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Bayesian estimation of reliability function for a changing exponential family model under different loss functions

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Absrtact: The paper deals with estimating shift point which occurs in any sequences of independent observations x_1, x_2, …, x_m, x_{m+1}, …, x_n of poisson and geometric distributions. This shift point occurs in the sequence when x_m i. e. m life data are observed. With known shift point 'm', the Bayes estimator on befor and after shift process means θ_1 and θ_2 are derived for symmetric and assymetric loss functions. The sensitivity analysis of Bayes estimators are carried out by simulation and numerical comparisons with R-programming. The results show the effectiveness of shift in sequences of both poisson and geometric distributions.

Keywords: Bayes Estimator, Exponential Family, Squared Error Loss Function, LinexLoss Function, General Entropy Loss Function, Precautionary Loss Function, Shift Point, Poisson and Geometric Distributions

1. Introduction

In some real life applications, like medical science and physical systems manufacturing the items are often subject to abrupt shifts in the failure rate function, which are observed due to major operations or specific activities, that is may observed at some point of time instability in the sequence of life times. These situations are for the times when the shift point, m, is known. there are many studies on shift point problem in a sequence of random variables. Hinkley (1970) studied the shift point problem and considered a sequence of independent continuous random variables.

Most authors' investigations are based on the work of Hinkley (1970). For example the shift point problem in a sequence of binomial variables is studied by Hinkley and Hinkley (1970); the shift point in a sequence of exponential and poisson variables are investigated by Worsley (1986); Haccou, Meelis and Geer (1988); Estimation of shift point in a homogeneous poisson process studied by Jandhyala and Fotopolus (1999) and Boudjelaba, MacGibbon and Sawyer (2001); Fotopolus and Jandhyala (2001). The study of homogeneous poisson process and continuous thime shift point problem in such poissonprocess has been carried out by some authors, For example use of cumulating sum (CUSUM) control charts and exponentially weighted charts are studied by Montgomery (2001) and Wu et. al. (2004). Lim et. al. (2002), Wu and Tiau (2005) and Zhang and Wu (2005) considered the applications ofCUSUM control charts. Bayes estimation of unknown shift point in geometric distribution is studied by Shah and Patel (2006); Many of statisticians like Chin and Broemeling (1980), Calabria and Pulcini (1994), Zacks (1983), Pandaya and Jani (2006), Shah and Patel (2007, 2009), Chib (2998), Altessimo and Corradi (2003) and Fiteni (2004) studied the shift point Models in Bayesian framework, and Bayesian estimation of shift point in poisson Model is studied by gorakhpour university's authors (2012).

In this paper the Bayes estimator of mean parameter θ_1 and θ_2, for the sequences, befor and after shift point 'm' of independent life times from poisson and geometric population are derived for symmetric and assymetric loss functions. squared error loss function, linex loss function, general entropy loss function, and precautionary loss function. A sensitivity analysis of these Bayes estimates has also been presented by simulation and numerical comparison study through R-programming.

2. Likelihood Functions

Let $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ ($n \geq 3$) be the sequences of observed discrete life times. First let observation $x_1, x_2, \ldots, x_n$ have come from poissondistribution, and $y_1, y_2, \ldots, y_n$ have come from geometric distribution with their probability mass functions(pmf) as

$$p(x) = \frac{e^{-\theta}\theta^x}{x!}; x=0, 1, \ldots, \theta > 0 \quad (1)$$

$$p(y) = \theta(1-\theta)^y; y=0, 1, \quad (2)$$

Let 'm' is shift point in the observation which breaks the distribution in two sequences, that is for poisson model ($x_1, x_2, \ldots, x_m$) & ($x_{m+1}, \ldots, x_n$), and for geometric model ($y_1, y_2, \ldots, y_m$) & ($y_{m+1}, \ldots, y_n$).

The probability mass functions of the above sequences are

$$p_1(x) = \frac{e^{-\theta_1}{\theta_1}^x}{x!}; x=0, 1, \ldots, \theta_1 > 0 \quad (3)$$

$$p_2(x) = \frac{e^{-\theta_2}{\theta_2}^x}{x!}; x=0, 1, \ldots, \theta_2 > 0 \quad (4)$$

and

$$p_1(y) = \theta_1(1-\theta_1)^y; y=0, 1, \quad (5)$$

$$p_2(y) = \theta_2(1-\theta_2)^y; y=0, 1, \quad (6)$$

The likelihood function of p. m. f. 's of the sequences are

$$L(\theta_1|\underline{x}) = \frac{e^{-m\lambda_1}{\lambda_1}^{s_{1m}}}{x_1!\ldots x_m!} \quad (7)$$

$$L(\theta_2|\underline{x}) = \frac{e^{-(n-m)\theta_2}{\theta_2}^{(s_{1n}-s_{1m})}}{x_{(m+1)}!\ldots x_n!} \quad (8)$$

Where $s_{1m} = \sum_{i=1}^{m} x_i$, $s_{1n} - s_{1m} = \sum_{i=m+1}^{n} x_i$ and

$$L\left(\theta_1\middle|\underline{y}\right) = {\theta_1}^m(1-\theta_1)^{s_{1m}} \quad (9)$$

$$L\left(\theta_1\middle|\underline{y}\right) = {\theta_2}^{(n-m)}(1-\theta_2)^{(s_{1n}-s_{1m})} \quad (10)$$

Where

$$s_{1m} = \sum_{i=1}^{m} y_i,\ s_{1n} - s_{1m} = \sum_{i=m+1}^{n} y_i \quad (11)$$

Then, the joint likelihood functions is given by

$$L(\theta_1\theta_2|\underline{x}) = \frac{e^{-m\theta_1}{\lambda_1}^{s_{1m}}}{x_1!\ldots x_m!} \times \frac{e^{-(n-m)\theta_2}{\theta_2}^{(s_{1n}-s_{1m})}}{x_{(m+1)}!\ldots x_n!} \quad (12)$$

$$L\left(\theta_1\theta_2\middle|\underline{y}\right) = {\theta_1}^m(1-\theta_1)^{s_{1m}} \times {\theta_2}^{(n-m)}(1-\theta_2)^{(s_{1n}-s_{1m})} \quad (13)$$

Suppose the marginal prior distributions of θ_1, θ_2 are natural conjugate priorfor poisson model:

$$g_1(\theta_1) = \frac{{b_1}^{a_1}}{\Gamma(a_1)}{\theta_1}^{a_1-1}e^{-b_1\theta_1};\ a_1, b_1 > 0 \quad (14)$$

$$g_2(\theta_2) = \frac{{b_2}^{a_2}}{\Gamma(a_2)}{\theta_2}^{a_2-1}e^{-b_2\theta_2};\ a_2, b_2 > 0 \quad (15)$$

and for geometric Model:

$$g_1(\theta_1) = \frac{\Gamma(a_1+b_1)}{\Gamma(a_1)\Gamma(b_1)}(\theta_1)^{a_1-1}(1-\theta_1)^{b_1-1};\ a_1, b_1 > 0 \quad (16)$$

$$g_2(\theta_2) = \frac{\Gamma(a_2+b_2)}{\Gamma(a_2)\Gamma(b_2)}(\theta_2)^{a_2-1}(1-\theta_2)^{b_2-1};\ a_2, b_2 > 0 \quad (17)$$

The joint prior distribution of θ_1, θ_2 and shift point 'm', according to Ganji(2010) by using the exponential family model

For poisson model:

$$g(\theta_1, \theta_2; m) = \frac{{b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)}{\theta_1}^{a_1-1}{\theta_2}^{a_2-1}\exp(-b_1\theta_1)\exp(-b_2\theta_2) \quad (18)$$

In case of, geometric model:

$$g(\theta_1, \theta_2; m) = \frac{\Gamma(a_1+b_1)}{\Gamma(a_1)\Gamma(b_1)} \times \frac{\Gamma(a_2+b_2)}{\Gamma(a_2)\Gamma(b_2)}(\theta_1)^{a_1-1}(1-\theta_1)^{b_1-1}(\theta_2)^{a_2-1}(1-\theta_2)^{b_2-1} \quad (19)$$

where $\theta_1, \theta_2 > 0$ and m=1, …, (n - 1)

The joint posterior density of θ_1, θ_2 and m is obtained by using equations (12), (18) for poissonmodel, and (13), (19) for geometric Model

$$p(\theta_1, \theta_2; m|\underline{x}) = \frac{e^{-(b_1+m)\theta_1}{\theta_1}^{(a_1+s_{1m}-1)}e^{-(b_2+n-m)\theta_2}{\theta_2}^{(a_2+s_{1n}-s_{1m}-1)}}{D(a_1, a_2, b_1, b_2, m, n)} \quad (20)$$

where

$$D(a_1, a_2, b_1, b_2, m, n) = \left[\frac{\Gamma(a_1+s_{1m})}{(b_1+m)^{(a_1+s_{1m})}} \times \frac{\Gamma(a_2+s_{1n}-s_{1m})}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m})}}\right] \quad (21)$$

$$p\left(\theta_1, \theta_2; m\middle|\underline{y}\right) = \frac{(\theta_1)^{(a_1+m-1)}(1-\theta_1)^{(b_1+s_{1m}-1)}(\theta_2)^{(a_2+n-m-1)}(1-\theta_2)^{(b_2+s_{1n}-s_{1m}-1)}}{D(a_1, a_2, b_1, b_2, m, n)} \quad (22)$$

where

$$D(a_1, a_2, b_1, b_2, m, n) = \left[\frac{\Gamma(a_1+m)\Gamma(b_1+s_{1m})}{\Gamma(a_1+m+b_1+s_{1m})} \times \frac{\Gamma(a_2+n-m)\Gamma(b_2+s_{1n}-s_{1m})}{\Gamma(a_2+n-m+b_2+s_{1n}-s_{1m})}\right] \quad (23)$$

The marginal posterior distribution of θ_1 , forpoissonmodel, by using the equations (12), (14) and dor geometric Model, by using the equations (13), (16)

$$p(\theta_1|\underline{x}) = \frac{\left[e^{-(b_1+m)\theta_1}\theta_1^{(a_1+s_{1m}-1)}\frac{\Gamma(a_2+s_{1n}-s_{1m})}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m})}}\right]}{D(a_1,a_2,b_1,b_2,m,n)} \quad (24)$$

$$p\left(\theta_1|\underline{y}\right) = \frac{(\theta_1)^{(a_1+m-1)}(1-\theta_1)^{(b_1+s_{1m}-1)}\times\frac{\Gamma(a_2+n-m)\Gamma(b_2+s_{1n}-s_{1m})}{\Gamma(a_2+n-m+b_2+s_{1n}-s_{1m})}}{D(a_1,a_2,b_1,b_2,m,n)} \quad (25)$$

the marginal posterior distribution of θ_2 , for poissonModel, by using the equations (12), (15) and for geometric Model, by using the equations (13), (17)

$$p(\theta_2|\underline{x}) = \frac{e^{-(b_2+n-m)\theta_2}\theta_2^{(a_2+s_{1n}-s_{1m}-1)}\frac{\Gamma(a_1+s_{1m})}{(b_1+m)^{(a_1+s_{1m})}}}{D(a_1,a_2,b_1,b_2,m,n)} \quad (26)$$

$$p\left(\theta_2|\underline{y}\right) = \frac{(\theta_2)^{(a_2+n-m-1)}(1-\theta_2)^{(b_2+s_{1n}-s_{1m}-1)}\times\frac{\Gamma(a_1+m)\Gamma(b_1+s_{1m})}{\Gamma(a_1+m+b_1+s_{1m})}}{D(a_1,a_2,b_1,b_2,m,n)} \quad (27)$$

3. Bayes Estimators under Symmetric and Asymmetric Loss Functions

In decision theory the loss criterion is specified in order to obtain best estimator. The simplest form of loss function is squared error loss function (SELF) which assigns equal magnitudes to both positive and negative errors. However this assumption may be inappropriate in most of the estimation problems. some time overestimation leads to many serious qonsequences. In such situation many outhors found the asymmetric loss functions, moreappropriate. In this paper we have considered some of the asymmetric loss functions, Linex loss function (LLF) suggested and studied by Varian (1975) , Zellner (1986) , Basu and Ebrahimi (1991) , General entropy loss function (GELF) by Calabria and Pulcini (1996) and Precautionary loss function (PLF) studied by Norstrom (1996). Such asymmetric loss functions are also studied by Ohtani (1995) , Parsian and Kirmani (2002) , Brases and Dette (2004) , Pandyaet. al. (2004) and Gorakhpour university's authors (2012).

3.1. Bayes Estimators under Squared Error Loss Functions (SELF)

From a decision theorical view point, in order to select value as representing on 'best' estimator , a loss function must be specified, In this section we consider SELF.

The Bayes estimate of a generic parameter θ based on a SELF is given by $L_1(\theta, d) = (\theta - d)^2$, where 'd' is a decision rule to estimate θ , is posterior mean.

The Bayes estimate $\hat{\theta}_{1BS}$ of θ_1 under SELF using marginal posterior density equation (24) for poisson model is given by

$$\hat{\theta}_{1BS} = \frac{\left[\frac{\Gamma(a_1+s_{1m}+1)}{(b_1+m)^{(a_1+s_{1m}+1)}}\times\frac{\Gamma(a_2+s_{1n}-s_{1m})}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m})}}\right]}{D(a_1,a_2,b_1,b_2,m,n)} \quad (28)$$

$$\hat{\theta}_{1BS} = \frac{D(a_1+1),a_2,b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)} \quad (29)$$

and equation (25) for geometric model is given by

$$\hat{\theta}_{1BS} = \frac{\left[\frac{\Gamma(a_1+m+1)\Gamma(b_1+s_{1m})}{\Gamma(a_1+m+b_1+s_{1m}+1)}\times\frac{\Gamma(a_2+n-m)\Gamma(b_2+s_{1n}-s_{1m})}{\Gamma(a_2+n-m+b_2+s_{1n}-s_{1m})}\right]}{D(a_1,a_2,b_1,b_2,m,n)} \quad (30)$$

$$\hat{\theta}_{1BS} = \frac{D((a_1+1),a_2,b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)} \quad (31)$$

The Bayes estimate $\hat{\theta}_{2BS}$ of θ_2 under SELF using marginal posterior density equation (26) for poisson model is given by

$$\hat{\theta}_{2BS} = \frac{\left[\frac{\Gamma(a_2+s_{1n}-s_{1m}+1)}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m}+1)}}\times\frac{\Gamma(a_1+s_{1m})}{(b_1+m)^{(a_1+s_{1m})}}\right]}{D(a_1,a_2,b_1,b_2,m,n)} \quad (32)$$

$$\hat{\theta}_{2BS} = \frac{D(a_1,(a_2+1),b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)} \quad (33)$$

The equation (27) for geometric model is given by

$$\hat{\theta}_{2BS} = \frac{\left[\frac{\Gamma(a_1+m)\Gamma(b_1+s_{1m})}{\Gamma(a_1+m+b_1+s_{1m})}\times\frac{\Gamma(a_2+n-m+1)\Gamma(b_2+s_{1n}-s_{1m})}{\Gamma(a_2+n-m+b_2+s_{1n}-s_{1m}+1)}\right]}{D(a_1,a_2,b_1,b_2,m,n)} \quad (34)$$

$$\hat{\theta}_{2BS} = \frac{D(a_1,(a_2+1),b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)} \quad (35)$$

3.2. Bayes Estimator under Linex Loss function (LLF)

The asymmetric loss function given by Varian (1975). known as Linex loss function (LLF), is defined by

$$L_2(\theta, d) = \exp[\gamma_1(d-\theta)] - \gamma_1(d-\theta) - 1 \,;\, \gamma_1 \neq 0 \quad (36)$$

Where d is the decision rule to estimate unknown parameter θ.

The Bayes estimate $\hat{\theta}_{1BL}$ of θ_1 using marginal posterior density equation (24) under LLF , forpoisson model is given by

$$\hat{\theta}_{1BL} = \frac{-1}{\gamma_1}\ln\left[\frac{\left[\frac{\Gamma(a_1+s_{1m}+1)}{(b_1+m+\gamma_1)^{(a_1+s_{1m}+1)}}\times\frac{\Gamma(a_2+s_{1n}-s_{1m})}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m})}}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right] \quad (37)$$

$$\hat{\theta}_{1BL} = \frac{-1}{\gamma_1}\ln\left[\frac{D(a_1,a_2,(b_1+\gamma_1),b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right] \quad (38)$$

The Bayes estimate of $\hat{\theta}_{2BL}$ of θ_2 using marginal posterior density equation (26) under LLF , forpoisson model is given by

$$\hat{\theta}_{2BL} = \frac{-1}{\gamma_1}\ln\left[\frac{\left[\frac{\Gamma(a_1+s_{1m})}{(b_1+m)^{(a_1+s_{1m})}}\times\frac{\Gamma(a_2+s_{1n}-s_{1m})}{(b_2+n-m+\gamma_1)^{(a_2+s_{1n}-s_{1m})}}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right] \quad (39)$$

$$\hat{\theta}_{2BL} = \frac{-1}{\gamma_1}\ln\left[\frac{D(a_1,a_2,b_1,(b_2+\gamma_1),m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right] \quad (40)$$

We can not use linex loss function for geometric loss function.

3.3. Bayes Estimator under General Entropy Loss Function

Occasionally, the use of symmetric loss function , namely SELF , was found inappropriate, since for example , an overestimation of reliability function usually much more serious than an underestimation. Here was considered asymmetric loss function namely general entropy loss function (GELF) proposed by Calabria and Pulcini (1994), is given by

$$L_3(\theta, d) = \left(\frac{d}{\theta}\right)^{\gamma_2} - \gamma_2 \ln\left(\frac{d}{\theta}\right) - 1;\ \gamma_2 \neq 0 \quad (41)$$

The Bayes estimate $\hat{\theta}_{1BE}$ of θ_1 , unde GELF using marginal posterior distribution equation (24) for poissonmodel, is given by

$$\hat{\theta}_{1BE} = \left[\frac{\left[\frac{\Gamma(a_1+s_{1m}-\gamma_2)}{(b_1+m)^{(a_1+s_{1m}-\gamma_2)}}\times\frac{\Gamma(a_2+s_{1n}-s_{1m})}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m})}}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{-\frac{1}{\gamma_2}} \quad (42)$$

$$\hat{\theta}_{1BE} = \left[\frac{D((a_1-\gamma_2),a_2,b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{-\frac{1}{\gamma_2}} \quad (43)$$

and equation (25) for geometric model is given by

$$\hat{\theta}_{1BE} = \left[\frac{\left[\frac{\Gamma(a_1+m+\gamma_2)\Gamma(b_1+s_{1m})}{\Gamma(a_1+m+\gamma_2+b_1+s_{1m})}\times\frac{\Gamma(a_2+n-m)\Gamma(b_2+s_{1n}-s_{1m})}{\Gamma(a_2+n-m+b_2+s_{1n}-s_{1m})}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{-1}{\gamma_2}} \quad (44)$$

$$\hat{\theta}_{1BE} = \left[\frac{D((a_1+\gamma_2),a_2,b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{-1}{\gamma_2}} \quad (45)$$

The Bayes estimate $\hat{\theta}_{2BE}$ of θ_2 , unde GELF using marginal posterior distribution equation (26) for poissonmodel, is given by

$$\hat{\theta}_{2BE} = \left[\frac{\frac{\Gamma(a_1+s_{1m})}{(b_1+m)^{(a_1+s_{1m})}}\times\frac{\Gamma(a_2+s_{1n}-s_{1m}-\gamma_2)}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m}-\gamma_2)}}}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{-\frac{1}{\gamma_2}} \quad (46)$$

$$\hat{\theta}_{2BE} = \left[\frac{D(a_1,(a_2-\gamma_2),b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{-\frac{1}{\gamma_2}} \quad (47)$$

and equation (27) for geometric model is given by

$$\hat{\theta}_{2BE} = \left[\frac{\left[\frac{\Gamma(a_1+m)\Gamma(b_1+s_{1m})}{\Gamma(a_1+m+b_1+s_{1m})}\times\frac{\Gamma(a_2+n-m-\gamma_2)\Gamma(b_2+s_{1n}-s_{1m})}{\Gamma(a_2+n-m-\gamma_2+b_2+s_{1n}-s_{1m})}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{-1}{\gamma_2}} \quad (48)$$

$$\hat{\theta}_{2BE} = \left[\frac{D(a_1,(a_2-\gamma_2),b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{-1}{\gamma_2}} \quad (49)$$

3.4. Bayes Estimators under Precautioanary Loss Function (PLF)

Norstrom (1996) introduced an alternative asymmetric loss function and also presented a general class of precautionary loss function with quadratic loss function as a special case. These loss functions approach infinitely near the origin to prevent the overestimation and thus giving conservative estimators, especially when low failure rates are being estimated which may lead to serious consequences.

A very useful and simple asymmetric precautionary loss function is given by

$$L_4(\theta, d) = \frac{(\theta-d)^2}{d} \quad (50)$$

The Bayes estimator $\hat{\theta}_{1BP}$ of θ_1 , under PLF using the marginal posterior distribution (24) for poisson model is given by

$$\hat{\theta}_{1BP} = \left[\frac{\left[\frac{\Gamma(a_1+s_{1m}+2)}{(b_1+m)^{(a_1+s_{1m}+2)}}\times\frac{\Gamma(a_2+s_{1n}-s_{1m})}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m})}}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{1}{2}} \quad (51)$$

$$\hat{\theta}_{1BP} = \left[\frac{D((a_1+2),a_2,b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{1}{2}} \quad (52)$$

and equation (25) for geometric model is given by

$$\hat{\theta}_{1BP} = \left[\frac{\left[\frac{\Gamma(a_1+m+2)\Gamma(b_1+s_{1m})}{\Gamma(a_1+m+b_1+s_{1m}+2)}\times\frac{\Gamma(a_2+n-m)\Gamma(b_2+s_{1n}-s_{1m})}{\Gamma(a_2+n-m+b_2+s_{1n}-s_{1m})}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right] \quad (53)$$

$$\hat{\theta}_{1BP} = \left[\frac{D((a_1+2),a_2,b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{1}{2}} \quad (54)$$

The Bayes estimator $\hat{\theta}_{2BP}$ of θ_2 , under PLF using the marginal posterior distribution (26) for poisson model is given by

$$\hat{\theta}_{2BP} = \left[\frac{\left[\frac{\Gamma(a_1+s_{1m})}{(b_1+m)^{(a_1+s_{1m})}}\times\frac{\Gamma(a_2+s_{1n}-s_{1m}+2)}{(b_2+n-m)^{(a_2+s_{1n}-s_{1m}+2)}}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{1}{2}} \quad (55)$$

$$\hat{\theta}_{2BP} = \left[\frac{D(a_1,(a_2+2),b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{1}{2}} \quad (56)$$

and equation (27) for geometric model is given by

$$\hat{\theta}_{2BP} = \left[\frac{\left[\frac{\Gamma(a_1+m)\Gamma(b_1+s_{1m})}{\Gamma(a_1+m+b_1+s_{1m})}\times\frac{\Gamma(a_2+n-m+2)\Gamma(b_2+s_{1n}-s_{1m})}{\Gamma(a_2+n-m+b_2+s_{1n}-s_{1m}+2)}\right]}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{1}{2}} \quad (57)$$

$$\hat{\theta}_{2BP} = \left[\frac{D(a_1,(a_2+2),b_1,b_2,m,n)}{D(a_1,a_2,b_1,b_2,m,n)}\right]^{\frac{1}{2}} \quad (58)$$

3.5. Numerical Comparison

We have generated the random samples of different sizes 10, 15, 20, 25, 30, 35, 40, 45, 50 with known shift points 5, 10, 15, 20, 25, 30, 35, 40, 45 repectively for each of poisson and geometric distributions. We also get θ = 2 in poissonmodel, and θ = 0. 2 in geometric model. The Bayes estimators of θ_1 and θ_2 for each distributions are calculated under Squared Error Loss Function, Linex Loss Function, General Entropy Loss Function and Precautionary Loss function by making programs in R-2. 13. 2 statistical software.

3.6. Sensitivity Analysis of Bayes Estimation

In this section we have studied the sensitivity of the Bayes estimators of θ_1 , $\theta_2$whth respect to the parameters of prior distribution a_1, a_2, b_1 and b_2 . We have computed the Bayes estimators of θ_1 and θ_2 under SELF, LLF, GELF and PLF. we have also considered different sample sizes n=10(05)50.

Table 1. shows the Bayes estimarors of θ_1 and θ_2 under SELF, LLF, GELF, PLF, and Loss functions of them that $L_1 = L(\hat{\theta}_{1BS}, \theta), L_2 = L(\hat{\theta}_{2BS}, \theta)$, $L_3 = L(\hat{\theta}_{1BL}, \theta), L_4 = L(\hat{\theta}_{2BL}, \theta), L_5 = L(\hat{\theta}_{1BE}, \theta), L_6 = L(\hat{\theta}_{2BE}, \theta), L_7 = L(\hat{\theta}_{1BP}, \theta), L_8 = L(\hat{\theta}_{2BP}, \theta)$.

n	m	$\hat{\theta}_{1BS}$	$\hat{\theta}_{2BS}$	$\hat{\theta}_{1BL}$	$\hat{\theta}_{2BL}$	$\hat{\theta}_{1BE}$	$\hat{\theta}_{2BE}$	$\hat{\theta}_{1BP}$	$\hat{\theta}_{2BP}$
10	5	2. 00	1. 83	1. 57	1. 61	1. 78	1. 61	2. 07	1. 89
15	10	2. 08	1. 11	1. 92	0. 98	1. 96	0. 89	2. 12	1. 83
20	15	3. 07	1. 11	2. 09	1. 23	2. 98	1. 18	3. 10	1. 47
25	20	2. 09	1. 97	2. 00	1. 73	2. 02	1. 75	2. 11	2. 04
30	25	1. 85	0. 68	1. 78	0. 60	1. 79	0. 46	1. 87	0. 75
35	30	2. 22	1. 83	2. 15	1. 61	2. 17	1. 61	2. 24	1. 89
40	35	1. 89	1. 40	1. 84	1. 23	1. 85	1. 18	1. 90	1. 46
45	40	1. 93	1. 54	1. 88	1. 36	1. 89	1. 33	1. 94	1. 61
50	45	2. 00	1. 54	1. 96	1. 36	1. 99	1. 33	2. 01	1. 61
n	**m**	**L_1**	**L_2**	**L_3**	**L_4**	**L_5**	**L_6**	**L_7**	**L_8**
10	5	0. 000	0. 029	0. 105	0. 240	0. 026	0. 081	0. 003	0. 005
15	10	0. 007	0. 784	0. 011	1. 169	0. 001	0. 804	0. 007	0. 563
20	15	1. 155	0. 784	3. 295	0. 752	0. 426	0. 399	0. 393	0. 191
25	20	0. 008	0. 001	0. 000	0. 119	0. 000	0. 003	0. 006	0. 001
30	25	0. 022	1. 727	0. 081	1. 855	0. 022	1. 968	0. 009	2. 060
35	30	0. 048	0. 029	0. 052	0. 240	0. 014	0. 081	0. 024	0. 005
40	35	0. 012	0. 360	0. 045	0. 752	0. 011	0. 399	0. 005	0. 191
45	40	0. 005	0. 209	0. 025	0. 562	0. 006	0. 261	0. 002	0. 093
50	45	0. 000	0. 209	0. 003	0. 562	0. 000	0. 261	0. 000	0. 093

Table 2, shows the Bayes estimarors of θ_1 and θ_2 under SELF, LLF, GELF, PLF, and Loss functions of them that $L_1 = L(\hat{\theta}_{1BS}, \theta), L_2 = L(\hat{\theta}_{2BS}, \theta)$, $L_3 = L(\hat{\theta}_{1BL}, \theta), L_4 = L(\hat{\theta}_{2BL}, \theta), L_5 = L(\hat{\theta}_{1BE}, \theta), L_6 = L(\hat{\theta}_{2BE}, \theta), L_7 = L(\hat{\theta}_{1BP}, \theta), L_8 = L(\hat{\theta}_{2BP}, \theta)$.

n	m	$\hat{\theta}_{1BS}$	$\hat{\theta}_{2BS}$	$\hat{\theta}_{1BE}$	$\hat{\theta}_{2BE}$	$\hat{\theta}_{1BP}$	$\hat{\theta}_{2BP}$
10	5	0. 21	0. 13	0. 17	0. 10	0. 22	0. 14
15	10	0. 26	0. 21	0. 23	0. 17	0. 24	0. 23
20	15	0. 43	0. 31	0. 41	0. 26	0. 44	0. 33
25	20	0. 17	0. 38	0. 16	0. 32	0. 17	0. 39
30	25	0. 23	0. 36	0. 22	0. 31	0. 23	0. 38
35	30	0. 19	0. 21	0. 19	0. 17	0. 19	0. 22
40	35	0. 23	0. 26	0. 22	0. 21	0. 23	0. 28
45	40	0. 26	0. 24	0. 25	0. 13	0. 26	0. 25
50	45	0. 28	0. 22	0. 27	0. 18	0. 28	0. 23
n	**m**	**L_1**	**L_2**	**L_3**	**L_4**	**L_5**	**L_6**
10	5	0. 000	0. 005	0. 056	0. 594	0. 002	0. 029
15	10	0. 003	0. 000	0. 053	0. 034	0. 048	0. 003
20	15	0. 052	0. 012	1. 735	0. 165	0. 129	0. 049
25	20	0. 000	0. 033	0. 079	0. 658	0. 003	0. 098
30	25	0. 001	0. 026	0. 027	0. 482	0. 006	0. 083
35	30	0. 000	0. 000	0. 018	0. 035	0. 000	0. 003
40	35	0. 000	0. 004	0. 017	0. 014	0. 004	0. 021
45	40	0. 004	0. 001	0. 133	0. 002	0. 015	0. 009
50	45	0. 007	0. 000	0. 231	0. 020	0. 022	0. 004

Table 1, shows the Bayes estimarors of θ_1 and θ_2 under SELF, LLF, GELF, PLF, and Loss functions of them that $L_1 = L(\hat{\theta}_{1BS}, \theta), L_2 = L(\hat{\theta}_{2BS}, \theta)$, $L_3 = L(\hat{\theta}_{1BL}, \theta), L_4 = L(\hat{\theta}_{2BL}, \theta), L_5 = L(\hat{\theta}_{1BE}, \theta), L_6 = L(\hat{\theta}_{2BE}, \theta), L_7 = L(\hat{\theta}_{1BP}, \theta), L_8 = L(\hat{\theta}_{2BP}, \theta)$.

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