American Journal of Science, Engineering and Technology
2019; 4(3): 48-54
http://www.sciencepublishinggroup.com/j/ajset
doi: 10.11648/j.ajset.20190403.12
ISSN: 2578-8345 (Print); ISSN: 2578-8353 (Online)

Magnetic Properties of Diluted Spinels $Mg_xNi_{1-x}Fe_2O_4$ Systems: Are Studied by Green's Functions and Mean Field Theories

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To cite this article:
Abdellah El Grini, Said Salmi, Abdelouahid Azouaoui, Ahmed Hourmatallah, Khalid Bouslykhane, Mohamed Hamedoun, Najib Benzakour. Magnetic Properties of Diluted Spinels $Mg_xNi_{1-x}Fe_2O_4$ Systems: Are Studied by Green's Functions and Mean Field Theories. *American Journal of Science, Engineering and Technology*. Vol. 4, No. 3, 2019, pp. 48-54. doi: 10.11648/j.ajset.20190403.12

Received: March 6, 2019; **Accepted**: May 5, 2019; **Published**: October 29, 2019

Abstract: Many-body Green's function, and mean field theories have been developed for magnetic systems $Mg_xNi_{1-x}Fe_2O_4$. We apply this theory to evaluate thermal magnetization and magnetic susceptibility for different values of magnetic field and dilution x, by considering all components of the magnetisation when an external magnetic field is applied in (x-z)-plane. The critical temperatures of $Mg_xNi_{1-x}Fe_2O_4$ systems in the range $0 \leq x \leq 1$ have been deduced. The Green's function results are compared with the results of high temperature series expansion technique, and experimental magnetic measurements.

Keywords: Spinels, Heisenberg Model, Exchange Interaction, Green's Function, Random-Phase Approximation (RPA)

1. Introduction

Materials in the pure spinel structure (AB_2O_4) are still the subject of extensive experimental and theoretical studies due to their wide variety of physical properties and potential applications in nanoscience and technology. The oxy-spinel ferrites solid solutions $Mg_xNi_{1-x}Fe_2O_4$ with x ranging from 0 to 1 provide an additional degree of freedom to tune their magnetic and electronic properties, which opens new opportunities for fabricating hybrid systems. Thanks to the existence of tetrahedral (A) and octahedral (B) crystallographic sublattices available for the metal ions, this structure has large flexibility in hosting various metal ions that are differently distributed between the two sublattices.

In this paper, we present theoretical approaches to study the magnetic behaviour of $Mg_xNi_{1-x}Fe_2O_4$, in particular the Green's function theory (GFT) [1, 2]. This technique makes use of the spectral theorem to connect correlation functions to the Green's functions. This contribution deals with the reorientation of the magnetization of thin ferromagnetic (FM) Heisenberg films as a function of the temperature or an external magnetic field. It is of particular importance to take into account collective excitations (magnons), which influence the magnetic properties of films more strongly than those of bulk magnet. Fröbrich et al. [3, 4] developed, for the first time, an improved method which enabled to calculate numerically more than one component of magnetization for a ferromagnetic system under random-phase approximation (RPA). With this method, they investigated the magnetization reorientation of (FM) films caused by dipole interaction and external field.

The field-induced reorientation of the magnetization is treated for all temperatures of interest. Since expectation values of all three components of the spin operator are considered, a corresponding set of Green's functions must be defined. The method used for the calculation of the expectation values, does not utilize only the eigenvalues but also the eigenvectors of the (non-symmetric) matrix governing the equations of motion for the Green's functions with the random-phase approximation (RPA) [5, 6]. The thermal magnetization and magnetic susceptibility are given for different values of magnetic field and dilution x. The

critical temperatures have been deduced. On the other hand, The values of exchange integrals $J_1(x)$ and $J_2(x)$ for each dilution x of $Mg_xNi_{1-x}Fe_2O_4$ is deduced by using mean field theory. The obtained values are given in Table 1. The Green's function results are compared with the results of high temperature series expansion technique (HTSE) [7], and experimental magnetic measurements [8].

2. Theories

2.1. Green's Functions Theory (GFT)

The Hamiltonian of the spin-S Heisenberg exchange interaction *J* between nearest neighbour lattice sites and under an external magnetic field B(Bx, By, Bz) is

$$H=-\sum_{\langle kl\rangle}J_{kl}\left(S_k^xS_l^x+S_k^yS_l^y\right)-\sum_k\left(B^xS_k^x+B^yS_k^y+B^zS_k^z\right) \quad (1)$$

The z-axis is perpendicular to the (x-y) plane. The external field component in the z-direction enhances the effect of the transverse field components $B^{\pm}$.

The transformations $S_i^{\pm}=S_i^x\pm iS_i^y$ and $B^{\pm}=B^x\pm iB^y$

$$H=-\sum_{\langle kl\rangle}\left\{\frac{1}{2}\left[S_k^+S_l^-+S_k^-S_l^+\right]+S_k^zS_l^z\right\}-\sum_k\left\{\frac{1}{2}\left[B^+S_k^-+B^-S_k^+\right]+B^zS_k^z\right\} \quad (2)$$

Where k and l denote the lattice sites and <kl> indicates summation over the nearest neighbours only.

As in reference [9], we introduce the set of thermal Green's functions in the spectral representation

$$G_{ij(\eta)}^{\alpha mn}(\omega)=S_i^{\alpha};\left(S_j^z\right)^m\left(S_j^-\right)^n{}_{\omega} \quad (3)$$

Where ω denotes the energy. and i, j the lattice sites and

$$\begin{cases}\alpha=+,-,z\\ \eta=\pm1\\ m\geq0\\ n\geq1\end{cases}$$

In order to treat the problem, one needs the following Green's functions

$$G_{ij(\eta)}^{\alpha mn}(\omega)=S_i^{\alpha};\left(S_j^z\right)^m\left(S_j^-\right)^n{}_{\omega};\quad \alpha=+,-,z \quad (4)$$

where $\eta=\pm1$ refer to the commutator ($\eta=-1$) or anti-commutator ($\eta=1$) Green'sfunctions, respectively, n≥1 and m≥ 0 are positive integers, *i* and *j* denote lattice sites.

The $G_{ij(\eta)}^{\alpha mn}$ are determined from the equations of motion in the spectral representation.

$$\omega G_{ij(\eta)}^{\alpha mn}(\omega)=\left[S_i^{\alpha},H\right]_{-1};\left(S_j^{\alpha}\right)^m\left(S_j^-\right)^n{}_{\omega} \quad (5)$$

With the inhomogeneities term is:

$$A_{ij(\eta)}^{\alpha mn}(\omega)=\left[S_i^{\alpha};\left(S_j^{\alpha}\right)^m\left(S_j^-\right)^n\right]=S_i^{\alpha}\left(S_j\right)^m\left(S_j\right)^n+\eta\left(S_j^z\right)^m\left(S_j^-\right)^nS_i^{\alpha}$$

The knowledge of the Green's functions allows the determination of the respective

Correlation functions by the spectral theorem [6, 9, 10].

$$C_{ij(\eta)}^{\alpha mn}(\omega)=\left(S_j^z\right)^m\left(S_j^-\right)^n\left(S_i^{\alpha}\right)^n$$

$$=\frac{i}{2\pi}\lim_{\delta\to O}\int_{-\infty}^{+\infty}\frac{d\omega}{e^{\beta\omega}+\eta}\left[G_{ij(\eta)}^{\alpha mn}(\omega+i\delta)+G_{ij(\eta)}^{\alpha mn}(\omega i\delta)\right] \quad (6)$$

Calculation of the commutators $\left[S_i^{\alpha},H\right]_{-1}$ yields the following set of equations of motion for the Green's functions

$$\omega G_{ij(\eta)}^{\alpha mn}=A_{ij(\eta)}^{\alpha mn}\pm J\sum_k\left(S_i^zS_k^{\pm};\left(S_j^z\right)^m\left(S_j^-\right)^n-S_k^zS_i^{\pm};\left(S_j^z\right)^m\left(S_j^-\right)^n\right)$$

$$\pm B^{\pm}G_{ij(\eta)}^{zmn}\pm B^zG_{ij(\eta)}^{\pm mn}$$

$$\omega G_{ij(\eta)}^{\alpha mn}=A_{ij(\eta)}^{\alpha mn}\pm\frac{J}{2}\sum_k\left(S_i^-S_k^+-S_k^-S_i^+;\left(S_j^z\right)^m\left(S_j^-\right)^n-\frac{1}{2}B^-G_{ij(\eta)}^{+mn}+\frac{1}{2}B^+G_{ij(\eta)}^{-mn}\right)$$

$$\pm B^{\pm}G_{ij(\eta)}^{zmn}\pm B^zG_{ij(\eta)}^{\pm mn} \quad (7)$$

The higher-order Green's functions, occurring on the right-hand sides, have to be decoupled in order to obtain a closed set of equations. For the exchange coupling terms, we apply a generalized Tyablikov (or RPA) [11, 12] decoupling, allows also for a finite value of the x, y(or $\pm$) components of the magnetization ($\alpha,\beta=+,-,z;i\neq k$).

$$S_i^{\alpha}S_k^{\beta};\left(S_j^z\right)^m\left(S_j^-\right)^n\approx S_i^{\alpha}G_{ij}^{\beta mn}+S_j^{\beta}G_{ij}^{\alpha mn} \quad (8)$$

The terms resulting from the single-ion anisotropy ($i=k$) have to be decoupled differently. An RPA-decoupling, as was proposed by Narath [12], is reasonable for exchange anisotropy $\left(\infty S_i^zS_k^z\right)$.

In reference [4] the proper inclusion of the single-ion anisotropy with Green's function techniques was thoroughly discussed in connection with the magnetic reorientation. Accordingly, we choose the Anderson-Callen [4, 13] decoupling for the treatment of the anisotropy terms:

$$S_i^{\pm}S_i^z+S_i^zS_i^{\pm}\approx2S_i^z\left(1-\frac{1}{2S}\left[S(S+1)-S_i^zS_i^z\right]\right)G_{ij}^{\pm mn} \quad (9)$$

Because we are interested in laterally periodic systems, we perform a Fourier transformation to the two-wave vector space k, introducing vectors for the Green's functions $G^{mn}(k;\omega)$, and for the inhomogeneities, A_{η}^{mn}. For spinels systems the Green's function vector G^{mn}reads.

$$(\omega\mathrm{I}-\Gamma)G_{\eta}^{mn}=A_{\eta}^{mn} \quad (10)$$

Where

$$G_\eta^{mn} = \begin{pmatrix} G_\eta^{+mn} \\ G_\eta^{-mn} \\ G_\eta^{zmn} \end{pmatrix} \quad ; \; A_\eta^{mn} = \begin{pmatrix} A_\eta^{+mn} \\ A_\eta^{-mn} \\ A_\eta^{zmn} \end{pmatrix} \tag{11}$$

And I is the unit matrix (3X3).

After applying the decoupling procedures (8) and (9), the Γ matrix obtained is:

$$\Gamma \begin{cases} \Gamma_{11} = B^z + S^z J\left(q-\gamma_k\right) \\ \Gamma_{13} = -\left[B^+ + S^+ J\left(q-\gamma_k\right)\right] \\ \Gamma_{22} = -\left[B^z + S^z J\left(q-\gamma_k\right)\right] \\ \Gamma_{23} = B^- + S^z J\left(q-\gamma_k\right) \\ \Gamma_{31} = -\frac{1}{2}\left[B^- + S^z J\left(q-\gamma_k\right)\right] \\ \Gamma_{32} = \frac{1}{2}\left[B^+ + S^z J\left(q-\gamma_k\right)\right] \\ \Gamma_{12} = \Gamma_{21} = \Gamma_{33} = 0 \end{cases}$$

Now we introduce a transformation which diagonalizes the matrix Γ.

$$R^- \Gamma R = \Omega = \begin{pmatrix} \omega_0 & 0 & 0 \\ 0 & \omega_+ & 0 \\ 0 & 0 & \omega_- \end{pmatrix}$$

Where

$$R = \begin{pmatrix} \frac{H^+}{H^z} & \frac{H^+}{H^z - E_k} & \frac{H^+}{H^z + E_k} \\ \frac{H^-}{H^z} & \frac{H^-}{H^z + E_k} & \frac{H^-}{H^z - E_k} \\ 1 & 1 & 1 \end{pmatrix}$$

Where R is the passage matrix from Γ to Ω diagonal matrix. The three eigenvalues are.

$$\begin{cases} \omega_0 = 0 \\ \omega_+ = E_k \\ \omega_- = -E_k \end{cases}$$

Where $E_k = \sqrt{H^+H^- + H^zH^z}$ is the magnons dispersion relation H^+ and H^- are given by.

$$\begin{cases} H^\pm = B^\pm + 2J\left(q-\gamma_k\right)S^\pm \\ H^z = B^z + 2J\left(q-\gamma_k\right)S^z \end{cases}$$

Where in reference [14], q is considered as the number of the nearest neighbours and.

$$\gamma_k = 4\left(cos\frac{k_x}{2}cos\frac{k_y}{2} + cos\frac{k_y}{2}cos\frac{k_z}{2} + cos\frac{k_x}{2}cos\frac{k_z}{2}\right)$$

for an fcc lattice.

Multiplying the equation of motion (10) from the left by L and inserting 1 = RL one finds.

$$L\left(\omega \mathrm{I} - \Omega\right)G_\eta^{mn} = LA_\eta^{mn} \tag{12}$$

Defining $\begin{cases} G_\eta^{mn} = LG_\eta^{mn} \\ A_\eta^{mn} = LA_\eta^{mn} \end{cases}$ We obtained

$$\left(\omega \mathrm{I} - \Omega\right)G_\eta^{mn} = A_\eta^{mn} \tag{13}$$

Where G_η^{mn} is a new vector of Green's functions, each component τ of which has only a single pole

$$G_\eta^{mn\tau} = \frac{A_\eta^{mn\tau}}{\omega - \omega_\tau} \tag{14}$$

With

$$\begin{cases} G_\eta^{mn} = LG_\eta^{mn} \\ A_\eta^{mn} = LA_\eta^{mn} \end{cases} \Rightarrow \begin{cases} RG_\eta^{mn} = G_\eta^{mn} \\ RA_\eta^{mn} = A_\eta^{mn} \end{cases} \tag{15}$$

The regular condition of Green's functions for $\omega = 0$ is

$$\lim_{\omega \to 0} \omega G_\eta^{\alpha mn} = 0 \tag{16}$$

The regular condition of Green's functions $G_{-1}^{zmn}\left(\omega, k\right)$ for $\omega = 0$ with $H^z \neq 0$ is

$$H^- A^{+mn} + H^+ A_{-1}^{-mn} + 2H^z A_{-1}^{zmn} = 0 \tag{17}$$

Evaluating the expression (17) for m =0 and n =1, we find together with the definitions in Equation (16).

$$\frac{H^\pm}{H^z} = \frac{B^\pm}{B^z} \tag{18}$$

Introducing the equation (18) in equation (17) we obtained

$$-2B^z A_{-1}^{z01} = A_{-1}^{+01}B^- + A_{-1}^{-01}B^+ \tag{19}$$

With

$$\begin{cases} A_{-1}^{+01} = 2S^z \\ A_{-1}^{+01} \\ A_{-1}^{z01} = -S^z \end{cases}$$

Introducing these expressions in equation (19) we obtained:

$$S^{\pm} = \frac{B^{\pm}}{B^z} S^z \tag{20}$$

The equations determining the correlation functions are:

$$C_{ij}^{\alpha mn} = \left\langle \left(S_j^z\right)^m \left(S_j^-\right)^n S_i^{\alpha} \right\rangle \tag{21}$$

The application of the spectral theorem [15, 11] to the t component of the single-pole

Green's function of Equation (15) then yields

$$C_{ij}^{\alpha mn} = \frac{i}{2\pi} \lim_{\delta \to O} \int_{-\infty}^{+\infty} \frac{d\omega}{e^{\beta\omega} - 1} \left[G_{-1}^{zmn}\left(\omega + i\delta k\right) + G_{-1}^{zmn}\left(\omega - i\delta k\right) \right] + D_k^{zmn} \tag{22}$$

Here D_k^{zmn} is the corrective term of correlation function obtained from Green's anti-commutator function.

$$D_k^{zmn} = \lim_{\omega \to 0} \frac{\omega}{2} G_+^{zmn}\left(\omega, k\right)$$

Using the relation between anticommutators and commutator correlation functions.

$$A_{+1}^{\alpha mn} = A_{-1}^{\alpha mn} + 2C_{-1}^{\alpha mn}$$

We find together with regularity condition:

$$D_k^{zmn} = \frac{C_k^{+mn} H^- H^z + C_k^{-mn} H^+ H^z + C_k^{zmn} \left(H^z\right)^2}{2E_k^2} \tag{23}$$

By using the expression of the equations determining the correlation functions.

$$G_-^{+mn}\left(\omega, k\right) = \frac{A^{+mn} \frac{1}{2} H^- \omega + A_{-1}^{-mn} \frac{1}{2} H^+ \omega + A_-^{zmn} \omega^2}{\omega\left(\omega - E_k\right)\left(\omega + E_k\right)}$$

and introducing (23) in Equation (22), we obtained [8, 9]

$$2H^+H^-C_k^{mn} - H^-H^zC_k^{+mn} - H^+H^zC_k^{-mn} = A^{+mn}\frac{1}{2}E_kH^-\left[\frac{E_k}{H^z} - coth\left(\frac{\beta E_k}{2}\right)\right]$$

$$+ A^{-mn}\frac{1}{2}E_k + \left[\frac{E_k}{H^z} + coth\left(\frac{\beta E_k}{2}\right)\right] \tag{24}$$

Evaluating the expression (17) for m =0 and n =2, we find together with the definitions in equation (12).

$$4B^zS^-S^- = 4B^xS^zS^- + \frac{B^x}{B^z}S^z \tag{25}$$

Evaluating the expression (18) for m =1 and n =1, we find together with the definitions in equation (13)

$$2B^zS^zS^- = \left(3S^zS^z - S^z - S\left(S+S\right)\right)B^x + S^-S^-B^x \tag{26}$$

Evaluating the expression (24) for m =0 and n =1, and introducing the equation (25) and (26) we obtained for S =1

$$S^zS^z - \frac{2}{1+\left(\frac{B^x}{B^z}\right)^2} + \frac{S^z}{2}\frac{\left[2-\left(\frac{B^x}{B^z}\right)^2\right]}{\sqrt{1+\left(\frac{B^x}{B^z}\right)^2}}\Phi\left(T\right) = 0 \tag{27}$$

where

$$\Phi\left(T\right) = \frac{1}{\pi^2}\int_0^{\pi} dk_x \int_0^{\pi} dk_y coth\left(\frac{\beta E_k}{2}\right) \tag{28}$$

Evaluating the expression (24) for m =1 and n =1 and S =1, and introducing the regular conditions of Green's functions for $\omega = 0$, we obtained

$$S^z = \left(1-\left(\frac{B^x}{B^z}\right)^2\right)\left(2-\left(\frac{B^x}{B^z}\right)^2\right) + 2S^zS^z\left(1+\left(\frac{B^x}{B^z}\right)^2\right) - 4$$

$$-\left[6S^zS^z - \left(1-\frac{B^x}{B^z}\right)^2 - S^z\left(2-\left(\frac{B^x}{B^z}\right)^2\right)\right]\sqrt{1+\left(\frac{B^x}{B^z}\right)^2}\,\Phi\left(T\right) = 0 \tag{29}$$

In reference [16], Callen also derives a closed form expression for the magnetization for general spin S from the solution of a differential equation. The result is

$$S^z = \frac{\left(S-\phi_k\right)\left(1+\phi_k\right)\left(1+\phi_k\right)^{2S+1} + \left(1+S+\phi_k\right)\phi_k^{2S+1}}{\left(1+\phi_k\right)^{2S+1} - \phi_k^{2S+1}} \tag{30}$$

With

$$\phi_k = \frac{1}{e^{\beta E_k} - 1} = \frac{1}{2} coth\left(\frac{\beta E_k}{2} - 1\right)$$

In the following, we restrict ourselves to an external magnetic field B connected to the (x-z)-plane. Because of the azimuthal symmetry in the case of uniaxial anisotropy, it issucient to deal with the z and x components of the magnetization ($\left\langle S^y \right\rangle = 0$ for $B^y = 0$).

The equations (27), (28), (29) and (30) have to be solved numerically in order to obtain $\left\langle S^z \right\rangle$ and $\left\langle S^zS^z \right\rangle$.

We have used equations (20) to (30) to calculate the thermal magnetization.

$$M\left(T\right) = \sqrt{\left\langle S^x \right\rangle^2 + \left\langle S^z \right\rangle^2} \tag{31}$$

Introducing equation (21) in this expression we obtained.

$$M(T)=\left(\sqrt{\left(\frac{B^x}{B^z}\right)^2+1}\right)\left\langle S^z\right\rangle \tag{32}$$

The equilibrium polar angle of the magnetization is determined.

$$\theta_0(T)=Arctg\left(\frac{S^x}{S^z}\right) \tag{33}$$

The knowledge of M(T) and θ_0 enables a non perturbative determination of the temperature dependence of the anisotropy coefficient.

Introducing the equation (20) in this expression we obtained

$$\theta_0(T)=Arctg\left(\frac{B^x}{B^z}\right) \tag{34}$$

The components of magnetic susceptibility are defined by

$$\begin{cases}\chi_{zz}=\dfrac{\left\langle S^z\left(B^z\right)\right\rangle-\left\langle S^z(0)\right\rangle}{B^z}\\ \chi_{xx}=\dfrac{\left\langle S^x\left(B^x\right)\right\rangle-\left\langle S^x(0)\right\rangle}{B^x}\end{cases} \tag{35}$$

2.2. Mean Field Theory (MFT) and Probability Law

The method of Holland and Brown [17], leads to simple relations between the paramagnetic Curie-temperature θp and the Néel temperature T_N respectively by:

$$\begin{cases}T_C(K)=\dfrac{5}{2k_B}[-4J_1(K)+2J_2(K)]\\ \theta_p(K)=\dfrac{5}{2k_B}[6J_1(K)+12J_2(K)]\end{cases} \tag{36}$$

where k_B is the Boltzmann's constant.

In the antiferromagnetic region, we have used the Néel temperature T_N formula given by:

$$T_N=\frac{2}{3}S(S+1)\lambda(\phi) \tag{37}$$

Where $\lambda(\phi)$ is the eigenvalue of the matrix formed by the Fourier transform of the exchange integral.

We have used the experimental results of T_N = 719 K [8] and Θp = −45 K [18] for $MgFe_2O_4$, and T_N = 860 K [8], Θp = −31 K [18] for $NiFe_2O_4$. The values of the exchange integrals of $MgFe_2O_4$ and $NiFe_2O_4$ are (J_1 = -6.4 K and J_2 = -9.8 K) and (J_1 = −14.67 K and J_2 = -14.33 K), respectively.

We have used the probability law [19] to determine the exchange integrals $J_1(x)$ and $J_2(x)$ in the whole range of concentration x=0 to 1:

$$J_{AA'}(x)=x^6J_A+6x^5(1-x)\left(J_A^5J_{A'}\right)^{1/6}+15x^4(1-x)^2\left(J_A^4J_{A'}^2\right)^{1/6}+20x^3(1-x)^3\left(J_A^3J_{A'}^3\right)^{1/6}$$
$$+15x^2(1-x)^4\left(J_A^2J_{A'}^4\right)^{1/6}+6x(1-x)^5\left(J_AJ_{A'}^5\right)^{1/6}+(1-x)^6J_{A'} \tag{38}$$

If $J_A(J_{A'})$ corresponds to the nn interactions of the opposite pure systems $AB_2O_4\left(A'B_2O_4\right)$ respectively. $J_{AA'}=J_1(x)$. If $J_A(J_{A'})$ corresponds to the nnn super-exchanges interactions of the opposite pure systems $AB_2X_4\left(A'B_2X_4\right)$ respectively $J_{AA'}=J_2(x)$. The obtained values of J_1 and J_2 for different dilution x are given in Table 1.

3. Results and Discussion

In this work, by employing the results of experiments and (HTSE) and GFT) theories, for a randomly diluted Heisenberg magnet, we have deduced the magnetic phase diagram of $Mg_xNi_{1-x}Fe_2O_4$ in the range 0 ≤ x ≤ 1. The obtained result in table 1 is presented in Figure 1. A single thermodynamic phase may appear, including the transition on paramagnetic order (PM) to the ferrimagnetic order (FerriM) in the range $0 \le x \le 1$ for the $Mg_xNi_{1-x}Fe_2O_4$ systems. In this figure, we have included, for comparison, the experimental results obtained by magnetic measurement. The passage from a phase to another is caused by the variation of exchange interactions [19].

In the other hand, we display results of our calculations for a Heisenberg Hamiltonian under an external magnetic field in the plane (x-z), with spin S =5/2 for a diluted normal spinel $Mg_xNi_{1-x}Fe_2O_4$ system having an fcc structure. we have presented in the figure 2 and figure 3 the thermal normalized magnetization $\langle\frac{S^z}{S}\rangle$ and magnetic susceptibility χ_{zz} respectively for diluted spinel system $Mg_xNi_{1-x}Fe_2O_4$ for B^x= 0, and B^z= 0,1T with values of dilution x = 0 to 1 and calculated by Green's function theory. The dependence of the thermal magnetization at the field B^z= 0.1 T for the different values of dilution x are observed. We see that the typical ferromagnetic behaviour is noticed only at T < T_N. The curves do not drop at T_C as may be expected for a pure ferromagnetic system but have only an inflexion at T_N. The values of Néel temperatures found in this work are roughly similar to the values found in reference [7] and [8]. We have compared the results obtained by different theory's.

Table 1. *Néel temperature: (K)(exp)[13], T_N(K)(GFT), T_N(K)(HTSE)), and the exchange integrals: $J_1(x)$ and $J_2(x)$ of $Mg_xNi_{1-x}F_2O_4$. as function of dilution x.*

x	T_N(K)(Exp) [8]	$J_1(x)$	$J_2(x)$	T_N(K)(GFT)	T_N(K)(HTSE).[7]
0	860	-14.67	-14.33	862	866
0.2	831	-12.54	-13.30	833	838
0.4	809	-10.67	-12.34	811	815
0.6	786	-9.04	-11.44	788	793
0.8	759	-7.62	-10.59	761	763
1	719	-6.4	-9.8	721	725

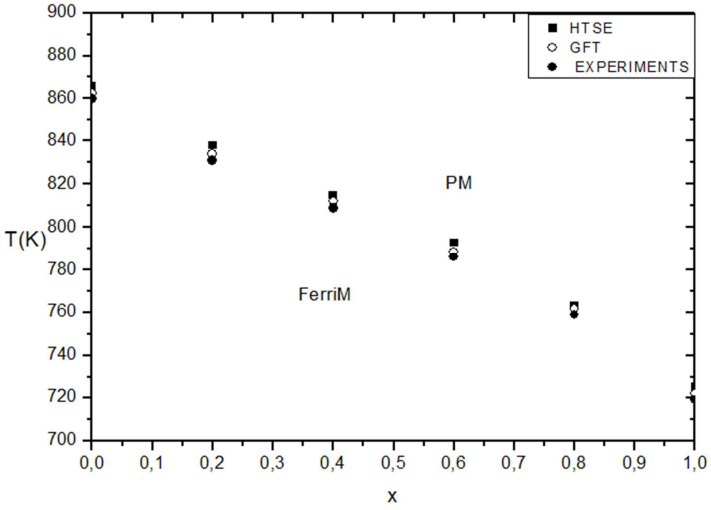

Figure 1. *agnetic phase diagram of $Mg_xNi_{1-x}Fe_2O_4$. A single phase is the transition of paramagnetic phase (PM) to ferrimagnetic (FerriM) phase in the range $0 \leq x \leq 1$. The solid squares are the results of (H.T.S.E) [7] theory. The open circles are the results of (G.F.T) theory. The solid circles represent the experimental points deduced by magnetic measurements [8].*

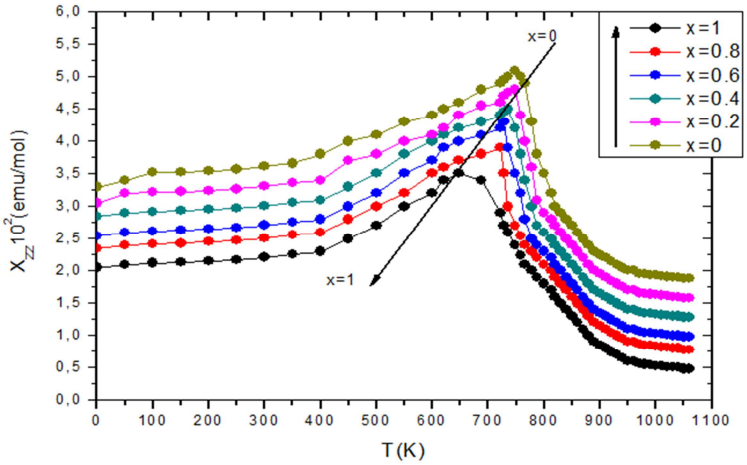

Figure 2. *mperature dependencies of the magnetic susceptibilities X_{ZZ} in spinel system $Mg_xNi_{1-x}Fe_2O_4$ for B^x=0, and B^z =0.1T, and with The values of dilution x = 0 to 1.*

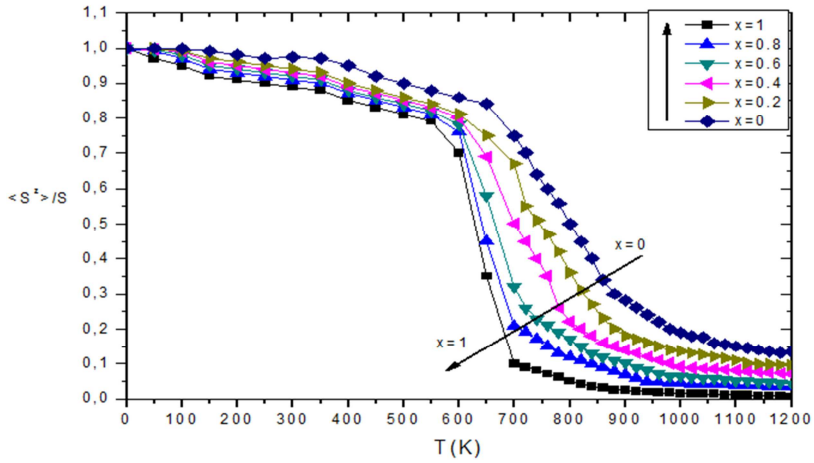

Figure 3. *ermal normalized magnetization $\langle \frac{S^z}{S} \rangle$ in spinel system $Mg_xNi_{1-x}Fe_2O_4$ for B^x=0, and B^z = 0.1T, and with the values of dilution x = 0 to 1.*

We find significant differences between results in the Green's function theory (GFT) and the high temperature series expansion (HTSE). We attribute this difference to the quantum fluctuations taken into account in the first theory but neglected in second treatment respectively, which are known to have a much greater influence on the magnetic

properties of spinel $Mg_xNi_{1-x}Fe_2O_4$.

4. Conclusion

The theoretical approach presented in this paper successfully captures the characteristic rapid reduction in magnetization near the Néel temperature. The thermal magnetization and the magnetic susceptibility are calculated with the help of many-body Green's function theory. The analysis of the magnetization evolution shows the existence of exchanges interaction effects on the magnetic order.

By applying the Green's function theory to the magnetic susceptibility $\chi(T)$ we have estimated the critical temperature T_N for each dilution x. The obtained magnetic phase diagram of spinels $Mg_xNi_{1-x}F_2O_4$ systems is presented in Figure 1. A single phase is the transition of paramagnetic phase (PM) to ferrimagnetic (FerriM) phase in the range $0 \leq x \leq 1$. In this figure, we have included, for comparison, the theoretical results obtained by (HTSE) theory [7], and the experimental results obtained by magnetic measurement [8], The obtained values of the critical temperature by Green's function theory (GFT), show some significant differences.

References

[1] S. B. Lazarev, M. Pantić, B. S. Tošić. Green's function theory of free electrons in thin films. Physica. A. 1997;246:53.

[2] O. Schute, F. Klose, W. Felch, Phys. Rev. B. 52 (1995) 6480.

[3] M. Farle, B. Mirwald Schulz, A. N. Anisimov, W. Platow, K. Baberschke, Phys. Rev. B. 55 (1997) 3708.

[4] P. Frobrich, J. Jensen, P. J. Kuntz, Eur. Phys. J. B. 13 (2000) 477.

[5] P. Frobrich and al, Eur. Phys. J. B. 18 (2000) 579.

[6] S. V. Tyablikov. Methods in the quantum theory of magnetism. Ukr. Mat. Zh. 1959;11:289.

[7] S. Salmi, M. El Azhari, A. El Grini, A. Hourmatallah, N. Benzakour, K. Bouslykhane and A. Marzouk. IOP Conf. Series: Materials Science and Engineering 186 (2017) 012016.

[8] Mahbubul Haque Md , Kazi Hanium Maria, Shamima Choudhury, Mahabub Alam Bhuiyan and.Hakim M A, *Journal of Ceramic Processing Research*. 14 (2013) 82–86.

[9] P. Frobrich et al.: Eur. Phys. J. B. 13, 744 2000.

[10] K. Elk, W. Gasser. Die methode der greenschenfunktionen in der festkorperphysik. Berlin: Akademie-Verlag; 1978.

[11] W. Nolting, Quanten theorie des Magnetismus, 2 Stuttgart: B. G. Teubner; 1986.

[12] A. Narath, Phys. Rev. 140, (1965) A854.

[13] F. B. Anderson, H. B. Callen, Phys. Rev. 136, (1964) A1068.

[14] F. Bloch. Z. Phys. 61 (1930) 206.

[15] K. Elk and W. Gasser, Die Methode der Greenschen Funktionen in der Festkorperphysik (Akademie-Verlag, Berlin, 1978).

[16] H. B. Callen, Phys. Rev. 130 (1963) 890.

[17] W. E. Holland and H. A. Broun (1972). Phys-Stat. Sol (a) 10,249.

[18] Mazhar U. Rana, Tahir Abbas. Materials Letters 57 (2002) 925–928.

[19] R. Masrour, M. Hamedoun, A. Benyoussef, A. Hourmatallah, K. Bouslykhane, N. Benzakour. Chin. Phys. Lett 2511(2008) 4117.