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Poisson Ridge Regression Estimators: A Performance Test

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Abstract: In Multiple regression analysis, it is assumed that the independent variables are uncorrelated with one another, when such happen, the problem of multicollinearity occurs. Multicollinearity can create inaccurate estimates of the regression coefficients, inflate the standard errors of the regression coefficients, deflate the partial t-tests for the regression coefficients, give false p-values and degrade the predictability of the model. There are several methods to get rid of this problem and one of the most famous one is the ridge regression. The purpose of this research is to study the performance of some popular ridge regression estimators based on the effects of sample sizes and correlation levels on their Average Mean Square Error (AMSE) for Poisson Regression models in the presence of multicollinearity. As performance criteria, average MSE of k was used. A Monte Carlo simulation study was conducted to compare performance of Fifty (50) k estimators under four experimental conditions namely: correlation, Number of explanatory variables, sample size and intercept. From the results of the analysis as summarized in the Tables, the MSE of the estimators performed better in a lower explanatory variables p and an increased intercept value. It was also observed that some estimators performed better on the average at all correlation levels, sample sizes, intercept values and explanatory variables than others.

Keywords: Multicollinearity, Ridge, Poisson, Estimators, Maximum Likelihood, Monte-Carlo Simulations, MSE

1. Introduction

In multiple regression analysis, predictions are made on one variable on the basis of several other variables. For example, suppose we were interested in predicting the level of exposure of an individual, variables such as age, gender, environment, academic qualifications and occupation might all contribute towards level of exposure. Another example could be when analyzing why people go to the cities; job opportunities, availability of basic amenities, schools and standard of living are important factors. In both examples, some or all covariates would be highly correlated. In the case of highly correlated explanatory variables, it is usually impossible to interpret the estimates of the individual coefficients. Such a problem is often referred to as the multicollinearity.

Multicollinearity exists whenever two or more of the predictor variables in a regression model are moderately or highly correlated. Multicollinearity can create inaccurate estimates of the regression coefficients, inflate the standard errors of the regression coefficients, deflate the partial t-tests for the regression coefficients, give false p-values and degrade the predictability of the model.

The Ridge Regression method is a well-known efficient remedial measure in the presence of multicollinearity. This method was first introduced by Schaeffer et al [16] and they have shown by both analytically and means of Monte Carlo simulations that this method has a smaller total Mean Square Error (MSE) than the OLS estimator. They suggested that a small positive number ($k > 0$) be added to the diagonal elements of the $X'X$ matrix from the multiple regression, and the resulting estimator is obtained as

$$\hat{\beta}_{RR} = (X'X + kI_p)^{-1}X'Y \tag{1}$$

and this is known as the ridge regression (RR) estimator. Schaeffer et al [16]

Different techniques for estimating the ridge parameter k have been suggested by a lots of researchers; [Hoerl and Kennard [6], Hoerl and Kennard [7], Algamal and Alanaz [3], Asar and Gen [4], Asar and Gen [5], Kaciranlar and Dawound [8], Kibra et al [12], Mansson and Shukur [13], Alkhamisi et al [1], Alkhamisi and Shukur [2], Muniz and Kibria [14], to mention but a few. However, the work on the estimation of the ridge parameter under the Poisson Regression model is limited. Schaeffer et al [16] worked on the Simple Poisson Regression model, Muniz and Kibria [14] considered the Poisson RR models, Kibria et al [11] generalized some estimators of the ridge parameters proposed for logistic regression by Kibria et al [12] for Poisson ridge regression and Zaldival [17] considered the performance of some Poisson Ridge Regression Estimators.

One of the standard statistical method for analyzing count data is the Poisson Regression (PR) model. This model has found a widespread use in microeconometrics when the dependent variable y of the regression model is Poisson distributed. In the presence of multicollinearity, when estimating the parameters for Poisson regression model using the maximum likelihood (ML) method, the estimated parameters become instable with high variance, resulting in an increase in the probability of conducting a Type II error in any hypothesis testing regarding the estimated parameters Kibria et al [11].

Several techniques for estimating the ridge parameter k have been suggested by a lot of researchers;

The purpose of this article is to determine the best estimators among 50 selected ridge parameters estimators (k), based on the effect of sample sizes and levels of correlation on the performance of the 50 selected estimators. The performance of the estimators is judged by the Average MSE of k.

2. Literature

Zaldivar [17] investigated some Ridge Regression (RR) estimators for estimating the ridge parameter k for the poisson regression model and proposed five new estimators. A simulation study was conducted to compare the performance of some of the estimators in literature with the newly proposed estimators using the average Mean Square Error (MSE), percentage of times Poisson Ridge Regression (PRR) outperforms MLE and the Average Mean Absolute Percentage Error (AMAPE) as performance criteria, these five new estimators performed well, producing small MSE values. A real life data was also used to illustrate the findings.

Kibria et al [11] conducted a simulation study of some biasing parameters for the ridge type estimation of Poisson regression, they generalized some estimators proposed for logistic regression by Kibria [12]. In the work they included the average value of k and the standard deviation of k as performance criteria aside the Mean Square Error (MSE), these performance criteria are very informative because if several estimators have equal estimated MSE, then those with low average value and low standard deviation of k should be preferred.

Muniz and Kibria [14] proposed a Poisson Ridge Regression (PRR) estimator, by means of Monte Carlo simulations they evaluated the traditional ML estimator and this new method using different estimators of the ridge parameter k. The result from the simulation study showed that the sample size, the value of the intercept, the number of independent variables and the correlation between the independent variables are important factors for the performance of the different estimation methods. The result also showed that the proposed PRR method, regardless which ridge estimator used, has a lower MSE than the ML method for all different situations that has been evaluated. Many researchers, Hoarl and Kennard [6], have worked on Regression and Poison estimator and their conclusion varied. Hoerl and Kennard [6], Hoerl and Kennard [7], Algamal and Alanaz [3], Asar and Gen [4], Asar and Gen [5], Kaciranlar and Dawound [8], Kibra et al [12], Mansson and Shukur [13], Alkhamisi et al [1], Alkhamisi and Shukur [2], Khalaf Ghazi [9], Kibria [10].

3. Methodology

This section starts by defining the Poisson Regression model and the traditional ML estimation method.

3.1. The Poisson Regression

The Poisson Regression is similar to the regular multiple regression except that the dependent variable (Y) is an observed count that follows the poisson distribution. Thus the possible values of (Y) are the non-negative integers: 0, 1, 2, 3, and so on. The model of this regression is given as:

$$y_i = u_i + \varepsilon_i \; (i = 1, 2, \dots, n) \tag{2}$$

Where y_i is an $n \times 1$ vector of responses that is poisson distributed, $u_i = \exp(x_i\beta)$ is an $n \times (p+1)$ data matrix with p explanatory variables, and ε_i is an $n \times 1$ vector of random errors.

The parameters of this model are estimated using the Maximum Likelihood (ML) method and the following iterated weighted least-square algorithm:

$$\hat{\beta}_{ML} = (X'\widehat{W}X)^{-1} X'\widehat{W}\hat{z}, \tag{3}$$

Where $\hat{z}$ is a vector where the ith element equals $\hat{z}_i = \log(\hat{\mu}_i) + \frac{y_{i-}\hat{\mu}_i}{\hat{\mu}_i}$ and $\widehat{W}$ is a matrix where the off- diagonal elements are equal to zero and the ith diagonal element is equal to $\hat{\mu}_i$. Where $\hat{\mu}_i = \exp(x_i\beta)$ and x_i is the i^{th} row of X.

3.2. Poisson Ridge Regression

In the presence of multicollinearity, the weighted matrix of cross products $(X'\widehat{W}X)$ is near singular. For this model, the following extension of the linear ridge regression estimator (RRE) was proposed by Mansson and Shukur [13].

$$\hat{\beta}_{RR} = (X'\widehat{W}X + kI_P)^{-1}\big(X'\widehat{W}X\hat{\beta}_{ML}\big). \tag{4}$$

They showed that the above estimator approximately reduces the increase of the weighted Sum of Squared Error. The reason for Poisson Ridge Regression (PRR) is to find a value of k that is large enough, but not so big that it causes a lot of bias. When such a k is found, then the MSE of the RIDGE regression (RR) estimator will be smaller than that of the ML estimator. The MSE of RR equals

$$E(L_{RR}^2) = E(\beta_{RR} - \beta)'^{(\beta_{RR} - \beta)} = \sum_{j=1}^{J} \frac{\lambda_j}{(\lambda_j + k)^2} + k^2 \sum_{j=1}^{J} \frac{\alpha_j}{(\lambda_j + k)^2} \quad (5)$$

While the MSE of the ML estimator is

$$E(L_{ML}^2) = E(\beta_{ML} - \beta)'(\beta_{ML} - \beta) = tr(X'\widehat{W}X)^{-1} = \sum_{j=1}^{J} \frac{1}{\lambda_j} \quad (6)$$

3.3. Some Methods for Estimating the Ridge Parameter (k)

Various methods have been suggested by different researchers for estimating the ridge parameter k.

Hoerl and Kennard [6] were the first to propose a ridge parameter estimator and their estimator formed the basis upon which other estimators were proposed. The first estimator is

$$k_1 = \hat{k}_{HK1} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{max}^2} \quad (7)$$

Where $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(y_i - \hat{\mu}_i)^2}{n-p-1}$, and $\hat{\alpha}_{max}$ is the maximum element of $\hat{\alpha}$, where $\hat{\alpha} = U'\hat{\beta}_{ML}$ and U is the matrix whose columns are the eigenvectors of $X'\widehat{W}X$.

Schaeffer et al [16] proposed the following for logistic ridge regression

$$k_2 = \hat{k}_{HKM} = \frac{1}{\hat{\alpha}_{max}^2} \quad (8)$$

Kibria (2003):

$$k_3 = \hat{k}_{GM} = \frac{\hat{\sigma}^2}{(\prod_{j=1}^{p} \hat{\alpha}_j^2)^{\frac{1}{P}}} \quad (9)$$

$$k_4 = \hat{k}_{MED} = Median\{m_j^2\}, \quad (10)$$

Where $m_j = \sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}}$.

Muniz et al [14] and Kibria et al [12] proposed k_5 to k_8 given as

$$k_5 \text{ max} = \left(\frac{1}{m_j}\right) \quad (11)$$

$$k_6 max = (m_j) \quad (12)$$

$$k_7 = \prod_{j=1}^{p} \left(\frac{1}{m_j}\right)^{\frac{1}{p}} \quad (13)$$

$$k_8 = median\left(\frac{1}{m_j}\right) \quad (14)$$

Kibria et al [11] proposed k_9 to k_{16} given as

$$k_9 = \prod_{j=1}^{p} (m_j)^{\frac{1}{p}} \quad (15)$$

$$k_{10} = median(m_j) \quad (16)$$

$$k_{11} = max\left(\frac{1}{q_j}\right) \quad (17)$$

$$k_{12} = max(q_j) \quad (18)$$

$$k_{13} = \prod_{j=1}^{p} \left(\frac{1}{q_j}\right)^{\frac{1}{p}} \quad (19)$$

$$k_{14} = \prod_{j=1}^{p} (q_j)^{\frac{1}{p}} \quad (20)$$

$$k_{15} = median\left(\frac{1}{q_j}\right) \quad (21)$$

$$k_{16} = median(q_j) \quad (22)$$

Where $q_j = \frac{\lambda_{max}}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_j^2}, where\ \lambda_{max}$ is the maximum eigenvalue of $X'\widehat{W}X$.

Zaldival [17] used the following in her work, 'on the performance of some Poisson Ridge estimators'

$$k_{17} = K_{Y1} = \frac{1}{p} \sum_{j=1}^{p} \sqrt{\frac{1}{\lambda_j \hat{\alpha}_j^2}} \quad (23)$$

$$k_{18} = K_{Y2} = \left(\prod_{j=1}^{p} \sqrt{\frac{1}{\lambda_j \hat{\alpha}_j^2}}\right)^{\frac{1}{p}} \quad (24)$$

$$k_{19} = K_{Y3} = median\left(\sqrt{\frac{1}{\lambda_j \hat{\alpha}_j^2}}\right), j = 1, 2, \ldots, p \quad (25)$$

$$k_{20} = K_{Y4} = max\left(\sqrt{\frac{1}{\lambda_j \hat{\alpha}_j^2}}\right), j = 1, 2, \ldots, p \quad (26)$$

$$k_{21} = K_{Y5} = median\left(\sqrt{\lambda_j \hat{\alpha}_j^2}\right), j = 1, 2, \ldots, p \quad (27)$$

$$k_{22} = K_{Y6} = max\left(\sqrt{\lambda_j \hat{\alpha}_j^2}\right), j = 1, 2, \ldots, p \quad (28)$$

$$k_{23} = K_{Y7} = \frac{1}{p} \sum_{j=1}^{p} \left(\sqrt{\lambda_j \hat{\alpha}_j^2}\right) \quad (29)$$

$$k_{24} = K_{Y8} = \frac{p}{\sum_{j=1}^{p} \sqrt{\left(\sqrt{\lambda_j \hat{\alpha}_j^2}\right)}} \quad (30)$$

$$k_{25} = K_{Y9} = \frac{p}{\sum_{j=1}^{p} \sqrt{\frac{1}{\lambda_j \hat{\alpha}_j^2}}} \quad (31)$$

where λ_j is the j^{th} eigenvalue of $X'\widehat{W}X$.

Muniz et al [14] proposed the following estimators, based on square root transformations:

$$k_{26} = \text{Max}\left\{\sqrt{\hat{\alpha}_j^2}\right\} \quad (32)$$

$$k_{27}=\text{Median}\left\{\sqrt{\hat{\alpha}_j^2}\right\} \quad (33)$$

$$k_{28}=\text{Max}\left\{\frac{1}{\sqrt{\hat{\alpha}_j^2}}\right\} \quad (34)$$

$$k_{29}=\left(\prod_{j=1}^{p}\sqrt{\hat{\alpha}_j^2}\right)^{1/p} \quad (35)$$

$$k_{30}=\left(\prod_{j=1}^{p}\sqrt{\frac{1}{\hat{\alpha}_j^2}}\right)^{1/p} \quad (36)$$

$$k_{31}=\text{Median}\left\{\frac{1}{\sqrt{\hat{\alpha}_j^2}}\right\} \quad (37)$$

$$k_{32}=\frac{p}{\sum_{j=1}^{p}\lambda_j\hat{\alpha}_j^2} \quad (38)$$

$$k_{33}=\frac{\lambda_{max}}{(n-p)+\lambda_{max}\hat{\alpha}_{max}^2} \quad (39)$$

$$k_{34}=\frac{1}{p}\sum_{j=1}^{p}\left(\frac{\lambda_j}{(n-p)+\lambda_j\hat{\alpha}_j^2}\right) \quad (40)$$

$$k_{35}=\text{Max}\left\{\frac{\lambda_j}{(n-p)+\lambda_j\hat{\alpha}_j^2}\right\} \quad (41)$$

$$k_{36}=\text{Median}\left\{\frac{\lambda_j}{(n-p)+\lambda_j\hat{\alpha}_j^2}\right\} \quad (42)$$

$$k_{37}=\prod_{j=1}^{p}\left\{\frac{\lambda_j}{(n-p)+\lambda_j\hat{\alpha}_j^2}\right\}^{1/p} \quad (43)$$

$$k_{38}=\frac{1}{\hat{\alpha}_{max}^2}+\frac{1}{\lambda_{max}} \quad (44)$$

$$k_{39}=\text{Max}\left\{\frac{1}{\hat{\alpha}_j^2}+\frac{1}{\lambda_j}\right\} \quad (45)$$

$$k_{40}=\frac{1}{Min\left\{\frac{1}{\hat{\alpha}_j^2}+\frac{1}{\lambda_j}\right\}} \quad (46)$$

$$k_{41}=\text{Max}\left\{\sqrt{\frac{(n-p)+\lambda_{max}\hat{\alpha}_j^2}{\lambda_{max}}}\right\} \quad (47)$$

$$k_{42}=\prod_{j=1}^{p}\left\{\sqrt{\frac{(n-p)+\lambda_{max}\hat{\alpha}_j^2}{\lambda_{max}}}\right\}^{1/p} \quad (48)$$

$$k_{43}=\text{Median}\left\{\sqrt{\frac{(n-p)+\lambda_{max}\hat{\alpha}_j^2}{\lambda_{max}}}\right\} \quad (49)$$

$$k_{44}=\prod_{j=1}^{p}\left(\frac{(n-p)+\lambda_{max}\hat{\alpha}_j^2}{\lambda_{max}}\right)^{1/p} \quad (50)$$

$$k_{45}=Max\left\{\sqrt{\frac{\lambda_{max}}{(n-p)+\lambda_{max}\hat{\alpha}_j^2}}\right\} \quad (51)$$

$$k_{46}=\prod_{j=1}^{p}\left\{\sqrt{\frac{\lambda_{max}}{(n-p)+\lambda_{max}\hat{\alpha}_j^2}}\right\}^{1/p} \quad (52)$$

$$k_{47}=\prod_{j=1}^{p}\left\{\frac{\lambda_{max}}{(n-p)+\lambda_{max}\hat{\alpha}_j^2}\right\}^{1/p} \quad (53)$$

$$k_{48}=\frac{2p}{\sum_{j=1}^{p}\lambda_{max}\hat{\alpha}_j^2} \quad (54)$$

$$k_{49}=Median\left\{\frac{2}{\lambda_{max}\hat{\alpha}_j^2}\right\} \quad (55)$$

$$k_{50}=\frac{2}{\lambda_{max}\left(\prod_{j=1}^{p}\hat{\alpha}_j^2\right)^{1/p}} \quad (56)$$

3.4. Performance of the Estimators

To investigate the performance of the Poisson Ridge RR and the Maximum Likelihood ML method, the Average MSE (AMSE) are obtained using the following equation:

$$\text{AMSE}=\frac{\sum_{i=1}^{r}SE_i}{r}=\frac{\sum_{i=1}^{r}(\hat{\beta}-\beta)_i'(\hat{\beta}-\beta)_i}{r} \quad (57)$$

where r = number of replicates = 1000

Where $\hat{\beta}$ is the estimator of β obtained from ML or PRR and SE is the squared error.

3.5. The Monte Carlo Simulation

This section consists of a brief description of how the data is generated and the factors varied in the simulation study.

3.6. The Design of the Experiment

The dependent variable (y) of the Poisson Regression Model is generated in R using pseudo-random numbers from the poisson distribution, with

$$\text{mean }(\mu_i)=e^{\beta_0+\beta_1x_{i1}+\cdots+\beta_px_{ip}};\ i=1, 2,\ldots,n \quad (58)$$

The parameter values in equ (4.1) are chosen so that $\sum_{j=1}^{p}\beta_j^2=1$ and $\beta_1=\beta_2=\cdots=\beta_p$. These are common restrictions in many simulation studies (zaldivar, 2018).

The degrees of freedom are defined as $df=n-p-1$. the value of $\beta_o=-1,0,1$.

The independent variables are generated as follows:

$$\mathcal{X}_{ij}=\sqrt{1-\rho^2}z_{ij}+\rho z_{ip};\ i=1,2,\ldots,n;\ j=1,2,\ldots,p \quad (59)$$

Where z_{ij} are pseudo-random numbers from the standard normal distribution, ρ^2 represents the degree of correlation. In the design of the experiment, three values of ρ^2 are considered which are 0.85, 0.90, 0.99. Other factors that were varied includes sample size (n) =50 and 200, number of explanatory variables (p) = 4 and 8 and intercept value (β_0) = -1, 0, 1.

4. Results / Findings

In this work, Multicollinearity was discussed. Some selected Ridge Regression (RR) estimators for estimating the ridge regression parameter k for Poisson Regression model were investigated for correlated variables. Monte-Carlo Simulation was carried out at two explanatory variables, $p = 4\ and\ 8$, three correlation levels $\rho = 0.85, 0.90, and\ 0.99$ and intercept, $\beta_o = -1, 0\ and\ 1$ with 50 selected estimators. For each combination, 1000 replications were performed, the performance of this estimators have been evaluated using the Average MSE.

From the simulated results, it was shown that some estimators are affected by sample sizes while some are not affected by sample sizes. For examples, at $\rho = 0.85, p = 4$ and $\beta_o = -1$, the MSE of the estimators are drastically affected by the sample size, while at $\rho = 0.90, and\ 0.99$ for $p = 4$ and $\beta_o = -1$, the estimators are not affected by the sample sizes. For $\rho = 0.85\ and\ 0.99, p = 4$ and $\beta_o = 0$, the MSE of the estimators are affected by the sample sizes, while it not affected by the sample sizes at $\rho = 0.90$. For $p = 4$, $\beta_o = 1$ and for all correlation levels, except for $\rho = 0.99$, the MSE of the estimators are not affected by the sample sizes.

For $p = 8$, $\beta_o = -1\ and$ for all correlation levels, the MSE of the estimators are not affected by the sample sizes. For $= 8$, $\beta_o = 0$, and $\rho = 0.85\ and\ 0.90$, the MSE of the estimators are not affected by the sample sizes except for $\rho = 0.99$. For $p = 8, \beta_o = 1$, the MSE of the estimators are not affected by the sample sizes at all levels of correlation.

Table 1. *Ranked Mean Square Error of estimators for p=4 and β=-1 at varying correlation levels and sample sizes.*

Estimators	ρ=0.85				ρ=0.90				ρ=0.99				
	Sample 50(0.85)	R1	Sample 200(0.85)	R2	Sample 50(0.90)	R3	Sample 200(0.90)	R4	Sample 50(0.99)	R5	Sample 200(0.99)	R6	Total MSE
k14	0.0002	3	0.6	32	0.0003	3	0	3	0	1	0	1	0.6005
k12	0.0002	2	0.003	13	0.0004	4	0	4	0	2	0	2	0.0036
k16	0.0002	4	12.5	43	0.0004	5	0	5	0	3	0	3	12.5006
k36	0.0001	1	0.35	31	0	1	0	1	0.0001	4	0	4	0.3502
k37	0.0004	5	2.37	38	0	2	0	2	0.0004	5	0	5	2.3708
k34	0.0067	9	0.007	18	0.0009	6	0	6	0.0064	10	0	6	0.021
k45	0.0181	17	0.005	16	0.0015	7	0	7	0.0172	16	0	7	0.0418
k46	0.0176	16	0.001	11	0.0069	12	0	11	0.0173	17	0	8	0.0428
k35	0.0265	21	0.29	30	0.0036	10	0	10	0.0254	22	0	9	0.3455
k47	0.0258	20	0	4	0.0114	16	0	12	0.0254	23	0	10	0.0626
k43	0.255	29	1.19	36	0.0015	8	0	8	0.242	26	0	11	1.6885
k42	0.255	27	0	6	0.0033	9	0	9	0.242	27	0	12	0.5003
k33	0.0257	19	1.85	37	0.0208	20	0.001	14	0.0253	21	0.001	13	1.9238
k41	0.255	28	0	7	0.0111	15	0.001	13	0.242	28	0.001	14	0.5101
k7	0.0054	8	1.19	35	0.0093	13	0.003	15	0.0044	8	0.003	15	1.2151
k8	0.0053	7	21.1	41	0.0094	14	0.004	16	0.0043	7	0.004	16	21.127
k40	0.0043	6	0	1	0.0054	11	0.005	17	0.0035	6	0.005	17	0.0232
k21	0.0084	10	0	2	0.0141	17	0.006	18	0.006	9	0.006	18	0.0405
k27	0.01	11	0.03	26	0.0179	19	0.008	20	0.0081	11	0.007	19	0.081
k29	0.0101	12	0.227	29	0.0175	18	0.007	19	0.0084	12	0.007	20	0.277
k18	0.0121	13	12.2	42	0.0209	21	0.009	21	0.01	13	0.008	21	12.26
k23	0.0143	14	0.009	20	0.0228	22	0.01	22	0.012	15	0.01	22	0.0781
k5	0.0164	15	0.005	15	0.0279	23	0.012	23	0.0115	14	0.012	23	0.0848
k19	0.021	18	0	3	0.0368	24	0.019	24	0.0176	18	0.019	24	0.1134
k17	0.0311	23	0	5	0.055	26	0.024	25	0.0234	20	0.024	25	0.1575
k26	0.031	22	0.006	17	0.0531	25	0.026	26	0.0217	19	0.025	26	0.1628
k22	0.0403	24	0.024	24	0.0648	27	0.029	27	0.0349	24	0.029	27	0.222
k20	0.0903	25	12.5	44	0.158	30	0.07	28	0.067	25	0.068	28	12.9533
k48	0.4	31	0	8	0.128	29	0.104	29	0.94	33	0.113	29	1.685
k25	0.212	26	0.07	27	0.11	28	0.111	30	0.269	29	0.115	30	0.887
k50	0.474	34	0	9	0.168	31	0.22	31	1.13	34	0.214	31	2.206
k24	0.272	30	0.019	23	0.172	32	0.227	32	0.352	30	0.219	32	1.261
k30	0.438	32	0.106	28	0.264	33	0.293	33	0.565	31	0.287	33	1.953
k31	0.458	33	0.025	25	0.38	35	0.353	34	0.592	32	0.342	34	2.15
k49	0.568	35	14.0	45	0.32	34	0.405	35	1.37	38	0.388	35	17.051
k9	0.95	36	0.731	33	0.588	36	0.609	36	1.29	35	0.597	36	4.765
k10	1	37	0.012	22	0.872	39	0.738	37	1.35	36	0.717	37	4.689
k4	1	38	27.4	50	0.872	40	0.738	38	1.35	37	0.717	38	32.077
k2	4.11	42	23.5	49	1.94	43	1.29	40	3.79	40	1.38	39	36.01
k38	4.11	41	0.001	12	1.94	42	1.29	39	12	44	1.38	40	20.721
k28	3.2	40	0.01	21	0.7	37	1.82	41	3.62	39	1.74	41	11.09
k32	1.77	39	0.007	19	0.77	38	2.38	42	4.15	41	2.3	42	11.377
k6	6.24	44	4.99	40	1.37	41	3.99	43	7.09	43	3.81	43	27.49
k1	18.5	45	28.5	51	7.64	44	5.62	44	17.7	45	6.02	44	83.98
k3	49.6	47	19.9	48	17.6	46	21.6	45	44.5	47	21.1	45	174.3
ML	89.0058	51	193.112	47	46.0752	51	88.049	47	76.0055	51	100.005	46	592.2525
k13	71.1	48	0.004	14	38.9	48	87	46	64.8	50	105	47	366.804

Estimators	ρ=0.85				ρ=0.90				ρ=0.99				
	Sample 50(0.85)	R1	Sample 200(0.85)	R2	Sample 50(0.90)	R3	Sample 200(0.90)	R4	Sample 50(0.99)	R5	Sample 200(0.99)	R6	Total MSE
k15	71.1	49	0.731	34	38.9	49	89	48	64.7	48	107	48	371.431
k11	71.1	50	4.07	39	39.7	50	89.4	49	64.7	49	107	49	375.97
k44	5.83	43	16.9	46	37	47	96.8	50	5.63	42	117	50	279.16
k39	22.2	46	0	10	13	45	166	51	21.2	46	157	51	379.4

Table 2. *Ranked Mean Square Error of estimators for p=4 and β=0 at varying correlation levels and sample sizes.*

Estimators	ρ=0.85				ρ=0.90				ρ=0.99				
	Sample 50(0.85)	R1	Sample 200(0.85)	R2	Sample 50(0.90)	R3	Sample 200(0.90)	R4	Sample 50(0.99)	R5	Sample 200(0.99)	R6	Total MSE
k36	0.0178	17	0	3	0	2	0	2	0	1	0	1	0.0178
k37	0.0002	1	0	1	0	1	0	1	0.0001	2	0	2	0.0003
k14	7.24	45	0	5	0.0004	4	0	4	0.0006	4	0	3	7.241
k16	7.23	44	0	4	0.0004	3	0	3	0.0007	5	0	4	7.2311
k12	7.25	46	0	6	0.0004	5	0	5	0.0008	6	0	5	7.2512
k34	0.0107	10	0	2	0.0009	6	0	6	0.0039	9	0	6	0.0155
k45	0.0036	6	0.0001	7	0.0016	7	0	7	0.0083	13	0	7	0.0136
k46	0.0059	7	0.0005	12	0.0069	12	0	11	0.0066	11	0.001	8	0.0209
k47	0.0087	9	0.0004	11	0.0111	16	0	12	0.0067	12	0.001	9	0.0279
k42	0.0026	5	0.0003	10	0.0035	9	0	9	0.0139	14	0.001	10	0.0213
k43	0.0017	2	0.0002	9	0.0016	8	0	8	0.014	15	0.001	11	0.0185
k35	0.013	11	0.0001	8	0.0037	10	0	10	0.0157	17	0.001	12	0.0335
k33	0.252	33	0.0013	14	0.0206	21	0.001	14	0.0244	23	0.001	13	0.3003
k41	0.123	28	0.0012	13	0.0116	17	0.001	13	0.0514	31	0.004	14	0.1922
k7	3.27	41	0.0043	16	0.0093	13	0.003	15	0.021	19	0.006	15	3.3136
k8	0.0077	8	0.0039	15	0.0094	14	0.004	16	0.0237	22	0.006	16	0.0547
k21	0.0441	19	0.0064	20	0.0136	18	0.006	18	0.0303	24	0.013	17	0.1134
k29	1.65	40	0.0081	22	0.0177	19	0.007	19	0.041	25	0.013	18	1.7368
k27	0.0167	16	0.0073	21	0.018	20	0.007	20	0.0455	26	0.013	19	0.1075
k40	0.0024	4	0.102	33	0.0046	11	0.005	17	0.0634	33	0.013	20	0.1904
k18	0.0163	15	0.0096	23	0.0211	22	0.009	21	0.0489	30	0.015	21	0.1199
k5	0.412	34	0.0127	25	0.0272	24	0.011	23	0.0457	27	0.02	22	0.5286
k23	0.0148	14	0.0118	24	0.0225	23	0.01	22	0.0697	34	0.027	23	0.1558
k19	0.0487	20	0.0152	26	0.0368	25	0.018	24	0.107	37	0.029	24	0.2547
k17	0.051	22	0.0231	27	0.0544	28	0.023	25	0.0998	36	0.034	25	0.2853
k26	0.0336	18	0.0241	28	0.0521	27	0.025	26	0.093	35	0.04	26	0.2678
k48	0.0134	12	0.0687	30	0.0109	15	0.069	28	0.0004	3	0.069	27	0.2314
k22	0.0647	23	0.0353	29	0.0655	29	0.031	27	0.225	38	0.084	28	0.5055
k20	0.104	26	0.0751	31	0.158	32	0.069	29	0.281	40	0.087	29	0.7741
k25	0.0756	24	0.0752	32	0.04	26	0.096	30	0.0053	10	0.098	30	0.3901
k50	0.179	30	0.153	34	0.0966	30	0.223	31	0.0018	7	0.225	31	0.8784
k24	0.0501	21	0.197	36	0.138	31	0.242	32	0.017	18	0.251	32	0.8951
k30	0.0763	25	0.24	38	0.193	33	0.294	33	0.0225	21	0.305	33	1.1308
k31	0.114	27	0.206	37	0.346	35	0.353	34	0.0217	20	0.363	34	1.4037
k49	0.0147	13	0.184	35	0.291	34	0.414	35	0.002	8	0.416	35	1.3217
k9	0.156	29	0.477	42	0.455	36	0.601	36	0.0539	32	0.622	36	2.3649
k10	0.225	32	0.41	40	0.808	39	0.731	37	0.0474	28	0.751	37	2.9724
k4	4.48	42	0.41	41	0.808	40	0.731	38	0.0474	29	0.751	38	7.2274
k38	1.61	39	1.73	45	1.51	43	1.03	40	2.41	44	0.962	39	9.252
k2	1.6	38	1.73	44	1.51	42	1.03	39	5.37	45	0.962	40	12.202
k28	0.478	35	4.87	46	0.745	38	1.89	42	0.313	41	1.89	41	10.186
k32	0.213	31	1.3	43	0.675	37	2.57	43	0.0146	16	2.57	42	7.3426
k1	6.96	43	7.06	48	5.54	45	4.18	45	12.3	47	3.75	43	39.79
k6	0.925	36	9.48	49	1.45	41	4.14	44	0.813	42	4.13	44	20.938
k44	0.0017	3	0.41	39	4.34	44	1.03	41	8.41	46	4.31	45	18.502
k15	36.9	50	0.0043	18	29.5	48	19.5	47	32.5	49	20.3	46	138.7
k3	1.52	37	13.5	50	11.9	46	21.3	48	0.275	39	21.5	47	69.995
k13	36.9	51	0.0043	19	29.5	49	18.7	46	84.7	50	24.1	48	193.9
k11	36.8	49	0.0043	17	32	50	25.5	49	32.2	48	25	49	151.5
ML	36.2	48	6.84	47	33.008	51	28.004	50	97.009	51	28.045	50	229.11
k39	14.7	47	13.7	51	13.4	47	171	51	2.27	43	169	51	384.07

Table 3. *Ranked Mean Square Error of estimators for p=4 and β=1 at varying correlation levels and sample sizes.*

Estimators	ρ=0.85				ρ=0.90				ρ=0.99				
	Sample 50(0.85)	R1	Sample 200(0.85)	R2	Sample 50(0.90)	R3	Sample 200(0.90)	R4	Sample 50(0.99)	R5	Sample 200(0.99)	R6	Total MSE
k14	0.0005	3	0	3	0.0003	3	0	3	0	1	0	1	0.0008
k12	0.0006	4	0	4	0.0003	4	0	4	0	2	0	2	0.0009
k16	0.0006	5	0	5	0.0003	5	0	5	0	3	0	3	0.0009
k36	0	1	0	1	0	1	0	1	0.0001	4	0	4	0.0001
k37	0.0001	2	0	2	0	2	0	2	0.0004	5	0	5	0.0005
k34	0.0046	6	0	6	0.0009	6	0	6	0.0067	10	0	6	0.0122
k45	0.0078	7	0	7	0.0014	7	0	7	0.0179	17	0	7	0.0271
k46	0.0164	9	0	9	0.0069	12	0	11	0.0179	18	0	8	0.0412
k47	0.0223	12	0	12	0.0116	16	0	12	0.0263	22	0	9	0.0602
k35	0.0185	10	0	10	0.0034	10	0	10	0.0264	23	0	10	0.0483
k43	0.0145	8	0	8	0.0014	8	0	8	0.251	27	0	11	0.2669
k42	0.0199	11	0	11	0.0032	9	0	9	0.251	28	0	12	0.2741
k33	0.0254	13	0.001	13	0.021	21	0.001	14	0.0262	21	0.001	13	0.0756
k41	0.0621	19	0.001	14	0.0107	15	0.001	13	0.252	29	0.001	14	0.3278
k7	0.028	14	0.004	15	0.009	13	0.003	15	0.0043	8	0.003	15	0.0513
k8	0.0323	15	0.004	16	0.0091	14	0.004	16	0.0041	7	0.004	16	0.0575
k40	0.058	18	0.005	17	0.0059	11	0.005	17	0.0034	6	0.005	17	0.0823
k21	0.0513	16	0.006	18	0.0144	17	0.006	18	0.0059	9	0.006	18	0.0896
k29	0.0574	17	0.007	19	0.0169	18	0.007	19	0.008	12	0.007	19	0.1033
k27	0.066	20	0.008	20	0.0172	19	0.007	20	0.0078	11	0.008	20	0.114
k18	0.0685	21	0.009	21	0.0202	20	0.008	21	0.0095	13	0.008	21	0.1232
k23	0.0909	23	0.01	22	0.0224	22	0.009	22	0.0113	14	0.009	22	0.1526
k5	0.0715	22	0.012	23	0.0285	23	0.012	23	0.0113	15	0.012	23	0.1473
k19	0.132	26	0.019	24	0.0356	24	0.019	24	0.0167	16	0.019	24	0.2413
k17	0.163	29	0.025	25	0.0549	26	0.024	25	0.0228	20	0.024	25	0.3137
k26	0.15	28	0.026	26	0.0546	25	0.025	26	0.0212	19	0.025	26	0.3018
k22	0.257	32	0.03	27	0.0624	27	0.027	27	0.0328	24	0.027	27	0.4362
k20	0.419	35	0.071	28	0.158	29	0.068	28	0.0654	25	0.067	28	0.8484
k25	0.102	24	0.116	30	0.146	28	0.124	29	0.206	26	0.123	29	0.817
k48	0.106	25	0.11	29	0.178	30	0.135	30	0.598	33	0.127	30	1.254
k24	0.168	30	0.228	32	0.193	31	0.213	31	0.274	30	0.207	31	1.283
k50	0.143	27	0.223	31	0.21	32	0.217	32	0.721	34	0.208	32	1.722
k30	0.256	31	0.299	33	0.307	33	0.288	33	0.434	31	0.283	33	1.867
k31	0.343	34	0.36	34	0.39	35	0.339	34	0.451	32	0.335	34	2.218
k49	0.257	33	0.411	35	0.33	34	0.377	35	0.869	35	0.369	35	2.613
k9	0.531	36	0.612	36	0.673	37	0.605	36	0.965	36	0.593	36	3.979
k4	0.738	39	0.747	37	0.885	39	0.716	37	1.01	37	0.705	37	4.801
k10	0.738	40	0.747	38	0.885	40	0.716	38	1.01	38	0.705	38	4.801
k2	4.33	43	1.38	40	2.42	43	1.7	41	3.34	40	1.51	39	14.68
k38	1.89	42	1.38	39	2.42	42	1.7	40	7.4	44	1.51	40	16.3
k28	0.673	38	1.85	41	0.648	36	1.7	39	3.42	41	1.64	41	9.931
k32	0.666	37	2.36	42	0.844	38	2.24	42	2.7	39	2.17	42	10.98
k6	1.34	41	4.06	43	1.28	41	3.71	43	6.66	43	3.58	43	20.63
k1	6.73	44	5.5	44	10.1	44	7.78	44	15.2	45	6.74	44	52.05
k3	13.8	46	21.4	48	21.4	46	21.8	45	39.5	47	20.8	45	138.7
ML	79.0084	51	23.005	49	54.0072	51	23.005	46	64.0058	51	42.005	46	285.0364
k13	66.8	50	18.8	46	37.9	48	86.6	47	58.4	49	47.7	47	316.2
k15	32	49	20.4	47	37.9	49	88.6	48	58.4	50	49.1	48	286.4
k11	30.9	48	25.6	50	38.6	50	88.7	49	58.2	48	50.7	49	292.7
k44	17.8	47	12.9	45	36.1	47	96.4	50	5.81	42	55.1	50	224.11
k39	11	45	168	51	13.2	45	154	51	18.9	46	147	51	512.1

Table 4. *Ranked Mean Square Error of estimators for p=8 and β=-1 at varying correlation levels and sample sizes.*

Estimators	ρ=0.85				ρ=0.90				ρ=0.99				
	Sample 50(0.85)	**R1**	**Sample 200(0.85)**	**R2**	**Sample 50(0.90)**	**R3**	**Sample 200(0.90)**	**R4**	**Sample 50(0.99)**	**R5**	**Sample 200(0.99)**	**R6**	**Total MSE**
k36	0.0001	4	0	1	0	1	0	1	0.0001	4	0.0001	1	0.0003
k37	0.0004	5	0.0001	2	0.0001	2	0	2	0.0004	5	0.0003	2	0.0013
k12	0	1	0.0004	3	0.0004	3	0	3	0	1	0.0004	3	0.0012
k14	0	2	0.0004	4	0.0004	4	0	4	0	2	0.0004	4	0.0012
k16	0	3	0.0004	5	0.0004	5	0	5	0	3	0.0004	5	0.0012
k34	0.0067	10	0.002	6	0.0022	7	0	7	0.0064	10	0.0052	6	0.0225
k40	0.0028	6	0.0048	7	0.0048	9	0.005	17	0.0034	6	0.006	7	0.0268
k8	0.0037	8	0.0095	12	0.0094	14	0.004	16	0.0043	7	0.0119	8	0.0428
k7	0.0037	7	0.0095	11	0.0093	13	0.004	15	0.0044	8	0.0119	9	0.0428
k46	0.0183	19	0.0078	9	0.007	11	0	8	0.0176	18	0.0126	10	0.0633
k45	0.0183	18	0.0051	8	0.0017	6	0	6	0.0175	16	0.014	11	0.0566
k21	0.0049	9	0.014	14	0.0138	17	0.006	18	0.0059	9	0.0182	12	0.0628
k47	0.0273	23	0.0122	13	0.0115	15	0	10	0.0259	23	0.0183	13	0.0952
k33	0.0271	22	0.0209	17	0.0213	21	0.001	11	0.0258	22	0.0196	14	0.1157
k35	0.0265	21	0.0079	10	0.0087	12	0	9	0.0252	21	0.0206	15	0.0889
k29	0.0071	11	0.0179	15	0.0176	18	0.008	19	0.0083	12	0.0224	16	0.0813
k27	0.0072	12	0.0183	16	0.0179	19	0.008	20	0.0081	11	0.0226	17	0.0821
k18	0.0083	13	0.0214	18	0.021	20	0.009	21	0.0098	13	0.0264	18	0.0959
k23	0.0101	15	0.0225	19	0.0228	22	0.011	22	0.0119	15	0.0269	19	0.1052
k5	0.0089	14	0.0278	20	0.0278	23	0.012	23	0.0111	14	0.034	20	0.1216
k19	0.0158	16	0.0376	21	0.0367	24	0.02	24	0.0175	17	0.0461	21	0.1737
k26	0.0166	17	0.0534	22	0.0535	25	0.025	26	0.0208	19	0.0657	22	0.235
k17	0.0186	20	0.0557	23	0.0549	26	0.024	25	0.0226	20	0.0686	23	0.2444
k22	0.0291	24	0.065	24	0.0663	27	0.033	27	0.0347	24	0.0688	24	0.2969
k25	0.209	26	0.0725	28	0.0778	28	0.128	29	0.203	26	0.185	25	0.8753
k20	0.048	25	0.162	29	0.161	29	0.069	28	0.0634	25	0.188	26	0.6914
k41	0.253	29	0.0698	26	0.0126	16	0.003	14	0.245	29	0.199	27	0.7824
k42	0.252	28	0.0698	25	0.0051	10	0.002	13	0.245	28	0.203	28	0.7769
k43	0.252	27	0.07	27	0.0038	8	0.002	12	0.245	27	0.204	29	0.7768
k24	1.06	30	0.187	30	0.207	30	0.48	30	0.307	30	0.405	30	2.646
k32	1.68	31	0.615	37	0.655	36	2.48	39	1.75	39	0.605	31	7.785
k30	1.72	32	0.289	32	0.321	31	0.762	31	0.493	31	0.67	32	4.255
k48	8.6	39	0.267	31	0.36	32	2.15	38	0.564	33	0.712	33	12.653
k31	1.79	33	0.389	34	0.426	33	0.815	32	0.517	32	0.718	34	4.655
k50	10.3	40	0.326	33	0.43	34	2.57	40	0.678	34	0.839	35	15.143
k28	3.28	34	0.748	38	0.775	38	1.97	36	3.41	40	0.949	36	11.132
k49	11.9	41	0.447	35	0.559	35	3.07	41	0.821	35	0.998	37	17.795
k9	3.83	35	0.606	36	0.732	37	1.64	33	1.11	36	1.35	38	9.268
k4	3.97	36	0.85	39	0.979	39	1.76	34	1.16	37	1.47	39	10.189
k10	3.97	37	0.85	40	0.979	40	1.76	35	1.16	38	1.47	40	10.189
k6	6.83	38	1.46	41	1.56	41	4.32	42	6.66	43	1.93	41	22.76
k44	19.7	43	4.19	44	8.4	45	2.13	37	9.6	44	3.25	42	47.27
k2	49.5	47	2.06	42	2.62	42	12.1	44	4.51	41	3.52	43	74.31
k38	49.5	48	2.06	43	2.62	43	12.1	45	4.51	42	3.52	44	74.31
k11	72	50	32.6	50	27.1	50	28.7	48	68.7	49	10.7	45	239.8
k3	48.9	46	23.9	47	12.1	47	33.6	50	46.8	47	11.2	46	176.5
k15	17.2	42	28.3	48	11.5	46	23.9	47	69.3	50	11.9	47	162.1
k39	21.3	44	13.7	46	14.5	48	166	51	21.7	46	13.5	48	250.7
k1	24.2	45	7.94	45	6.74	44	7.37	43	17.5	45	13.9	49	77.65
k13	71.8	49	28.3	49	21.1	49	20.9	46	68.7	48	20	50	230.8
ML	92.1	51	49.2	51	41.4	51	28.8	49	83.8	51	36.3	51	331.6

Table 5. *Ranked Mean Square Error of estimators for p=8 and β=0 at varying correlation levels and sample sizes.*

Estimators	ρ=0.85				ρ=0.90				ρ=0.99				
	Sample 50(0.85)	**R1**	**Sample 200(0.85)**	**R2**	**Sample 50(0.90)**	**R3**	**Sample 200(0.90)**	**R4**	**Sample 50(0.99)**	**R5**	**Sample 200(0.99)**	**R6**	**Total MSE**
k36	0	1	0	1	0.0001	1	0	1	0.0001	1	0	1	0.0002
k37	0.0001	2	0	2	0.0002	2	0.0001	2	0.0002	2	0	2	0.0006
k14	0.0003	3	0	3	0.0004	3	0.0004	3	0.0006	4	0	3	0.0017
k16	0.0003	4	0	4	0.0004	4	0.0004	4	0.0008	5	0	4	0.0019
k12	0.0004	5	0	5	0.0004	5	0.0004	5	0.0009	6	0	5	0.0021
k34	0.0022	7	0	7	0.0039	7	0.0022	6	0.0042	9	0	6	0.0125
k45	0.0017	6	0	6	0.0019	6	0.0057	8	0.0089	13	0	7	0.0182
k35	0.0087	12	0	9	0.0155	18	0.0087	10	0.0165	15	0	8	0.0494
k46	0.0067	11	0	8	0.007	11	0.0082	9	0.0069	10	0.002	9	0.0308
k47	0.0109	15	0	10	0.0113	15	0.0126	14	0.007	11	0.002	10	0.0438
k42	0.0048	10	0.002	13	0.0066	10	0.0779	28	0.0168	16	0.002	11	0.1101
k43	0.0037	8	0.002	12	0.006	9	0.0781	29	0.0154	14	0.003	12	0.1082
k33	0.0201	21	0.001	11	0.0212	22	0.0209	18	0.0249	22	0.003	13	0.0911
k41	0.0118	17	0.002	14	0.0123	16	0.0778	27	0.0584	30	0.004	14	0.1663
k40	0.0048	9	0.005	17	0.0053	8	0.0049	7	0.0629	32	0.005	15	0.0879
k18	0.0208	22	0.008	21	0.0206	21	0.0218	19	0.0586	31	0.009	16	0.1388
k7	0.0092	13	0.003	15	0.0091	12	0.0096	11	0.022	19	0.013	17	0.0659
k8	0.0093	14	0.004	16	0.0097	13	0.0096	12	0.0229	20	0.015	18	0.0705
k21	0.0134	18	0.006	18	0.0135	17	0.014	15	0.0313	23	0.018	19	0.0962
k19	0.0365	25	0.019	24	0.0364	25	0.0383	22	0.124	36	0.018	20	0.2722
k29	0.0173	19	0.007	19	0.0176	19	0.0182	16	0.0449	24	0.024	21	0.129
k27	0.0178	20	0.007	20	0.0195	20	0.0185	17	0.045	25	0.028	22	0.1358
k17	0.0535	28	0.024	25	0.0532	27	0.0562	25	0.113	35	0.031	23	0.3309
k5	0.0266	24	0.012	23	0.027	24	0.0273	21	0.0492	28	0.034	24	0.1761
k23	0.0226	23	0.011	22	0.0268	23	0.0228	20	0.0704	33	0.042	25	0.1956
k26	0.0509	27	0.025	26	0.0535	28	0.0521	24	0.102	34	0.064	26	0.3475
k48	0.0111	16	0.093	29	0.0108	14	0.0107	13	0.0004	3	0.07	27	0.196
k25	0.0418	26	0.105	30	0.0409	26	0.0439	23	0.0079	12	0.099	28	0.3385
k20	0.155	32	0.069	28	0.155	32	0.161	32	0.287	40	0.106	29	0.933
k22	0.0774	29	0.049	27	0.123	30	0.0685	26	0.274	39	0.164	30	0.7559
k50	0.101	30	0.218	31	0.0944	29	0.0932	30	0.002	7	0.23	31	0.7386
k24	0.141	31	0.233	32	0.138	31	0.133	31	0.0171	17	0.271	32	0.9331
k30	0.2	33	0.29	33	0.191	33	0.187	33	0.0244	21	0.33	33	1.2224
k31	0.361	35	0.349	34	0.342	35	0.331	35	0.0217	18	0.39	34	1.7947
k49	0.304	34	0.402	35	0.283	34	0.275	34	0.002	8	0.427	35	1.693
k9	0.475	36	0.596	36	0.446	36	0.437	36	0.0577	29	0.677	36	2.6887
k4	0.846	39	0.725	37	0.791	39	0.766	38	0.0475	26	0.81	37	3.9855
k10	0.846	40	0.725	38	0.791	40	0.766	39	0.0475	27	0.81	38	3.9855
k38	1.5	42	1.19	39	6.97	43	1.49	40	1.56	41	0.989	39	13.699
k2	1.5	43	1.19	40	6.97	44	1.49	41	5.76	44	0.989	40	17.899
k28	0.732	38	1.82	41	0.724	38	4.67	43	6.11	45	1.79	41	15.846
k32	0.662	37	2.42	42	0.627	37	0.63	37	0.256	38	2.48	42	7.075
k1	5.48	44	4.89	44	25.4	47	5.53	44	5.19	43	3.87	43	50.36
k6	1.43	41	3.97	43	1.4	41	9.28	45	13.2	47	3.88	44	33.16
k44	9.06	45	69.4	50	4.2	42	4.24	42	8.28	46	4	45	99.18
k13	29.5	49	63.5	47	46.3	48	28.8	48	88.2	51	17.4	46	273.7
k15	29.5	48	64.8	48	46.3	49	28.8	49	18.7	49	18.2	47	206.3
ML	48.1	51	28.4	46	84.1	51	30.3	50	19.1	50	20.7	48	230.7
k3	12.5	47	21	45	11.6	45	11.3	46	0.249	37	22.1	49	78.749
k11	31.1	50	66.1	49	47.2	50	32.1	51	17	48	24.2	50	217.7
k39	12.4	46	163	51	12.7	46	13.4	47	2.99	42	151	51	355.49

Table 6. *Ranked Mean Square Error of estimators for p=8 and β=1 at varying correlation levels and sample sizes.*

Estimators	ρ=0.85				ρ=0.90				ρ=0.99				
	Sample 50(0.85)	R1	Sample 200(0.85)	R2	Sample 50(0.90)	R3	Sample 200(0.90)	R4	Sample 50(0.99)	R5	Sample 200(0.99)	R6	Total MSE
k36	0	1	0.0001	1	0.0001	1	0.0001	1	0	1	0	1	0.0003
k37	0	2	0.0002	2	0.0003	2	0.0004	2	0	2	0.0001	2	0.001
k12	0	3	0.0004	3	0.0004	3	0.0004	3	0	3	0.0004	3	0.0016
k14	0	4	0.0004	4	0.0004	4	0.0004	4	0	4	0.0004	4	0.0016
k16	0	5	0.0004	5	0.0004	5	0.0004	5	0	5	0.0004	5	0.0016
k34	0	6	0.0032	6	0.0052	7	0.0061	7	0	6	0.0022	6	0.0167
k40	0.005	10	0.0051	7	0.0057	8	0.006	6	0.005	10	0.0048	7	0.0316
k45	0.013	17	0.0074	8	0.0023	6	0.0163	11	0.017	18	0.0057	8	0.0617
k46	0.013	18	0.009	9	0.008	9	0.014	10	0.017	17	0.008	9	0.069
k35	0	7	0.0125	12	0.0206	19	0.024	17	0	7	0.0087	10	0.0658
k7	0.004	8	0.0098	10	0.0095	11	0.0115	8	0.004	8	0.0095	11	0.0483
k8	0.004	9	0.0098	11	0.0095	12	0.0116	9	0.004	9	0.0095	12	0.0484
k47	0.019	19	0.0138	13	0.0138	14	0.0201	14	0.026	23	0.0124	13	0.1051
k21	0.006	11	0.0148	14	0.0153	15	0.0178	12	0.006	11	0.0141	14	0.074
k29	0.007	12	0.0186	15	0.0179	17	0.0218	15	0.007	12	0.018	15	0.0903
k27	0.008	13	0.0187	16	0.018	18	0.0219	16	0.008	13	0.018	16	0.0926
k33	0.019	20	0.0214	17	0.0238	21	0.0178	13	0.026	22	0.0208	17	0.1288
k18	0.009	14	0.0221	18	0.0211	20	0.0256	18	0.008	14	0.0214	18	0.1072
k23	0.01	15	0.0237	19	0.0242	22	0.0263	19	0.01	15	0.0231	19	0.1173
k5	0.012	16	0.0293	20	0.0307	23	0.033	20	0.012	16	0.0285	20	0.1455
k19	0.02	21	0.038	21	0.037	24	0.0448	21	0.02	19	0.0362	21	0.196
k26	0.026	23	0.0564	22	0.0597	26	0.0632	22	0.025	21	0.055	22	0.2853
k17	0.025	22	0.0575	23	0.0575	25	0.0665	23	0.024	20	0.0554	23	0.2859
k22	0.029	24	0.067	24	0.0677	27	0.0665	24	0.028	24	0.0663	24	0.3245
k41	0.17	29	0.103	26	0.017	16	0.232	27	0.23	29	0.0776	25	0.8296
k42	0.17	27	0.103	25	0.0104	13	0.236	28	0.229	27	0.0779	26	0.8263
k43	0.17	28	0.104	27	0.0092	10	0.237	29	0.229	28	0.0781	27	0.8273
k25	0.144	26	0.114	28	0.188	29	0.212	26	0.164	26	0.0816	28	0.9036
k20	0.069	25	0.166	29	0.166	28	0.178	25	0.062	25	0.164	29	0.805
k24	0.386	30	0.288	30	0.529	30	0.486	30	0.464	30	0.226	30	2.379
k30	0.615	31	0.461	31	0.864	32	0.802	32	0.763	31	0.352	31	3.857
k48	1.11	33	0.527	32	1.34	35	0.977	34	1.44	33	0.411	32	5.805
k31	0.661	32	0.534	33	0.908	33	0.834	33	0.805	32	0.448	33	4.19
k50	1.33	35	0.631	34	1.6	36	1.16	36	1.72	38	0.495	34	6.936
k49	1.59	38	0.765	36	1.87	37	1.36	37	2.04	39	0.633	35	8.258
k32	2.38	40	0.646	35	0.621	31	0.587	31	2.33	40	0.657	36	7.221
k9	1.29	34	1.36	38	1.93	38	1.61	38	1.59	34	0.754	37	8.534
k28	1.83	39	0.858	37	1.17	34	1.05	35	1.69	37	0.822	38	7.42
k4	1.38	36	1.51	39	2.03	39	1.7	39	1.68	35	0.987	39	9.287
k10	1.38	37	1.51	40	2.03	40	1.7	40	1.68	36	0.987	40	9.287
k6	4	41	2.11	41	2.57	41	2.12	41	3.68	41	1.61	41	16.09
k2	4.86	42	3.4	42	9.18	42	4.83	43	6.3	42	2.77	42	31.34
k38	4.86	43	3.4	43	9.18	43	4.83	44	6.3	43	2.77	43	31.34
k44	18.9	46	7.03	44	19.1	48	3.12	42	23.7	49	4.28	44	76.13
k1	21.3	48	8.51	45	12.6	46	19.4	49	17.9	46	7.27	45	86.98
k11	23.6	49	13.1	49	19.6	50	11	45	23.1	48	10.3	46	100.7
k13	17.6	45	9.95	47	19.5	49	25.1	51	17.2	45	10.8	47	100.15
k39	163	51	14.5	50	14.5	47	11.3	46	144	51	15.5	48	362.8
k15	17.4	44	9.84	46	10.3	44	15	48	16.4	44	36.6	49	105.54
k3	19.9	47	11.8	48	11.5	45	12.9	47	18.6	47	49	50	123.7
ML	35.6	50	22.3	51	25.7	51	19.4	50	27.5	50	52.2	51	182.7

5. Conclusion

Simulations were varied by sample size (n). The sample sizes used ere $n = 50$ and $n = 200$. Generally, the larger the sample size, the smaller the MSE values. It is also important to note that the sample size affects the performance of individual k estimators. With $n = 50$ and $p = 4$, the k estimators with the lowest total MSE values were k_{37}, k_{36}, k_{34}, k_{45}, k_{46}, k_8, k_{35}, k_{47}, k_{40}, and k_{21}. With $n = 50$ and $p = 8$, the k estimators with the lowest total MSE values were k_{36}, k_{37}, k_{14}, k_{16}, k_{12}, k_{34}, k_7, k_8, k_{45}, k_{40}, k_{46}, k_{21}, k_{35}, k_{29}, and k_{27}. While with $n = 200$ and $p = 8$, the k estimators with the lowest total MSE values were k_{36}, k_{37}, k_{12}, k_{14}, k_{16}, k_{34}, k_{40}, k_{45}, k_{46}, k_7, k_{35}, k_8, and k_{47}.

Simulations were also varied by the number of explanatory variables (p), with $p = 4$ and 8. It was discovered that the total MSE increases with an increase in p, therefore the PRR is best used when the number of explanatory variable is small.

Estimators were judged by their total MSE value for the different sample sizes, correlation, number of explanatory variable and intercept. The estimators that produced the lowest total MSE values were selected and they are as follows: k_{34}, k_{40}, k_{45}, k_{35}, k_{36}, k_{12}, k_{47}, k_{21}, k_{37} and k_8.

In conclusion, the MSE of the estimators performed better in an increased explanatory variables p and an increased intercept value. It was also observed that k_{36}, k_{12}, k_{37}, k_{14} and k_{16} performed better on the average at all correlation levels, sample sizes, intercept values and explanatory variables

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