
An Algorithm for a Sub-Nyquist Rate AM and FM Software-Defined Radio Based on the Market Paradigm

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Abstract: Software-defined radio accomplishes both modulation and demodulation processes using software. While this has a number of advantages, which includes flexibility, interoperability, sustainability, and adaptability, the requirement for sampling the signal for digital processes toward adequate recovery often involves the use of a fast but expensive analogue-to-digital converter (ADC). This, in a way translates to higher cost and requirement for bigger storage. This paper presents a method of switched signal recovery at uniform sampling rates that are less than the frequently over-estimated Nyquist rate employed. In particular, an algorithm for achieving this was implemented for an AM wave, under-sampled at varied uniform rates up-to twice the carrier rate, and then demodulated using the Market Paradigm. Furthermore, the slope detector was also implemented by including a differentiator after the sampling stage of the algorithm. The simulated results showed that the algorithm was able to recover the message signal at sampling rates far less than twice the carrier rate without the need for any additional hardware. Specifically, the best value of the Spurious Free Dynamic Range (SFDR) obtained for the recovered message signal was 20dB at a sampling rate of less than 20% of the Nyquist rate for the carrier signal.

Keywords: Software-Defined Radio, Sampling, Big Data, Market Paradigm, Agent-Based Detection, Wireless Networks

1. Introduction

A radio is any kind of device that wirelessly transmits or receives signals in the radio frequency (RF) part of the electromagnetic spectrum in order to facilitate the transfer of information. In today's world, radios exist in a multitude of items such as cell phones, computers, vehicles and television systems (Sharma, 2009).

In a traditional radio, all the signal processes such as frequency translation, filtering, demodulation, etc., are implemented using hardware. Although this approach has proven to be practical for a very wide range of applications, there are cases in which the ability to alter radio functionality at run-time is highly desirable, but cannot be accomplished without altering the hardware design and structure. Interoperability with the existing legacy systems, ability to operate with region-specific communication standards, and readiness for future communication protocols are few of the benefits for desiring a reconfigurable system. Traditional radio lacks the capability to accomplish these plans without changing the hardware and many times the structure too, and

thus results in higher production cost and minimal flexibility in supporting multiple waveform standards (Lehr *et al.*, 2002). Traditional radio systems, subsequently, lacks the flexibility to adapt to newer standards or to change the relative proportion of transceiver channel modules without changing all the hardware.

Software-defined radio (SDR) was proposed by Mitola III (1992) to signal the shift from hardware designed dominated radio systems to systems where major part of the functionality is defined in software. SDR is characterized as a radio whose physical layer behaviour can be significantly altered through changes to its software (ShajedulHassan and Balister, 2005). It uses software to modulate and demodulate radio signals (Millhaem, 2006). While this has a number of advantages, which include flexibility, interoperability, sustainability, and adaptability, the requirement for sampling the signal at twice its maximum frequency to fulfill Shannon sampling theorem (Nyquist criterion) has to be met for adequate recovery. Consequently, this often involves the use of a fast but expensive analogue-to-digital converter (ADC), which in a way translates to higher cost and requirement for bigger storage. The major motivation of this theorem lies in the

ability to shift processing tasks from the analog to the digital or switched domain. Digital signal processing (DSP) is therefore a major driving force that supports the wide popularity of sampling at the Nyquist rate (Proakis and Manoulis, 1992; and Mishali and Eldar, 2010 and 2011). Following this, conversion speeds that are twice the signal's maximum frequency have recently become more and more difficult to obtain, and consequently, alternatives to high rate sampling are expected to continue to draw considerable attention in both academia and industry (e.g., Kumar and Reddy, 2014). It is also important to note that sampling, as a solution, must be explored in line with the laws of large numbers (e.g., Ross, 2009) in order to successfully transmit and receive a message over a channel.

Sampling below the Nyquist rate otherwise known as sub-Nyquist sampling have prominent advantages over the conventional Nyquist theorem in that the DSP operations are carried out at low input rate, and storage may not require a preceding compression stage. Demodulation is one of the methods used to achieve this. In the method, the incoming signal is multiplied by the carrier frequency of a band of interest, so as to shift contents to the origin, and then sampled uniformly in time. This technique however needs analog preprocessing (Proakis and Salehi, 1995) that adds to the hardware complexity. Under-sampling refers to uniform sampling of bandpass signals at a rate lower than the maximum frequency. In this technique, uniform sampling of a bandpass input $x(t)$ whose information band lies in the frequency range (f_u, f_l) are sampled at a rate of f_s that obeys:

$$\frac{2f_u}{k} \leq f_s \leq \frac{2f_l}{k-1} \quad (1)$$

for some integer $1 \leq k \leq f_u/B$, (where $k=1$ corresponds to Nyquist rate). This ensures that aliases of the positive and negative contents do not overlap for sampling frequency greater or equal to twice the bandwidth of the signal, i.e., $f_s = 2B$ (Landau, 1967; and Vaughan et al., 1991). However, apart from the fact that a significant higher rate is likely to be required in practice to cope with design imperfections due to possible deviations in the values of f_s , f_l , f_u not every ADC device fits an under-sampling system: only those devices whose front-end analog bandwidth exceeds f_u are viable.

Periodic non-uniform sampling on the other hand allows for minimal rates that approach values equal to the bandwidth of the signal without complicated analog preprocessing (Kohlenberg, 1953; and Lin and Vaidyanathan, 1998). But the method adds to the hardware complexity as a set of time-delay elements are required beside the ADC.

This paper presents a method of signal recovery at sampling rates that are less than the Nyquist rate for switched signals commonly associated with sampled data systems, and AM and FM signals. In particular, an FM wave would be sampled at uniform rate, and then the intelligence will be recovered and other advantages indicated.

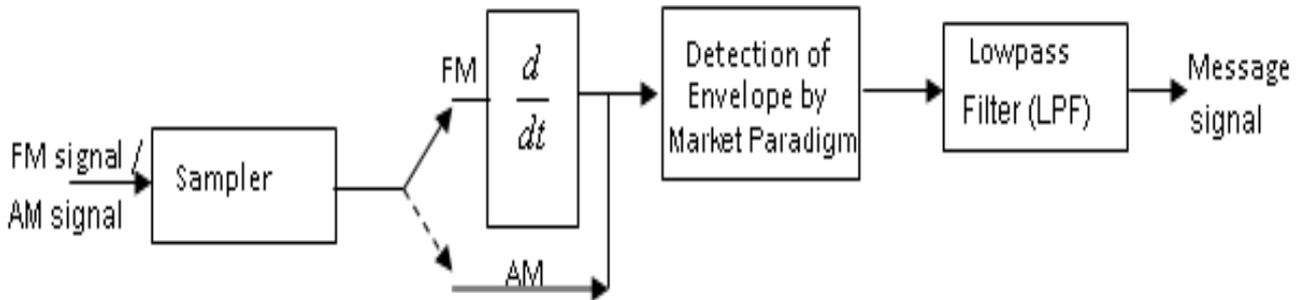


Figure 1. Model for Proposed Software-Based Demodulation of FM and AM Signals.

2. Theoretical Development

Consider a generalized modulated or switched signal shown in Figure 1 and expressed as

$$S(t) = A_c(t) \cos(2\pi f_c t + \phi(t)) \quad (2)$$

$A_c(t)$ is the instantaneous amplitude, which is assumed constant for an FM signal. For AM signal: $A_c(t) = A_c + m(t)$. $\phi(t)$ is the phase, which also for an AM is assumed constant, but for FM signal: $\phi(t) = K_f \int_0^t m(t) dt$ while $m(t)$ is the message signal.

The generalized modulated signal represented by (2) is

sampled uniformly at the rate of

$$T_s = \frac{n}{r(f_c + hf_m)} \quad (3)$$

where n , r and h are integers ($r=2$ corresponds to the Nyquist rate, $h=1$ for narrow band FM with only one (1) pair of sidebands), and the maximum frequency deviation is $2hf_m$ (for wideband FM signal with h significant pairs of sidebands).

For $f_c \gg f_m$, we can equally assume that:

$$T_s = \frac{n}{rf_c} \quad (4)$$

without loss of generality, and therefore,

$$S[n] = A \cos \theta(nT_s) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad (5a)$$

for an FM wave;

$$S[n] = A_c(nT_s) \cos \theta \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad (5b)$$

for an AM wave.

Use is normally made of a slope detector, a frequency demodulation technique, which can readily combine FM and AM demodulation processes with a suitable modification in hardware. A simple approximation of the slope detector is an ideal differentiator followed by an envelope detector. The slope detector produces an output that may be assumed to be proportional to the instantaneous frequency deviation of the input signal. Therefore using the slope detector method of demodulation affords us the possibility of using a common sampler and differentiator circuit for both FM and AM signals (Figure 1).

The Market Paradigm is an algorithm developed to detect the envelope of sampled AM signal. In the analysis of interacting agents' scenario, such as market-mediated economic interactions or strategic interactions in game theoretic settings with few agents, as explained by Yesufu and Yesufu (2003) and Yesufu and Oladimeji (2008), it has been realized that the market paradigm is a good tool for analyzing time series data, strategic system analysis and network analysis. Hence, the series, X_t , is a general form for a stationary series given by

$$X_t = r_t \cdot X_{t-1} + e_t \quad (6)$$

where e_t , the error term, has zero mean and covariance, and a constant variance and r_t , the root of the series, has a value of unity for random walk. Accordingly, we have

$$X_{t-1} = X_t - \frac{1}{2}(X_t \cdot X_{t-1})(X_t) \quad (7)$$

On rearranging (7), we have

$$X_t = \frac{2(X_t - X_{t-1})}{X_t \cdot X_{t-1}} \quad (8)$$

By comparing (6) with (8) for convergence, we have

$$r_t = 2 \cdot \frac{(X_t - X_{t-1})}{(X_t)(X_{t-1})^2} \quad (9)$$

The value of r_t indicates the motivation or amount of information present by adding a new term, X_t , to the series S after the term X_{t-1} . In other words this is the characteristic root of the time series at the time t. $S[n]$ is a discrete time signal, and so the differentiation operation for the FM wave was done in digital domain, using backward difference

equation,

$$S' [n] \approx \frac{1}{T_s} [S(nT_s) - S(n-1)T_s] \quad (10)$$

The instantaneous frequency $f_i(t)$,

$$f_i(t) = f_c + \Delta f m(t) = \theta'(t) \quad (11)$$

is proportional to the difference between consecutive data samples. The envelope of the differentiated output $S' [n]$ was detected using market paradigm as in (10). The output of the differentiator is a time series. The message can be extracted by a low pass filtering (Otolorin, 2013). The summary of the algorithm is as shown in Table 1.

Table 1. Algorithm for the demodulation of AM and FM signals based on the market paradigm.

Step1: input A_m, A_c, f_m, f_c, K_f
Step2: generate a sinusoidal signal with the parameter in step1 above.
Step 3: generate the carrier signal.
Step 4: generate the modulated signal.
Step 5: sample the modulated signal in step 4 above using appropriate sampling rate
$T_s = \frac{n}{rf_c}$
$S[n] = A \cos \theta(nT_s) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$
For FM wave
$S[n] = A_c(nT_s) \cos \theta \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$
For AM wave
Step 6: differentiate the sampled signal using numerical difference equation
$S' [n] \approx \frac{1}{T_s} [S(nT_s) - S(n-1)T_s]$
Step 7: detect the envelope of the differentiated output in step 6 above using market paradigm algorithm.
$r_t = 2 * \frac{(X_t - X_{t-1})}{(X_t)(X_{t-1})^2}$
Step 8: find the reciprocal of the output in step 7
$m(t) = \frac{1}{r_t}$
Step 9: filter the recovered signal.

Table 2. SFDR Values in dB for Final Output of Demodulated AM signal at Different Sampling Rates.

Sampling Rate, f_s (Hz)	Normalized Value of Baseband Signal Amplitude, BB_{peak}	Normalized Value of SH_{peak}	SFDR in dB
0.1 f_c	0.18883962	0.131096738	3.170024641
0.2 f_c	0.19273360	0.145999306	2.412132955
0.3 f_c	0.00552357	0.005293273	0.369907719
0.4 f_c	0.19844341	0.025300372	17.89019548
0.5 f_c	0.00485271	0.001198534	12.14668133
0.6 f_c	0.01107818	0.010611077	0.374174837
0.8 f_c	0.00101180	0.027836617	-28.790482

Sampling Rate, f_s (Hz)	Normalized Value of Baseband Signal Amplitude, BB_{peak}	Normalized Value of SH_{peak}	SFDR in dB
$1.0 f_c$	0.00485872	0.001211051	12.06719343
$1.5 f_c$	0.01631337	0.020656931	-2.05044048
$2.0 f_c$	1.00000000	0.125454018	18.03030846

Table 3. SFDR Values in dB for Final Output of Demodulated FM signal at Different Sampling Rates.

Sampling Rate, f_s (Hz)	Normalized Value of Baseband Signal Amplitude, BB_{peak}	Normalized Value of MSH_{peak}	SFDR in dB
$0.1 f_c$	0.000000173	0.000000057	9.6869
$0.2 f_c$	0.000000438	0.000000776	-4.9590
$0.3 f_c$	0.000034172	0.000051506	-3.5638
$0.4 f_c$	0.000821177	0.000078208	20.4237
$0.5 f_c$	0.000184884	0.000042799	12.7094
$0.6 f_c$	0.000136323	0.000158879	-1.3300
$0.8 f_c$	0.002799009	0.042900221	-23.7091
$0.9 f_c$	0.081096614	0.042170239	5.6799
$1.0 f_c$	0.003854422	0.002940377	2.3511
$1.5 f_c$	0.646000000	0.572968293	1.0392
$2.0 f_c$	1.000000000	0.084895802	21.4223

3. Results and Discussion

This work considered both AM and FM demodulation based on sub-Nyquist sampling rate. We have assumed a single tone modulated system. The message frequency was chosen to be 20Hz, while the carrier waveform has frequency of 2 kHz. Other parameters were chosen to generate the baseband, carrier and FM signal waves. Modulation indices of 0.1 to 1.2 were tested by varying the frequency deviation constant, K_f between 0 and 5 while keeping the amplitude of

modulating signal constant. The generated FM wave equation was sampled at $0.1f_c$ to $1f_c$, $1.5f_c$ and $2f_c$.

The performance of the developed algorithm was evaluated using the Spurious Free Dynamic Range (SFDR) of the recovered message signal. At each sampling rate, the normalized amplitude at frequency of the baseband signal (BB_{peak}) and that of the most significant harmonic (MSH_{peak}) were determined and tabulated as shown in Tables 2 and 3 for AM demodulation and FM demodulation processes respectively. The corresponding values of SFDR (dB) were computed and the graph of the sampling rate against the SFDR (dB) was plotted for the two cases as shown in Figures 2 and 3, respectively. It can be observed from Figure 4 that $0.4f_c$ is the optimum sampling rate for both FM and AM. Hence, a sampler at $0.4f_c$ (that is at 20% of Nyquist) can be used for both FM and AM waves.

The market paradigm suitably uses the slope detection method of FM demodulation to achieve interoperability between AM and FM, as well as other sampled data systems. The results show that obtaining the conveyed intelligence in carrier switched systems, which have been processed to meet certain transmission properties, will definitely require that sampling should be done at rates that are far less than the carrier rate but at least twice the highest frequency in the message signal. The ploy in modulation is to switch a signal at a given carrier rate, which is to achieve the channel-

efficient transfer of information by a reversible means to take a sample of the population (i.e., considerably reduce the sample size required to represent a message signal) before transmitting it.

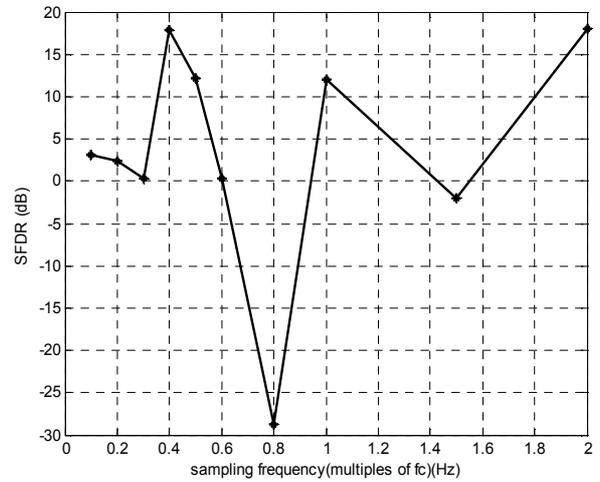


Figure 2. Plot of SFDR for Demodulated AM Signal.

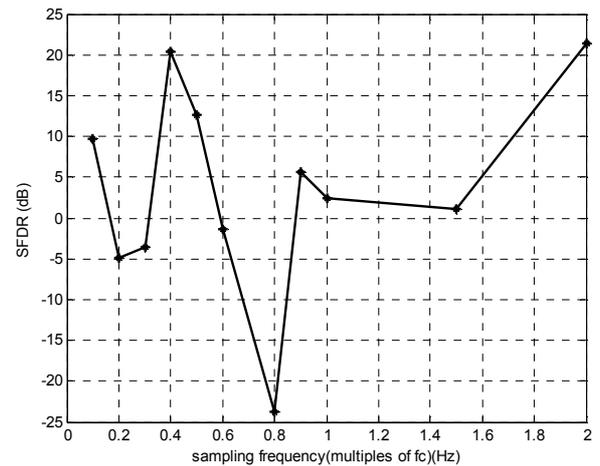


Figure 3. Plot of SFDR (db) for Final Output of Demodulated FM Signal.

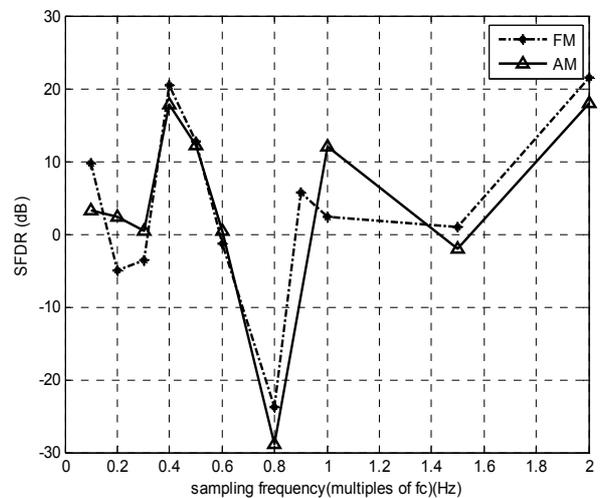


Figure 4. Plot of SFDR (dB) for Demodulated FM and AM Signals.

4. Conclusion

This paper presented the development of an algorithm for the sub-Nyquist rate sampled data processing of AM and FM signals. In this case, the sampling rate was found to be appropriate, being focused on the message signal rather than the carrier signal. This was possible because AM and FM signals are switched and processed waveforms that have inherent features of small-sized but representative random samples of a population of nodes in a wireless network. The entire population, in this case, is the intelligence to be recovered from the AM and FM signals. This is consistent with the law of large numbers as well as with the ability of the Market Paradigm to effectively detect the envelope of an AM waveform at sampling rates well below the carrier rate. The developed algorithm is suitable for the emergence of cheaper software-defined radio receivers and more secured wireless networks based on the possibility of using a much lower sample size for faster intrusion detection and other related implementation of cleverer schemes in digital radios.

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