American Journal of Physics and Applications
2018; 6(1): 22-25
http://www.sciencepublishinggroup.com/j/ajpa
doi: 10.11648/j.ajpa.20180601.15
ISSN: 2330-4286 (Print); ISSN: 2330-4308 (Online)

Energy Levels Calculations in ^{23}Mg/^{23}Na and ^{47}Cr/^{47}V Mirror Nuclei

Saeed Mohammadi, Alireza Ghayeni, Asiyeh Roushan

Department of Physics, Payame Noor University, Tehran, Iran

Email address:
Smohammadi1958@yahoo.co.uk (S. Mohammadi)

To cite this article:
Saeed Mohammadi, Alireza Ghayeni, Asiyeh Roushan. Energy Levels Calculations in ^{23}Mg/^{23}Na and ^{47}Cr/^{47}V Mirror Nuclei. *American Journal of Physics and Applications*. Vol. 6, No. 1, 2018, pp. 22-25. doi: 10.11648/j.ajpa.20180601.15

Received: December 18, 2017; **Accepted**: December 19, 2017; **Published**: January 17, 2018

Abstract: Coulomb Displacement Energies in mirror nuclei ^{23}Mg, ^{23}Na and ^{47}Cr, ^{47}V have been calculated using shell model code OXBASH [1] and compared with experimental results. The calculations were carried out in the USD model space with the W Hamiltonian [2]. This code which is based on one of the most applicable nuclear models, the shell model, deals with evaluating energy levels in nuclei. A comparison had been made between computational results in this paper and the available experimental data to test theoretical shell model description of nuclear structure in mirror nuclei. The energy states of mirror nuclei are almost identical, except for the small effects due to Coulomb interaction where the symmetry in being broken. The calculated energy spectrum is in good agreement with the available experimental data.

Keywords: Mirror Nuclei, OXBASH Code, Shell Model Structure, Model Space, Energy Levels

1. Introduction

The energy states of mirror nuclei (nuclei with the same mass number and the number of protons in one of them equals the number of neutrons in the other) are almost identical, except for the small effects due to Coulomb interaction where the symmetry in being broken. The study of this symmetry breaking reveals details of the mirror nuclei structure. This shift in mirror symmetry will be observed mostly as a function of spin, where the protons and/or neutrons rearrange themselves in new shell model orbits and hence cause changes in Coulomb energy differences. These effects, known as Coulomb Displacement Energies (CDE), have been the subject of several studies in nuclear structure physics [3-5].

Obtaining the nuclear structure and energy levels of nuclei is one of the criteria to improve investigations of nuclei properties. Nuclear models have the property to help us to better understanding of nuclear structure which contains main physical properties of nuclei, and shell-model is one of the most prominent and successful nuclear models [6-12]. This model can be compared with the electron shell model for atoms. As atomic behavior and properties can be described with valance electrons which exist out of a closed shell, similarly, valance nucleons (protons or neutrons) in a nucleus which are placed out of close shells (with magic numbers 2, 8, 20, 28, 50, 82 and 126) play important roles in determining nuclear properties. Nuclei with magic numbers are very stable and have completely different properties comparing with their neighbors.

One of the most attractive features of the spherical shell model is its relative simplicity for calculations in a strongly restricted configuration space. If the space is sufficiently truncated (i.e. beyond closed shells), one can perform exact shell model calculations which make the comparison with experiments more transparent and hence more attractive also for experimentalists. However, with any reasonable model space truncation one is always left with the problem of determining an effective residual interaction for the nucleons in the considered orbits, usually assumed to be a two-body force [2]. Clearly, the smaller the number of orbits considered the smaller the number of two-body matrix elements one has to deal with. Since it is a nontrivial problem to establish the two-body matrix elements for a shell model calculation, the uncertainties might increase with a larger shell mode space, although in principle one should get better agreement with experiment.

It has to be recalled that ^{23}Mg / ^{23}Na and ^{47}Cr / ^{47}V are mirror nuclei in the SD shell (see figure 1), and that their level schemes should be identical in that configuration

space. Considering the number of valance nucleons of these isotopes, the suitable space for related calculations of these nuclei is SD space. The SD model space includes the $1s_{1/2}$, $1d_{5/2}$ and $1d_{3/2}$ orbitals and 13 different interaction potentials which are considered in this model space are: SDBA, KUOSD, KUOSDM, BKUOSD, PW, CW, W, CWH, KUOSDH, SDM, W, HBUSD, HBUMSD and SU3SD [2].

For light nuclei, there are several standard effective interactions for the *p* and *SD* shells, respectively [13, 14]. Analysis of neutron-rich SD nuclei has been of high interest in recent years as they present new aspects of nuclear structure. Traditional shell-model studies have recently received a renewed interest through large scale shell-model computing in no-core calculations for light and medium nuclei. It is now therefore fully possible to work to large-scale shell-model data and study the excitation levels for larger systems. In these systems, inert core is assumed and space is determined by considering shell gaps. Figure 1 shows the PF and SD model spaces according to shell model theory.

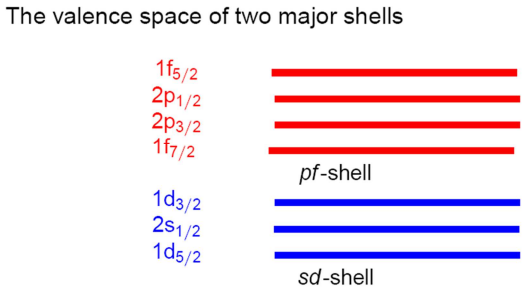

Figure 1. *SD and PF shell model spaces.*

2. Calculations

In order to calculate the nuclear structure properties of both ground and excited states based on the nuclear shell model one needs to have wave functions of those states. These wave functions are obtained by using the shell-model code OXBASH [1]. OXBASH code is a computer program that is described with a set of model spaces and interactions to apply in shell model calculations with high dimensions. This program is a set of commands for carrying out shell-model calculations with dimensions up to about 100,000 in the J-T scheme and about 2,000,000 in the M-scheme. We applied the 2005-8 version of this code which can be used on any Windows PC without the use of any other software [1].

In order to use this code one should specify the model space and interaction. In other words after choosing appropriate model space which is chosen considering valance nucleons, this code constructs a set of possible ground states and then makes JT matrix based on linear combination of ground states which give suitable T and J values. Finally by choosing the desirable interaction potential it constructs the Hamiltonian of the problem and carries out the calculations and as a default gives 10 lowest energies. In this paper, the energy levels of ^{23}Mg / ^{23}Na and ^{47}Cr / ^{47}V mirror nuclei have been calculated using the code OXBASH. Considering the number of valence nucleons for these nuclei, SD model space is the suitable model for these calculations which assign separate orbitals for protons and neutrons. This model consists of $d_{5/2}$, $s_{1/2}$ and $d_{3/2}$ valence orbitals. The code OXBASH for Windows has been used to calculate the nuclear structure for the above nuclei by employing the SD model space with the W effective interaction [2].

3. ^{23}Mg/^{23}Na Mirror Nuclei

In this section, some results concerning ground and excitation energies of the ^{23}Mg/^{23}Na Mirror Nuclei are presented. Table 1 shows data for ^{23}Mg isotope and table 2 shows data for ^{23}Na isotope. The first column is spin of states, column two the calculated energies by OXBASH code, columns three and four the measured energies and their respective errors [4].

Table 1. *Data for ^{23}Mg. All energies are in MeV.*

J	E (OXBASH)	E (Exp)
1=J=3/2	0.424	0.028
2=J=7/2	2.312	2.015
3=J=1/2	4.278	4.354
4=J=3/2	5.583	5.456
5=J=5/2	5.599	5.656
6=J=5/2	6.619	6.899
7=J=3/2	7.136	7.258

Table 2. *Data for ^{23}Na. All energies are in MeV.*

J	E (OXBASH)	E (Exp)
1=J=5/2	1.568	0.439
2=J=7/2	2.312	2.076
3=J=9/2	2.625	2.703
4=J=5/2	3.246	3.914
5=J=7/2	4.172	4.784
6=J=11/2	5.285	5.534
7=J=3/2	7.136	6.947
8=J=1/2	8.345	8.663

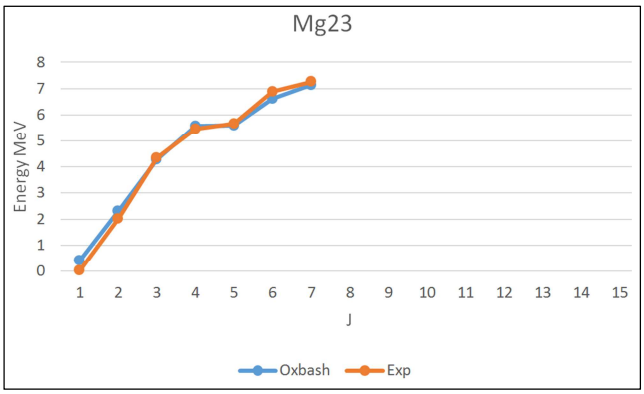

Figure 2. *Calculated and measured energies of ^{23}Mg.*

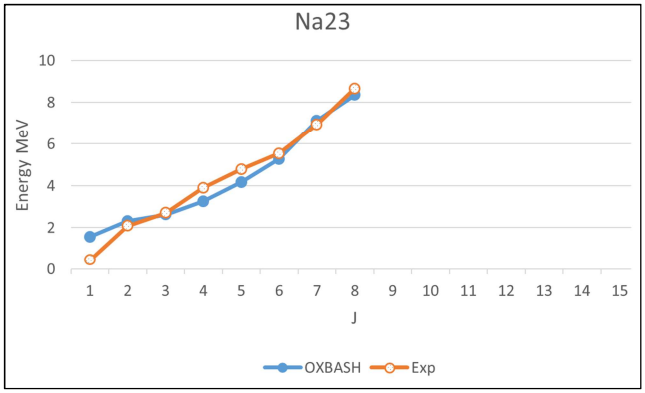

Figure 3. *Calculated and measured energies of ^{23}Na.*

Figures 2 and 3 shows calculated and measured energies of ^{23}Mg and ^{23}Na respectively. Due to very small measured errors, they have not shown here. As it can be seen, there is relatively good agreement between calculations made by OXBASH code and measured energies.

4. ^{47}Cr/^{47}V Mirror Nuclei

Here are presented some results concerning ground and excitation energies of the ^{47}Cr/^{47}V Mirror Nuclei. Table 3 shows data for ^{47}Cr isotope and table 4 shows data for ^{47}V isotope. The first column is spin of states, column two the calculated energies by OXBASH code, columns three and four the measured energies and their respective errors [15].

Table 3. *Data for ^{47}Cr. All energies are in MeV.*

J	E (OXBASH)	E (Exp)
1=J=9/2	0.473	0.471
2=J=5/2	1.568	1.333
3=J=7/2	2.312	2.406
4=J=3/2	3.421	3.43
5=J=9/2	3.852	3.747
6=J=3/2	4.166	4.169
7=J=7/2	5.413	5.409

Table 4. *Data for ^{47}V. All energies are in MeV.*

J	E (OXBASH)	E (Exp)
1=J=3/2	0.424	0.259
2=J=1/2	1.998	1.66
3=J=5/2	2.838	2.439
4=J=7/2	2.922	2.81
5=J=5/2	3.246	3.359
6=J=3/2	4.166	3.958
7=J=1/2	5.928	6.041
8=J=5/2	6.202	6.229
9=J=1/2	6.594	6.692

Figures 4 and 5 shows calculated and measured energies of ^{47}Cr and ^{47}V respectively. Due to very small measured errors, they have not shown here. As it can be seen, there is relatively good agreement between calculations made by OXBASH code and measured energies.

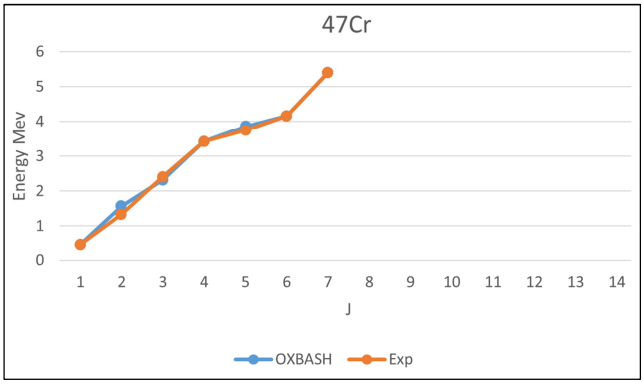

Figure 4. *Calculated and measured energies of ^{47}Cr.*

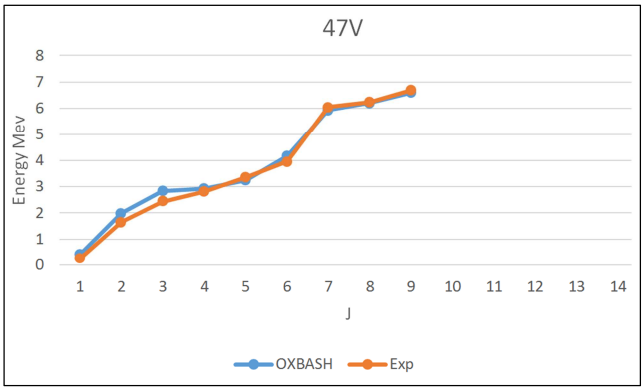

Figure 5. *Calculated and measured energies of ^{47}V.*

5. Conclusions

Coulomb Displacement Energies in mirror nuclei ^{23}Mg / ^{23}Na and ^{47}Cr / ^{47}V (the Z of the first nucleus must equal the N of the second and thus the N of the first equals the Z of the second) have been calculated using shell model code OXBASH and compared with experimental results. The calculations were carried out in the in the USD model space with the W Hamiltonian. The results show that the CDE of mirror nuclei which is the difference between binding energy of the mirror nuclei is not constant and there is some changes with increase in excitation energy of the nuclei due to rearranging of the nucleons.

References

[1] OXBASH for Windows, B. A. Brown, A. Etchegoyen, N. S. Godwin, W. D. M. Rae, W. A. Richter, W. E. Ormand, E. K. Warburton, J. S. Winfield, L. Zhao and C. H. Zimmerman, MSU_NSCL report number 1289.

[2] The Empirical $(1f_{7/2})^n$ Model, W. Kutschera, B. A. Brown and K. Ogawa, La Rivista del Nuovo Cimento 11, (1978).

[3] J. A. Cameron, et al, High-spin states in the mirror nuclei ^{49}Cr and ^{49}Mn, Phys. Lett. B 235 (1990) 235.

[4] C. D. O'Leary, et al, *Mirror Symmetry Up to the Band Termination in ^{49}Mn and ^{49}Cr,* Phys. Rev. Lett. 79, 4349 (1997).

[5] S. M. Lenzi, Coulomb energy differences in mirror nuclei, Journal of Physics: Conference Series 49 (2006) 85–90.

[6] K. S. Krane, Introductory Nuclear Physics, 3rd edition, John Wiley & Sons, 1988.

[7] S. Mohammadi, S. Arbab, E. Tavakoli. Energy Levels Calculations of 22-23Na and 24-26Mg Isotopes Using Shell Model Code OXBASH. *American Journal of Modern Physics.* Special Issue: Many Particle Simulations. Vol. 4, No. 3-1, 2015, pp. 27-31. doi: 10.11648/j.ajmp.s.2015040301.16.

[8] Saeed Mohammadi, Monna Chobbdar. Energy Levels Calculations of 32Cl and 33Cl Isotopes. *American Journal of Modern Physics.* Special Issue: Many Particle Simulations. Vol. 4, No. 3-1, 2015, pp. 10-14. doi: 10.11648/j.ajmp.s.2015040301.13.

[9] S. Mohammadi, A. Heydarzade, V. Ragheb. Energy Levels Calculations of 28, 30 Na and 26, 28 Al Isotopes Using Shell Model Code OXBASH, *American Journal of Modern Physics.* Special Issue: Many Particle Simulations. Vol. 4, No. 3-1, 2015, pp. 32-35. doi: 10.11648/j.ajmp.s.2015040301.17.

[10] S. Mohammadi, F. Bakhshabadi. Calculation of the Energy Levels of Phosphorus Isotopes (A=31 to 35) by Using OXBASH Code. *American Journal of Modern Physics.* Special Issue: Many Particle Simulations. Vol. 4, No. 3-1, 2015, pp. 15-22. doi: 10.11648/j.ajmp.s.2015040301.14.

[11] Saeed Mohammadi, Maryam Mounesi. Energy Levels Calculations of 20Ne and 21Ne Isotopes. *American Journal of Modern Physics*. Special Issue: Many Particle Simulations. Vol. 4, No. 3-1, 2015, pp. 36-39. doi: 10.11648/j.ajmp.s.2015040301.18.

[12] Saeed Mohammadi, Hassan Rostam Nezhad. Energy Levels Calculations of 26Al and 29Al Isotopes. *American Journal of Modern Physics.* Special Issue: Many Particle Simulations. Vol. 4, No. 3-1, 2015, pp. 1-4. doi: 10.11648/j.ajmp.s.2015040301.11.

[13] D. J. Dean, et al, Effective interactions and the nuclear shell-model, Progress in Particle and Nuclear Physics 53 (2004) 419–500.

[14] N. A. Smirnova, et al, Shell structure evolution and effective in-medium NN interaction, PHYSICAL REVIEW C 86, (2012) 034314.

[15] Table of Isotopes, edited by R. B. Firestone, 8th edition, John Wiley & Sons, 1990.