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Research Article

Linear Filtering and Total Variation De-noising of Composite Noises: Different Architectures and Simulation Study

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Abstract: Combined techniques consisting of traditional linear filtering and total variation de-noising are studied to de-noise images polluted with composite noises from different sources. Series and parallel approaches are compared theoretically. Some further discussions are given about parameters and their effect on the optimization problem. A simultaneous method is proposed and simulated using the convex optimization toolbox, comparing different alternative methods with respect to signal to noise ratio and error covariance matrix. The proposed method could be used to enhance the smoothing performance.

Keywords: Image Processing, Total Variation De-noising, Total Variation Smoothing, Linear Filter, Combined Linear Filtering and Total Variation De-noising

1. Introduction

The need for efficient image restoration methods has grown with the massive production of digital images and movies of all kinds, often taken in poor conditions. No matter how good cameras are, an image improvement is always desirable to extend their range of action. A digital image is generally encoded as a matrix of grey level or color values. In the case of a movie, this matrix has three dimensions, the third one corresponding to time. The two main limitations in image accuracy are categorized as blur and noise. Blur is intrinsic to image acquisition systems, as digital images have a finite number of samples and must satisfy the Shannon-Nyquist sampling conditions. The second main image perturbation is noise. A good quality photograph (for visual inspection) has about 256 grey level values, where 0 represents black and 255 represents white. Measuring the amount of noise by its standard deviation, an index of image quality could be defined as the so-called SNR (signal to Noise ratio). Noises may come from different sources including optical noises, noises from the process of imaging, noises from image processing algorithms and other.

The aim of any de-noising algorithm is to find an image namely $\hat{x}$, close enough to the original noisy image and smooth enough as well. There are two major challenges in the filtering problem. The first is to eliminate added noise thoroughly and the second is to preserve the main features of the original image. De-noising algorithms are categorized to linear and nonlinear schemes. In linear schemes a linear filter is used to elicit the original data by removing noises usually based on frequency content of data and noise. Therefore, the frequency distance between noise and data is a crucial factor determining the linear filter performance. Nonlinear techniques are most suited to applications where the noise and data frequency contents cannot be distinguished with certainty. These techniques may include using of filters or optimization techniques. Among nonlinear filtering schemes, total variation de-noising (TVD) has been given extensive attention. Total variation (TV) is a widely used smoothing method in signal and image processing, especially when the signal to be recovered is known to have a sparse derivative (or sparse gradients), i.e., when the signal is piecewise constant (PWC).

TV de-noising is defined in terms of a convex optimization problem involving a quadratic data fidelity term and a convex regularization term. Interestingly, for 1-D TV de-noising, the exact solution can be obtained using very fast direct algorithms terminating in a finite number of steps. However for 2D signals it could be a computationally heavy optimization problem.

TVD is based on the assumption that noise has excessive and possibly significant (in magnitude) variations and a high value of total variation, that is, the sum of the absolute gradient of the noise is high. In contrast, this value is much lower relatively for the original data set. Therefore, reducing the total variation of the signal subject to noises in such a way that the filtered signal is a close match to the original signal, removes unwanted data or noises. Preserving important details such as edges and keeping the image as clear as possible is the challenge to be solved when designing a TVD filter. This de-noising technique has advantages over simple linear / nonlinear techniques such as linear smoothing or median filtering which reduce noise at the cost of smoothing edges away to a greater or lesser degree. By contrast, total variation de-noising is remarkably effective at simultaneously preserving edges while smoothing away noise in flat regions, even at low signal-to-noise ratios. TVD was introduced by Rudin et al. in 1992. Different modifications are proposed consequently.

TVD solves an optimization problem to derive the filtered image. Utilization of TVD consists of solving the optimization problem to find the filtered image close to the original one. Any algorithm that solves the optimization problem can be used to implement TVD, though the solution is not trivial as TVD cost function is non-differentiable. Total variation is used not just for de-noising, but for more general signal restoration problems, including de-convolution, interpolation, in-painting, compressed sensing, etc. Unlike conventional filters, the TVD method is not designed based on a qualitative awareness of edges, but rather, on the basis of quantitative description of edges with respect to total variation of image defined via the gradient vector. The TV model is originally designed for analog or continuous signals. The equation of optimality criterion, associated with the TV functional is a nonlinear partial differential equation. When applying TVD to a digital image, one has to choose carefully the numerical scheme to take care of the nonlinearity.

When there are band limited high pass noises, it may be appropriate to have a combination of those techniques. Total Variation De-noising (TVD) was introduced first in [1] and then extended for several other applications (e.g. in paint, image reconstruction, etc.) for example in [2-4]. TVD as an optimization – based de-noising technique enjoying a significant level of versatility. It does not require knowledge of noise spectrum and is simple to adjust its parameters. On the other hand, TVD has a deteriorating effect on image edges and also is potentially disrupting as it could blend colors from different zones of an image. In order to reduce deteriorating effect of TVD on edges, it is proposed in this paper to combine it with linear traditional filters. This method is used in [5] for processing of speech signals. However it is not addressed in the literature for images.

TV de-noising requires solving an optimization problem either via numerical or analytical optimization techniques. Higher degrees for differentiation in TVD are studied in [6]. Different algorithms are proposed and compared in the literature [7-11, 23-26]. A drawback of TVD is that there is only one adjustable parameter (relative weight in the cost function) to be manipulated in order to get better results. Selection of this parameter has a crucial effect on the performance of TVD. In [28] an automated method is proposed for selection of the weighting scalar in order to maximize the filtering performance. The optimal weighting scalar value differs significantly for different images. Especially when noises do not suite into a white uncorrelated process model, TVD may have poor results. For a study on the noise behavior in TVD imaging see [27]. As a result adding some extra parameters may be beneficial to make TVD more flexible and efficient. These parameters could be linear filter parameters or weighting scalars for relative emphasis on TVD or filter. Therefore, combined approaches could be looked at, as extensions of TVD, adding extra parameters to this method.

This paper is organized as follows: In the second section, formulation of the combined approaches and the motivation of using this approach are presented. In the third section, simulation results are gathered. In the fourth and fifth sections, future extensions and conclusions are stated.

2. Formulation of Combined Approaches

This paper studies (via simulations) the effect and effectiveness of different combined methods using TVD and linear filtering. The followings are the considered approaches. Most of filtering/de-noising algorithms are suited for specific types of noises. White noises are usually considered in analysis and there is no specific criterion to re-design or adjust the filter /de-noising algorithm parameters to adapt it for a new class of noise. In fact, composite noises may deteriorate the performance of any algorithm designed for general noises. When the noises are composite, it should be some adjustable design parameters or flexible algorithms to compensate for noises and eliminate/ reduce noise levels effectively. One approach for this purpose is to use different algorithms combined with each other. A combined method, if properly adjusted, could benefit from different methods strength points while avoiding their drawbacks. The advantage of combined approaches is their simplicity and the availability of different existing algorithms to apply to the images. The proposed method could be used where the noise behavior is complicated and a single method is inadequate to eliminate noises. It could also be used for enhancement of image quality without need to delve into details of filter design techniques by simply adjusting a few weighting coefficients. A combined technique is the simultaneous or consecutive application of TVD with a traditional linear/ nonlinear filter to reduce noise effects and improve image quality. When a single method is sufficient to perform the required processing, weighting parameters of the auxiliary filter either in a series; parallel or simultaneous setting could be set to zero to result in the single method result. Therefore, the combined technique provides a generalization of the two methods. The following combinations are possible:

A. Series combination.

In this setting, shown in figure 1, a linear filter is followed / preceded by TVD algorithm which could be looked at as a nonlinear filter.

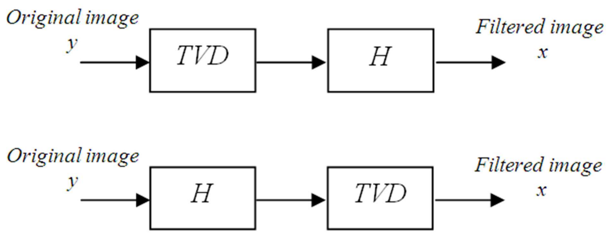

Figure 1. *Series combinations.*

A series combination could be used when a portion of image noise is known to exhibit specific frequency spectrum. (i.e. noise spectrum is concentrated within a specific, almost known frequency band) also it may be used to emphasize on the image edges before/after applying TVD, as TVD is known for its deteriorating effect on edge quality. The first series form could be stated as:

$$\underset{z}{argmin}\|z - Hy\| + \|\Delta z\| \quad , x = Hz$$

In this formulation, the image is filtered by a linear filter before being processed by TVD. This combination is best suited to applications where the noise spectrum is bandwidth limited and a linear filter is known to reduce the noise. TVD is used afterwards to enhance the filtering and improve edges.

The second series form is modeled as follows:

$$\underset{x}{argmin}\|x - Hy\| + \|\Delta x\|$$

In this setting, a TVD is first applied and a filter (either linear or nonlinear) is used consequently. This technique could be used specially when TVD demolishes or deteriorates edges and an edge detection filter could be used after application if TVD.

B. Parallel combination.

Parallel combination is a compromise between both methods. Images are matrices containing pixels with different numeric values representing each pixel color or brightness. Those matrices could be summed up or averaged with relative weights to balance the effects of different methods. By adjusting the weights, one may get better results combining different methods strength points.

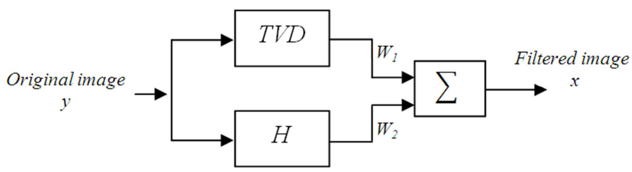

Figure 2. *Parallel combination.*

Parallel combinations could be formulated as:

$$\underset{z}{argmin}\|z - Hy\| + \|\Delta z\| \quad , x = w_1 z + w_2 Hy$$

Weighting parameters, w_1 and w_2 are design parameters emphasizing on TVD or linear filter. When $w_1 \gg w_2$ one should expect a result very similar to linear filtering and when $w_1 \ll w_2$, the result is very close to TVD. In order to preserve image basic feature, weighting scalars should be chosen such that:

$$\sum w_i = 1$$

C. Simultaneous methods.

Motivated by series and parallel combinations, one may use other combinations in integrated settings to get enhanced results. A formulation is proposed in [5] for 1-D signals as follows:

$$\arg\min_{x}\left\{\frac{1}{2}\| H(y-x)\|_2^2 + \lambda_0 \| X \|_1 + \lambda_1 \| Dx \|_1\right\}$$

To the best of the authors knowledge there is still no work for combined image processing schemes. The problem is formulated as:

$$minimize\|DX\|_1 + \lambda\|X - Y\| + \gamma\|H(X - Y)\|$$

In which:

$$Y = \underbrace{\begin{bmatrix} 1/N & \dots & 1/N & 0 & 0 & \dots 0 \\ 0 & 1/N & \dots & 1/N & 0 & \dots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & 1/N \end{bmatrix}}_{H} X$$

The software tool used for solving the optimization problem is CVX tool of [15, 16], developed to be used in conjunction with MATLAB. Optimization takes a few minutes for a 2.5 GHz @Intel double core processor and requires that the original image is broken into smaller parts to be feasible for optimization problem.

3. Simulations

Simulation is performed via Matlab CVX tool box developed in [15] and the image processing tool box. The following table summarizes the results of implementation on Lena picture shown below.

Table 1. *Results of implementation: Error matrix covariance.*

Error matrix Covariance						
λ	TVD	Series (TVD+F)		Parallel (TVD/F)		Proposed Method (γ = λ)
		Median filter	Mean filter	Median filter	Mean filter	Mean filter
.3	1.74	1.57	2.48	1.67	2.07	1.31
.6	1.49	1.48	2.31	1.55	1.98	1.30
.9	1.53	1.52	2.34	1.58	1.99	1.31
1.2	1.57	1.55	2.37	1.60	2.01	1.32
1.5	1.62	1.61	2.41	1.60	2.03	1.34

Table 2. *Results of implementation: SNR.*

Signal to Noise Ration (SNR)						
λ	**TVD**	**Series (TVD+F)**		**Parallel (TVD/F)**		**Proposed Method ($\gamma = \lambda$)**
		Median filter	**Mean filter**	**Median filter**	**Mean filte**	**Mean filter**
.3	7.46	7.52	4.30	8.30	5.23	9.35
.6	7.83	7.87	4.40	8.86	0.82	9.56
.9	7.94	7.42	4.48	8.45	0.95	9.78
1.2	7.32	7.00	4.56	8.24	0.82	9.95
1.5	7.12	6.98	4.51	8.0	0.79	9.83

The following pictures are included for visual comparison and evaluations.

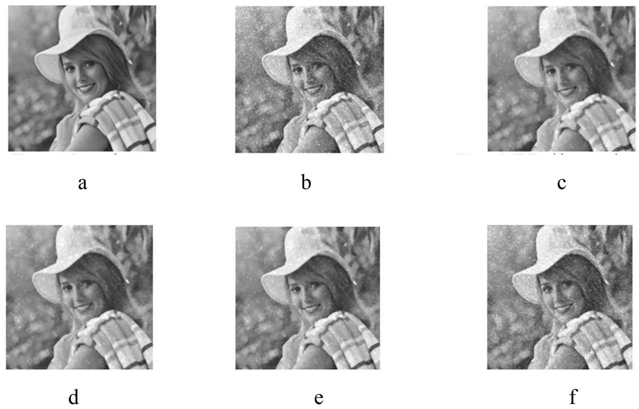

Figure 3. *a.original picture; b. Picture with noise; c. TVD applied, composite noise; d. Series, linear filter after TVD; e. Series, with linear filter before TVD; f. Parallel method.*

The following observations are made:

- Increasing the weighting coefficientλ, causes the image to lose its edges and become vague.
- In presence of colored noises which are made by applying Sobel and mean filters to white noises, application of TVD is not effective and it should be accompanied by a filter. When the noise is high pass, combined low pass filter with TVD yields better results than TVD or low pass filters alone.
- In almost all cases, λ values about 0.6 yield the best results.
- The best results are derived for the proposed method combining low pass filter and TVD simultaneously. Sobel filter deteriorated image quality after a TVD application. The result of combined mean and edge sharpening filters does not significantly improve TVD.
- No significant difference is detected between applying series filter before or after the application of TVD.

4. Conclusion

In his paper, combined techniques consisting of traditional filtering and total variation de-noising are studied to de-noise images polluted with composite noises from different sources. Series and parallel combinations are introduced and different aspects are discussed. Some further discussions are given about parameters and their effect on the optimization problem. A simultaneous method is proposed for the combination of linear filters with TVD. Simulations are included using MATLAB CVX toolbox.

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