (and asymptotes) have the general point with value of the relative module $\sqrt{2}$ at the relative frequency equal $1/\sqrt{2}$. The existence region of the module from above is limited by the second asymptote at low frequencies, and from below the first, and for frequencies is higher critical on the contrary.

Note that the product of frequency stable points on the value of the module is equal to the product of the resonance frequency to the characteristic impedance. It is possible that this fact will be useful to solve the main problem.

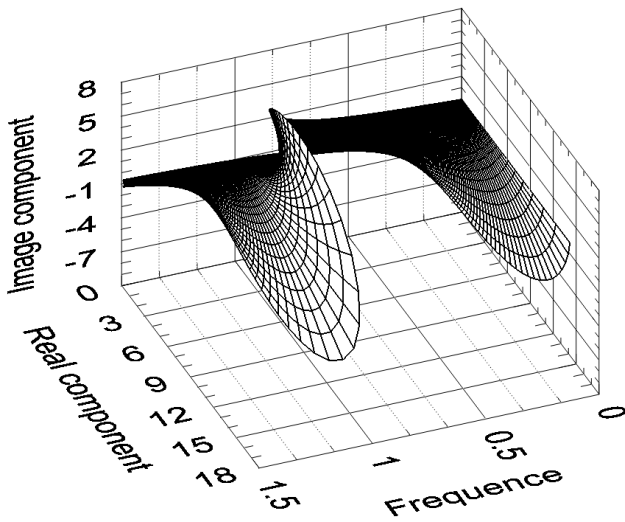


Figure 5. Surface of the phase charts (type B).

The phase surface of the impedance (type B) is given depending on the frequency. The chart is formed by semi-circles which radius is equal $\frac{1}{|2\bar{\omega}(1-\bar{\omega}^2)|}$, and the center is displaced on the distance $\frac{2\bar{\omega}^2-1}{|2\bar{\omega}(1-\bar{\omega}^2)|}$. At the critical point the phase chart represents the notdisplaced semi-circle with a radius equal $\sqrt{2}$.

3.2. The Imaginary Part of the Impedance

The components of the impedance calculated depending on the relative frequency and the relative resistance in the course of studying this issue. The frequency was normalized by the resonance frequency and the resistance and impedance of components at.

$$Im = \frac{\bar{\omega}(1-\bar{r}^2-\bar{\omega}^2)}{(1-\bar{\omega}^2)^2 + \bar{r}^2 \bar{\omega}^2},$$

where $\bar{\omega}$ is normalized frequency, \bar{r} - is the normalized resistance. The computation results are shown in Figure 6. Features include a special point located at the resonant frequency. The imaginary component does not depend on the resistance and equal to -1 at this point.

The three asymptotes form the region of existence of imaginary components

1. asymptote $-\frac{1}{\bar{\omega}}$, when $\bar{r} \rightarrow \infty$,
2. asymptote $\frac{\bar{\omega}}{1-\bar{\omega}^2}$, when $\bar{r} \rightarrow 0$,
3. cut the y-axis from 0 до $-\infty$.

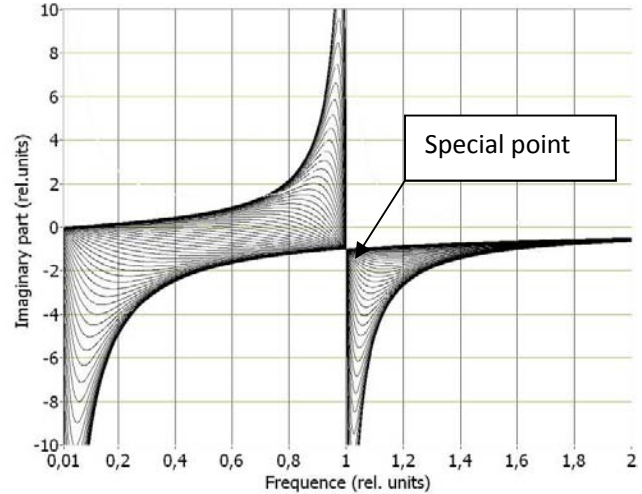


Figure 6. Dependence of the imaginary components of impedance against frequency and normalized resistance.

Two asymptotes are marked by fat lines in Figure 6.

Special points investigated in these characteristics: the extreme of the imaginary components of the frequency. The results are shown in Figure 7. The maximum of the imaginary part of impedance is present only in the range of relative resistances from 0 to 1. The minimum of imaginary components is present throughout the range of relative resistances. The characteristic point associated with the factor $\sqrt{2}$, which arises in the analysis of the dependence of impedance on frequency [8]. The minimum is the resonance frequency at this value of relative resistance, as well as at zero resistance value. When the resistance is $1/\sqrt{2}$, the minimum is at the maximum possible frequency in excess of approximately 1.1719 times the resonant frequency. The last digit occurs in the numerical solution of the cubic equation.

3.3. The Real Part of the Impedance

The real component of the impedance of such a circuit - Re , normalized by the characteristic loop resistance, is calculated according to the formula

$$Re = \frac{\bar{r}}{(1-\bar{\omega}^2)^2 + \bar{r}^2 \bar{\omega}^2}$$

where $\bar{\omega}$ and \bar{r} are the normalized parameters introduced earlier. The result of calculation (Figure 8)

The two asymptotes form the region of existence of real component:

1. asymptote $-\frac{1}{2\bar{\omega}|1-\bar{\omega}^2|}$
2. the x-axis,

The first asymptote has a local minimum at the relative

frequency $\frac{1}{\sqrt{3}}$. The real part of the impedance is equal $\frac{3\sqrt{3}}{4}$ at this frequency.

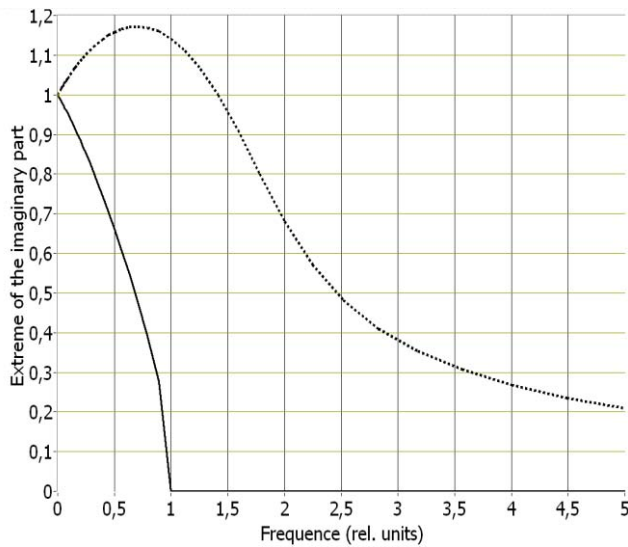


Figure 7. The dependence of the frequency maximum of the imaginary components of impedance against normalized resistance (solid line) and minimum (dotted line).

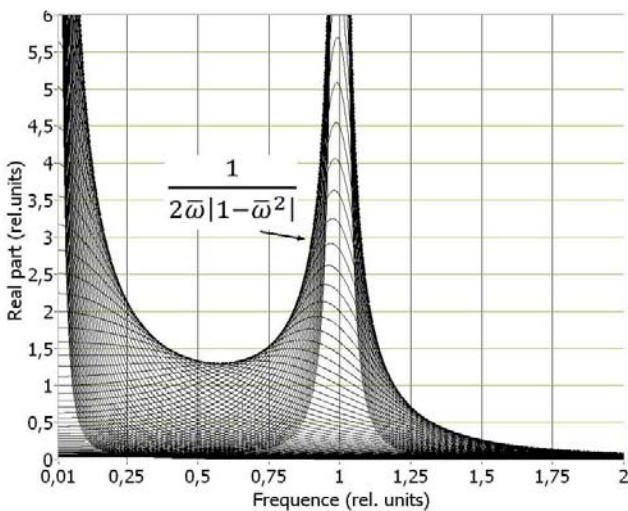


Figure 8. Real part of impedance RCL circuit.

A local minimum occurs when the value of the relative resistance is equal to $\frac{2}{\sqrt{3}}$.

3D diagram of the real part of the impedance is shown in Figure 9. In the diagram, a logarithmic scale is used for relative resistance.

The shape of the contour is determined by the magnitude of real component of impedance.

3.4. Comparison of Natural Frequencies and Maximum Impedance

Note that the frequency of maximum impedance does not coincide with the natural frequency of the circuit.

Calculations of the maximum impedance and frequency of natural (damped) oscillations of the circuit shown in Figure 10.

As can be seen from the figure, these dependences differ significantly from each other. The region of existence of maximum impedance corresponds to the segment of the abscissa from 0 to $\sqrt{1+\sqrt{2}}$, while damped oscillations in the circuit is possible with resistance values from 0 to 2 problem.

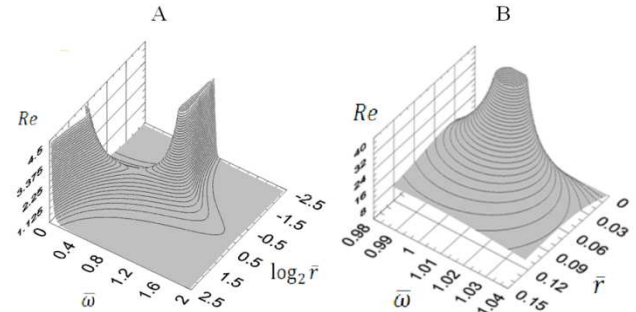


Figure 9. Diagrams the real component of the impedance against frequency and relative resistance of the circuit (large variables ($\tilde{r} \gg 1$) and small ($\tilde{r} \ll 1$) - right).

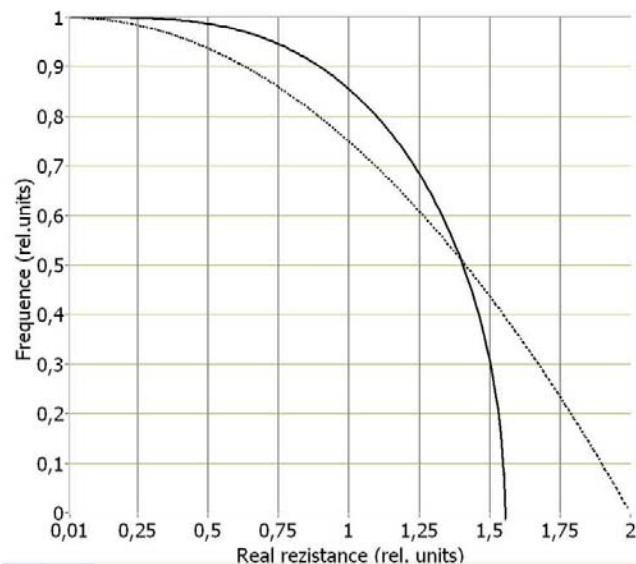


Figure 10. The dependence of the frequency of maximum impedance (solid line) and natural frequency (dotted line) of the normalized resistance.

All calculations and plotting were performed using the program LabView 8.0.

Since the observed effect stability remained unnoticed by researchers, we can expect that it will find application in measurement technology in the development of fundamentally new methods of measurement.

In particular, it is possible to offer additional method of measuring the resonant frequency at which you need to fix the frequency stability, sequentially connecting the coil and additional resistance. After determining the frequency stability of the module, the resonant frequency can be found by simple multiplication. The large value of the error in the difference of the measured frequency from the frequency stability can provide greater accuracy than measuring the maximum signal at the resonant frequency.

Method of determining the reactive component of high-resistance resistors serial connection involves investigational

resistor and incremental variable resistor. Then this circuit is connected in parallel with a capacitor. Point stability is determined when the incremental change in the magnitude of the resistor. Reactive component calculated by the frequency stability.

4. Conclusions

The properties of the system studied, which reveals the stability of the impedance on the path to understanding the nature of this effect. Regardless of the decision of the main question, obviously, was found the effect of a new fundamental knowledge in electrical engineering. It deserves to be known to specialists in this field.

Existence region of the impedance module are various for contours of different types. The lower bound of the module is defined for the contour of type B. In the result of research of the surfaces formed by the phase charts it is shown that charts have a semi-circle form in the wide range of frequencies. Basic distinction of the the phase charts is revealed for the contour different models. This results is obviously important for researches of the chains containing the oscillatory contour and also when studying electrical equipment and radio engineering.

The founded effect may find application in measurement technology in the development of new techniques, for example, additional monitoring of the resonant frequency or measurement of the reactive component of the high-resistance resistors

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