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Neighbor designs: A new approach of local control

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Abstract: It is well known that randomization, replication and local control play important role in experimental design. Neighbor designs provide a tool for local control in situation where experimental units are influenced by neighboring units. A neighbor design is called one-dimensional if neighbor effects are controlled in only one way, i.e., either in row or in column direction. In two-dimensional design, neighbor effects are controlled in both ways (rows and columns). In this paper the concept of neighbor designs, its types and importance is discussed with examples. Models of Neighbor effects for different situations are also discussed.

Keywords: One-Dimensional Neighbor Designs, Two-Dimensional Neighbor Designs, Circular Design

1. Introduction

Randomization, replication and local control play important role in experimental design. Usually randomization reduces bias, replication provides experimental error estimate and local control makes the design efficient by minimizing the experimental error. Neighbor designs provide a tool for local control in biometrics, agriculture, horticulture and forestry. There are situations where experimental units are influenced by neighboring units for example in agriculture, the response on a given plot may be affected by treatments on neighboring plots as well as by the treatment applied to that plot. Observations are not independent in such type of experiments. Usual statistical techniques cannot be applied here because variability arises in experimental results due to neighbor effects, hence leading to substantial losses in efficiency. Neighbor designs are recommended for the cases where the performance of treatment is affected by the neighboring treatments. “In such situations, it is impossible to achieve orthogonality between direct effects and neighbor effects”, [Bailey and Druilhet (2004)].

2. Neighbor Designs

Neighbor designs provide a tool for local control in biometrics, agriculture, horticulture and forestry. Neighbor effects (either natural or due to layout of plots) can deprive the results from its representativeness. Therefore it is important to control it through design or analysis. Design is a specification of the preparations made before the measurement e. g. allocation of treatments to experimental units and analysis is drawing conclusions about the treatments from the result of the experiment. Neighbor designs are used to study the neighbor effects among neighboring units. Neighbor effects can be caused by differences in height, root vigor, or germination date of plant in agriculture. Similarly fertilizer, irrigation, or pesticide applied on one plot may cause neighbor effect to its adjacent plots. “In certain situations, neighbor effect may depend upon direction (shade, wind, etc.), which motivates to distinguish between left and right neighbors”, [Hamad et al. (2010)]. In neighbor designs, treatments are allotted in such a way that every treatment may occur equally often with every other treatment as neighbor in order to ensure that no treatment is unduly disadvantaged by its neighbors. Neighbor designs can be divided mainly into two types:

a) One-dimensional neighbor designs
b) Two-dimensional neighbor designs

We discuss each case separately with some examples from our daily life in order to show the importance of neighbor designs.

2.1. One-Dimensional Neighbor Designs

A neighbor design is called one-dimensional if neighbor effects are controlled in only one way, i.e., either in row or in column direction. One-dimensional neighbor designs are used in circular plates in biometrics and in block design setup in the field of agriculture where each block is a single line of plots and blocks are well separated. Border plots are needed on both ends of every block to make the design circular because "the effect of having no treatment as a neighbor differs from the neighbor effect of any treatment", [Azais and Druilhet (1997)]. Treatment on border plots is same as the treatment on the inner plot at the other end of the block, but response from these border plots is not included in response variable. Number of inner plots of any block is its length. These border plots do not add too much to the cost of one-dimensional experiments. "A bordered block will said to be circular if the level of factor in the left border or right border is the same as the level of factor in the right-end inner plot or the left-end inner plot, respectively", [Monod (1992)].

Example 1

One-dimensional design with bordered plots in agriculture for five treatments in 5 blocks of size 4 is given below. A line of treated border plots on each side is required for the neighbor effect of edges. These border plots provide the neighboring treatments for plots 1 and 4, but are not used for measuring response variables. Following neighbor design have parameters $v = 5$, $b = 5$, $k = 4$ and $\lambda = 2$. In this design, for a particular treatment, other treatments occur as neighbor once to the left and once to the right.

Blocks

1 5 | 2 3 1 5 | 2
2 3 | 5 4 1 3 | 5
3 4 | 2 5 3 4 | 2
4 1 | 4 3 2 1 | 4
5 4 | 5 1 2 4 | 5

v = Total number of treatments,
b = Total number of blocks,
k = Block size
λ = Number of occurrence of particular treatment with the rest of treatments as neighbor.

Example 2

An example of one-dimensional neighbor circular design can be seen in the biometrics where a test called the ouchterlony gel diffusion test is used to investigate the relationship between antigens and their individual components on circular plates. The technique involves the cultivating samples of different antigens in a suitable culture medium (such as agar gel) around an antibody (antiserum). The central well contains the antibody (antiserum) and the surrounding wells contain the various antigens to be observed. The components of the antigens and antibody (antiserum) diffuse radically from the sources, where the antigens and antibody meet, an immune complex is formed which gives a visible white precipitate line on the meeting front if the antibody has components produced in response to the antigens. If there is no common component between antibody and antigen under observation, no precipitate line is formed. Considering the effect of the antibody (antiserum) on multiple antigens at a time, multiple precipitate lines may be observed, the lines converge in between neighboring antigens if they have the same components reacting with antibody, a spur continuation of the precipitate line of the first antigen is observed as the components of the antibody (antiserum) that do not react with the second antigen diffuse through the precipitate line of the second antigen to give a continuation of the line of the first antigen. A continuous white line reveals a full identity; a spur continuation of the precipitate line reveals a partial identity and if there is no line, it represents no identity. For such placement more than one plate may be required. Clearly, in order to determine the relationships of the antigens completely, it is necessary to have each antigen next to each of the others in turn. Figure below shows an arrangement of five antigens around an antibody on a plate. Each antigen has two neighbors and generally v kinds of antigens are placed on b plates of size k.

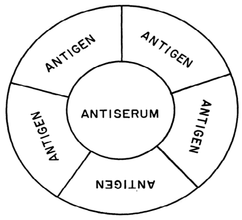

Figure 1.

This test requires suitable ordered sets of antigens. Neighbor designs are appropriate for such type of experiments.

Example 3

Another example of one-dimensional neighbor circular design can be seen in the agriculture on mountains where crops are cultivated on terraces in such a way that these form a natural circular blocks, each cultivated plot is nearest neighbor to the next and hence along with the effect of its own it may yield the effects of its neighboring plots.

2.2. Two-Dimensional Neighbor Designs

In two-dimensional design, neighbor effects are controlled in both ways (rows and columns). Two-dimensional neighbor designs exist for bordered and un-bordered plots. In an un-bordered plot setup, designs are regarded as linear in each direction. Complete Latin and quasi complete Latin square are examples of un-bordered neighbor design. Whereas in bordered two-dimensional neighbor designs, neighbor effects of the outer boundary of block are taken into account. The reason for border block is that the response on one plot may be affected by treatments on neighboring plots as well as by the treatment applied to that plot. For instance in experiment on insecticides, plots receiving control or ineffective treatments may cause cross infection to neighboring plots. In such cases direction also has a bearing on the neighbor effect for example of prevailing wind or sunlight.

A neighbor design with border plots may happen in the form of torus. We can call such toruses, pseudo-circular neighbor designs. A "pseudo-circular two-dimensional neighbor design" is a design in which, aspects of neighbor effects have been presented in row circular and column circular blocks. In such design setup, either extra parameters or bordered plots are needed for the east, west and north, south neighbor effect of edges. These bordered plots are not used for measuring response variables. A pseudo-circular two-dimensional neighbor design with bordered plots for four treatments is given below.

Example

A line of bordered plots on each side is required for the neighbor effect of edges. Every treatment in this design appears equal number of times in rows and columns, i.e., r = 12. Every pair of two distinct treatments occurs as neighbor λp times in rows and λq times in columns. Here in this case λp = 8 and λq = 8. The following block gives a nearest neighbor balanced design with parameters v = 4, r = 12, λp = 8 and λq = 8.

	2	3	1	0	3	0	2	1	
2	1	2	0	3	0	1	3	2	1
0	2	3	1	0	2	3	1	0	2
3	0	1	3	2	1	2	0	3	0
0	3	0	2	1	2	3	1	0	3
2	0	1	3	2	0	1	3	2	0
1	2	3	1	0	3	0	2	1	2
	1	2	0	3	0	1	3	2	

3. Response Models for Neighbor Designs

Neighbor designs work in a quite optimistic way, where the neighbor effects are additive. They do not avoid all bias due to neighbors but provide some robustness and minimize some measures of the bias of the estimated treatment differences. In literature many models of one-dimensional and two-dimensional neighbor designs have been proposed.

3.1. Models for One-Dimensional Neighbor Designs

One-dimensional neighbor designs considered here are assumed to be circular. Without loss of generality, we define left and right neighbors for a fixed direction; say clock wise. In certain situations, neighbor effect may depend upon direction (shade, wind, etc.), which motivates to distinguish between left and right neighbors. There are different types of models related to neighbor effects. One-sided neighbor effects can be observed in sunflowers experiments where tall plants shade the plot on their north side or in pesticide or fungicide experiments where a part of the treatment may spread to the plot immediately downwind. Similarly two-side neighbor effects can be seen in plants with an important root system, such as potatoes, varieties which germinate earlier will establish their roots and take nutrients from adjoining plots on both sides. The model with one sided neighbor effect;

$$Y_{ij} = \mu + \tau_{(i,j)} + \beta_j + \varphi_{(i-1,j)} + \varepsilon_{ij} \qquad \text{(M1)}$$

The model with un-differentiated two-sided neighbor effect;

$$Y_{ij} = \mu + \tau_{(i,j)} + \beta_j + \varphi_{(i-1,j)} + \varphi_{(i+1,j)} + \varepsilon_{ij} \qquad \text{(M2)}$$

The model with differentiated two-sided neighbor effect;

$$Y_{ij} = \mu + \tau_{(i,j)} + \beta j + \varphi_{(i-1,j)} + \rho_{(i+1,j)} + \varepsilon_{ij} \qquad \text{(M3)}$$

Y_{ij} is response from the ith plot in the jth block, μ is overall mean, $\tau_{(i,j)}$ is direct effect of the treatment in the ith plot of jth block, β_j is effect of the jth block, $\varphi_{(i-1,j)}$ is left neighbor effect due to the treatment in the (i-1)th plot of jth block, $\varphi_{(i-1,j)}$, φ (i+1,j) is Un-differentiated neighbor effect due to the treatment in the (i-1)th plot and (i+1)th plot of jth block, i.e., neighbor effect due to left treatment is same to the neighbor effect of right treatment, $\rho_{(i+1,j)}$ is right neighbor effect due to the treatment in (i+1)th plot of jth block, $\varphi_{(i-1,j)}$, $\rho_{(j+1,j)}$ is differentiated neighbor effect due to the treatment in the (i-1)th plot and (i+1)th plot of jth block, i.e., neighbor effect due to left treatment is different to the neighbor effect of right treatment, and $\varepsilon_{ij} \sim N(0, \sigma^2)$, i.e., error terms are independently and normally distributed with mean zero and variance σ^2.

Neighbor effects of treatments are additive in given models. It has been proposed that direct effects and neighbor effects are not additive when a treatment has itself as a neighbor, Bailey and Druilhet (2004). Model (1), (2) and (3) were given by Azais et al. (1993). They also presented analysis procedures for these models. Azais and Druilhet (1997) showed that neighbor balanced designs are optimal under these models even when neighbor effects are not taken into account. Filipiak and Markiewicz (2005, 2007) showed the optimality of neighbor balanced designs under model (1) and model (3).

3.2. Models for Two-Dimensional Neighbor Designs

The Model for two-dimensional neighbor designs is considered here for left, right, upward and downward neighbors. The design is assumed to be bordered on all four sides. The model is a response from the plot in row f and column g whose treatment is h.

$$Y_{fghijkm} = \mu + \check{r}_f + \check{c}_g + \tau_h + \varphi_i + \varphi_j + \varphi_k + \varphi_m + \varepsilon_{fghijkm} \qquad \text{(M4)}$$

$Y_{fghijkm}$ is response from the f_{th} row and g_{th} column of h_{th} treatment having neighbors of i,j,k,m directions, $\check{r}_f$ is effect of row f (f = 1,2,..,r), $\check{c}_g$ is effect of column g (g = 1,2,..,c), τ_h is effect of treatment h (h = 1,2,..,v), φ_i, φ_j, φ_k, φ_m are nearest neighbor effect of treatment in four main directions (i = East, j = West, k = North, m = South). Federer and Basford (1991) presented this model (4) for field arrangements and develop a statistical analysis for design of this type. If we delete g, k and m from the model (4) and consider $\check{r}_f$ as block effect then we will get the model (2) and if we delete i or j

(any one only) then we will get model (1) of Azais et. al (1993). Morgan and Uddin (1991) also suggested a model for torus lattice.

Chai and majumdar (2000) considered a following model for field trial of v treatments in b blocks of size k plots and supposed that plots within blocks are affected by a stochastic fertility process x.

$$Y_{ij} = \mu + \tau_{d(i,j)} + \beta_j + x_{ij} + \varepsilon_{ij} \qquad (M_5)$$

Y_{ij} is response from the ith plot in the jth block, $\tau_{d(i,j)}$ is effect of the treatment d assigned to the ith plot of jth block by design, and x_{ij} is fertility process of the ith plot in the jth block.

They suggested the outline for the analysis of this model under the first difference nearest neighbor method and assuming that x_{ij}-x_{i+1j} are uncorrelated.

4. Discussion

Neighbor designs have not yet achieved sufficient attention to be used on a routine basis. The absence of proper randomization theory in neighbor designs is great hindrance in the proper assessment of estimates. Besag and Kempton (1986) recommended that experiments should be randomized while applying neighbor designs. AzaÏs (1987) suggested four operations in the construction of neighbor designs for randomization purpose. He suggested for inter-block randomization, randomly and independently permutation of each plot in each block, random allocation of numbers (0,1,2,…, v-1) to each treatment and circularity condition for border plots. In neighbor designs, great confusion arises over the different causes of association between neighboring units and specification of appropriate models, [Besag and Kempton (1986)]. An expert can select an appropriate model according to the causes of association and can take decision about the selection of directional or non-directional neighbor designs. A neighbor design is said to be non-directional if the influence of left and right neighboring plots are equal otherwise the design will be considered directional.

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