American Journal of Physics and Applications
2017; 5(5): 73-79
http://www.sciencepublishinggroup.com/j/ajpa
doi: 10.11648/j.ajpa.20170505.13
ISSN: 2330-4286 (Print); ISSN: 2330-4308 (Online)

Charge Exchange of Proton Sodium Atom Collision

Sabbah A. Elkilany[1, 2]

[1]Department of Mathematics, Faculty of Science, University of Dammam, Dammam, Kingdom of Saudi Arabia

[2]Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafrelsheikh, Egypt

Email address:
sabbelkilany@yahoo.com

To cite this article:
Sabbah A. Elkilany. Charge Exchange of Proton-Sodium Atom Collision. *American Journal of Physics and Applications.* Vol. 5, No. 5, 2017, pp. 73-79. doi: 10.11648/j.ajpa.20170505.13

Received: June 22, 2017; **Accepted**: August 7, 2017; **Published**: October 11, 2017

Abstract: The coupled static approximation is modified for the first time to make it applicable to multi-channels problem of the collision of the proton by alkali atom. The possibility of producing more hydrogen during the proton-alkali atom collision is investigated. The formation of hydrogen $H(1s)$ and excited hydrogen (in 2s- and 2p-states) of the scattering of the proton by sodium atom is treated for the first time to test the convergence of our method. The modified method is used to calculate the total cross-sections of seven partial waves ($0 \le \ell \le 6$, where ℓ is the total angular momentum) at incident energies between 50 and 1000 keV. Our p-Na results and those determined by previous authors are in reasonable agreement.

Keywords: Inelastic Scattering, Proton-Alkali, Proton-Sodium, Hydrogen Formation, Excited Hydrogen Formation, Cross-Sections

1. Introduction

The appearance of intermediate states in atomic and nuclear reactions is considered as the most interesting phenomenon since the development of quantum theory. The development of the femtosecond laser has tremendously enhanced the experimental identification of these states in chemical and physical reactions. Most theoretical treatments of electron-, positron-, and proton-atom interactions are based on calculations of differential, partial and total cross-sections as functions of incident energies using different approximations. Choudhury and Sural [1] have studied proton-alkali atom (Na, K, Rb, Cs) collision in the wave formation of impulse approximation at the energies ranging from 50 to 500 keV.. Daniele et al. [2] have been calculated the total cross-sections for high energy proton scattering by alkali atom using eikonal-approximation. Ferrante and Fiordilino [3] have been used the eikonal-approximation to investigate high-energy proton collision with alkali atom. Ferrante et al. [4] have also studied the total hydrogen formation cross-sections in proton scattered with alkali atom using Oppenheimer Brinkman Kramers (OBK) approximation. Tiwari [5] have been reported the differential and total cross-sections of hydrogen formation of the collision of protons by lithium and sodium atoms using the Coulomb-projected Born approximation.

The aim of the present work is to explore the possibility of producing more hydrogen during the proton-sodium atom collision. For this reason, it is important to investigate the inelastic scattering of proton with sodium atom. In the present paper, the coupled static approximation used by Elkilany ([6]-[9]) will be modified to make it applicable to discuss the multi-channels problem (n=4) of the collision of proton with sodium atom at intermediate energies of the projectile. A numerical method technique will generalized to solve the obtained multi-coupled equations. Throughout this paper Rydberg units have been used and the total cross-sections are expressed in units of πa_0^2 ($=8.8\times10^{-17}\,cm^2$) and energy units of keV.

2. Theoretical Formalism

The multi-channels inelastic scattering of protons by alkali atoms can be sketched by(see Figure 1)

$$p+A=\begin{cases} p+A & \text{Elastic channel first channel)} \\ H(n\ell)+A^{+} & H(n\ell)\ \text{formation channels}\ \ (n\text{-1})\text{-channels)} \end{cases} \quad (1)$$

where p is the proton, A is alkali target atom, $H(n\ell)$ is

hydrogen formation of $n\ell-$states and n is the number of open channels.

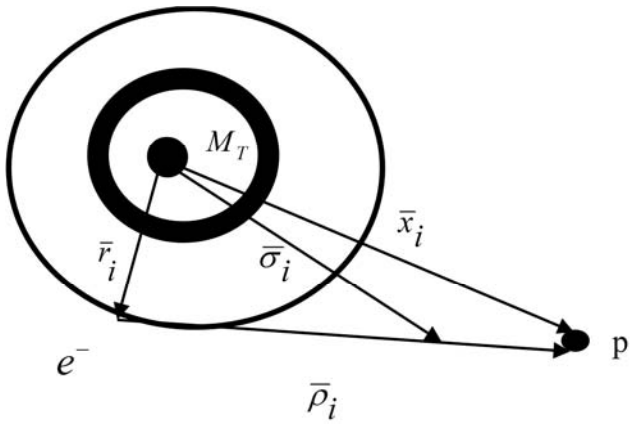

Figure 1. *Configuration space of* p−Atom *scattering:* $\bar{x}_i$ and $\bar{r}_i$ *are the position vectors of the projectile proton and the valence electron of the target with respect to the center of mass of the target,* $\bar{\rho}_i$ *is the position vector of the projectile proton with respect to the valence electron of the target,* $\bar{\sigma}_i$ *is the position vector of the center of mass of* H *from the target,* M_T *= mass of the nucleus of the target.*

The Hamiltonian of the elastic channel is given by:

$$\begin{aligned} \mathrm{H}=\mathrm{H}^{(1)} &= \mathrm{H}_T-\frac{1}{2\mu_1}\nabla_{x_1}^2+V_{\mathrm{int}}^{(1)}(x_1) \\ &=-\frac{1}{2\mu_T}\nabla_{r_1}^2-\frac{2}{r_1}+V_c(r_1)-\frac{1}{2\mu_1}\nabla_{x_1}^2+V_{\mathrm{int}}^{(1)}(x_1), \end{aligned} \tag{2}$$

where H_T is the Hamiltonian of the target atom.

μ_T is the reduced mass of the target atom.

The Hamiltonian of the (n-1)-rearrangement channels are expressed by:

$$\begin{aligned} \mathrm{H}=\mathrm{H}^{(i)} &= \mathrm{H}_i-\frac{1}{2\mu_i}\nabla_{\sigma_i}^2+V_{\mathrm{int}}^{(i)}(\sigma_i) \\ &=-\frac{1}{2\mu_i}\nabla_{\rho_i}^2-\frac{2}{\rho_i}-\frac{1}{2\mu_i}\nabla_{\sigma_i}^2+V_{\mathrm{int}}^{(i)}(\sigma_i), \\ & i=2,3,4,.....n \end{aligned} \tag{3}$$

where H_i, $i=2,3,4,.....n$ are the Hamiltonians of the hydrogen formation atoms, $H(nl)$, respectively. μ_i, $i=2,3,4,.....n$ are the reduced masses of (n-1)-channels, respectively. $V_c(r_1)$ is a screened potential and $V_{\mathrm{int}}^{(1)}(x_1)$ is the interaction potential of the first channel and are given by:

$$V_c(r_1)=V_{cCoul}(r_1)+V_{cex}(r_1) \tag{4}$$

where $V_{cCoul}(r_1)$ and $V_{cex}(r_1)$ are the Coulomb and exchange parts of the core potential, respectively(see ref. [8]), and

$$V_{\mathrm{int}}^{(1)}(x_1)=\frac{2}{x_1}-\frac{2}{\rho_1}+V_{cCoul}(x_1) \tag{5}$$

where $V_{cCoul}(x_1)=-V_{cCoul}(r_1)$

and $V_{\mathrm{int}}^{(i)}(\sigma_i)$, $i=2,3,4,.....n$, are the interaction potentials of the (n-1)-hydrogen formation channels, respectively and are given by:

$$\begin{aligned} V_{\mathrm{int}}^{(i)}(\sigma_i) &= \frac{2}{x_i}-\frac{2}{r_i}+V_{cCoul}(x_i) \\ &+V_{cCoul}(r_i)+V_{cex}(r_i), \quad i=2,3,4,.....n \end{aligned} \tag{6}$$

The total energies E of the n-channels are defined by:

$$E=E_i+\frac{1}{2\mu_i}k_i^2, \quad \mathrm{i}=1,2,3.....n \tag{7}$$

where $\frac{1}{2\mu_1}k_1^2$ is the kinetic energy of the incident proton relative to the target and $\frac{1}{2\mu_i}k_i^2$, $i=2,3,4,.....,n$ are the kinetic energy of the center of mass of the hydrogen formation atoms, $H(n\ell)$, respectively, with respect to the nucleus of the target. E_1 is the binding energy of the target atom, and E_i, $\mathrm{i}=2,3,4,.....,n$ refer to the binding energies of the hydrogen formation atoms, respectively.

In the multi-channels coupled-static approximation (MCSA), it is assumed that the projections of the vector $(\mathrm{H}-E)|\Psi\rangle$ onto the bound state of the n-channels are zero. Thus, the following conditions:

$$\langle\Phi_i|(\mathrm{H}-E)|\Psi\rangle=0, \quad i=1,2,3,....,n \tag{8}$$

are satisfied. The total wavefunction $|\Psi\rangle$ is expressed by

$$\Psi=\sum_{i=1}^{\mathrm{n}}|\Phi_i\psi_i\rangle, \tag{9}$$

$$\psi_1=\sum_{\ell}\ell(\ell+1)f_\ell^{(1)}(x_1)Y_\ell^0(\hat{x}_1), \tag{10}$$

$$\psi_i=\sum_{\ell}\ell(\ell+1)g_\ell^{(i)}(\sigma_i)Y_\ell^0(\hat{\sigma}_i), \quad i=2,3,.....n \tag{11}$$

where $f_\ell^{(1)}(x_1)$ and $g_\ell^{(i)}(\sigma_i)$, $i=2,3,.....n$ are the radial wavefunctions of the elastic and the hydrogen formation atoms, respectively, corresponding to the total angular momentum ℓ. $Y_\ell^0(x_1)$ and $Y_\ell^0(\hat{\sigma}_i)$ $i=2,3,.....,n$ are the related spherical harmonics. $\hat{x}_1$ and $\hat{\sigma}_i$, $i=1,2,3,.....,n$ are the solid angles between the vectors $\hat{x}_1$, $\hat{\sigma}_i, i=2,3,.....n$ and the z-axis, respectively. ψ_i, $i=1,2,3,.....n$ are the

corresponding scattering wavefunction of the n-channels, respectively. Φ_1 is the wavefunction for the valence electron of the target atom which is calculated using ref. [10]. Φ_i, $i=2,3,4,.....,n$ are the wavefunctions of the hydrogen formation atoms, $H(n\ell)$, respectively, which are defined using hydrogen like wavefunction.

The multi-channels coupled static approximation (MCSA) (eq. (8)) can be solved by considering the *n*- integro-differential equations

$$[\frac{d^2}{dx_1^2}-\frac{\ell(\ell+1)}{x_1^2}+k_1^2]f_\ell^{(1)}(x_1) = 2\mu_1 U_{st}^{(1)}(x_1)f_\ell^{(1)}(x_1)+\sum_{\alpha=2}^{n} Q_{1\alpha}(x_1), \tag{12}$$

$$[\frac{d^2}{d\sigma_i^2}-\frac{\ell(\ell+1)}{\sigma_i^2}+k_i^2]g_\ell^{(i)}(\sigma_i) = 2\mu_i U_{st}^{(i)}(\sigma_i)g_\ell^{(i)}(\sigma_i)+\sum_{\alpha=1}^{n}{}' Q_{i\alpha}(\sigma_i), \quad i=2,3,.....,n, \tag{13}$$

where the prime on the sum sign means that $i\neq\alpha$, and

$$Q_{1\alpha}(x_1)=\int_0^\infty K_{1\alpha}(x_1,\sigma_\alpha)g_\ell^{(\alpha)}(\sigma_\alpha)d\sigma_\alpha, \quad \alpha=2,3,....,n \tag{14}$$

$$Q_{i1}(\sigma_i)=\int_0^\infty K_{i1}(\sigma_i,x_1)f_\ell^{(1)}(x_1)dx_1, \quad i=2,3,....,n \tag{15}$$

$$Q_{i\alpha}(\sigma_i)=\int_0^\infty K_{i\alpha}(\sigma_i,\sigma_\alpha)g_\ell^{(\alpha)}(\sigma_\alpha)d\sigma_\alpha, \quad i,\alpha=2,3,.....,n, \quad i\neq\alpha \tag{16}$$

the Kernels $K_{i\alpha}$, $i=1,2,3,.....,n,\ i\neq\alpha$ are expanded by:

$$K_{1\alpha}(x_1,\sigma_\alpha)=2\mu_1\ (8x_1\sigma_\alpha)\iint \Phi_1(r_1)\Phi_\alpha(\rho_\alpha)\cdot [-\frac{1}{2\mu_\alpha}(\nabla_{\sigma_\alpha}^2+k_\alpha^2)+V_{\text{int}}^{(\alpha)}]Y_\ell^o(\hat{x}_1)Y_\ell^o(\hat{\sigma}_\alpha)d\hat{x}_1 d\hat{\sigma}_\alpha, \quad \alpha=2,3,...n, \tag{17}$$

$$K_{i1}(\sigma_i,x_1)=2\mu_i\ (8\sigma_i x_1)\iint \Phi_i(\rho_i)\Phi_1(r_1) [-\frac{1}{2\mu_1}(\nabla_{x_1}^2+k_1^2)+V_{\text{int}}^{(1)}]Y_\ell^o(\hat{\sigma}_i)Y_\ell^o(\hat{x}_1)d\hat{\sigma}_i d\hat{x}_1, \quad i=2,3,.....n, \tag{18}$$

$$K_{i\alpha}(\sigma_i,\sigma_\alpha)=2\mu_i\ (8\sigma_i\alpha_\alpha)\iint \Phi_i(\rho_i)\Phi_\alpha(\rho_\alpha) [-\frac{1}{2\mu_\alpha}(\nabla_{\sigma_\alpha}^2+k_\alpha^2)+V_{\text{int}}^{(\alpha)}]Y_\ell^o(\hat{\sigma}_i)Y_\ell^o(\hat{\sigma}_\alpha)d\hat{\sigma}_i d\hat{\sigma}_\alpha, \quad i,\alpha=2,3,.....n, \ i\neq\alpha. \tag{19}$$

The number eight appearing in the preceding equations refers to the Jacobian of the transformations $\int dr_i\to 8\int dx_i$ and $\int d\rho_i\to 8\int d\sigma_i$.

The static potentials $U_{st}^{(1)}(x_1)$ and $U_{st}^{(i)}(\sigma_i)$, $i=2,3,.....,n$ are defined by

$$\left.\begin{aligned} U_{st}^{(1)}(x_1)&=<\Phi_1(r_1)\left|V_{\text{int}}^{(1)}\right|\Phi_1(r_1)>\\ U_{st}^{(i)}(\sigma_i)&=<\Phi_i(\rho_i)\left|V_{\text{int}}^{(i)}\right|\Phi_i(\rho_i)>\end{aligned}\right] \tag{20}$$

The multi-channels coupled integro-differential equations (12, 13) are inhomogeneous equations in x_i, and σ_i, $i=1,2,3,.....,n$ and are possessing the general form

$$(\varepsilon-\mathrm{H}_0)|\chi>=|\eta\rangle \tag{21}$$

where ε is $k_i^2(i=1,2,.....,n)$.

H_0 is $-\frac{d^2}{dx_1^2}+\frac{\ell(\ell+1)}{x_1^2}$ or $-\frac{d^2}{d\sigma_i^2}+\frac{\ell(\ell+1)}{\sigma_i^2}$, $i=2,3,...,n$.

$|\chi\rangle$ is $\left|f_\ell^{(1)}(x_1)\right\rangle$ or $\left|g_\ell^{(i)}(\sigma_i)\right\rangle$, $i=2,3,...,n$.

$|\eta\rangle$ is the right-hand side of the coupled integro-differential equations, respectively.

The solution of the multi-channel coupled integro-differential eqs. (12, 13) are given (formally) by Lippmann-Schwinger equation in the form

$$|\chi>=\left|\chi_0\right\rangle+G_0|\eta> \tag{22}$$

where G_0 is Green operator $(\varepsilon-\mathrm{H}_0)^{-1}$ and $\left|\chi_0\right\rangle$ is the solution of the homogeneous equation

$$(\varepsilon-\mathrm{H}_0)\left|\chi_0\right\rangle=|0\rangle\ , \tag{23}$$

Using the partial wave expansions of the Green operator G_0, the solutions of (12, 13) are given formally by

$$f_\ell^{(1,j)}(x_1)=\{\delta_{j1}+\frac{1}{k_1}\int_0^\infty \tilde{g}_\ell(k_1x_1)[2\mu_1U_{st}^{(1)}(x_1)f_\ell^{(1,j)}(x_1) +\sum_{\alpha=2}^{n} Q_{1\alpha}^{(j)}(x_1)]dx_1\}\tilde{f}_\ell(k_1x_1) +\{-\frac{1}{k_1}\int_0^\infty \tilde{f}_\ell(k_1x_1) [2\mu_1U_{st}^{(1)}(x_1)f_\ell^{(1,j)}(x_1)+\sum_{\alpha=2}^{n} Q_{1\alpha}^{(j)}(x_1)]dx_1\}\tilde{g}_\ell(k_1x_1), \quad j=1,2,3,.....,n \tag{24}$$

$$g_\ell^{(i,j)}(\sigma_i)=\{\delta_{ji}+\frac{1}{k_i}\int_0^\infty \tilde{g}_\ell(k_i\sigma_i)[2\mu_iU_{st}^{(i)}(\sigma_i)g_\ell^{(i,j)}(\sigma_i) +\sum_{\alpha=1}^{n}{}' Q_{i\alpha}(\sigma_i)]d\sigma_i\}\tilde{f}_\ell(k_i\sigma_i)+\{-\frac{1}{k_i}\int_0^\infty \tilde{f}_\ell(k_i\sigma_i) [2\mu_iU_{st}^{(i)}(\sigma_i)g_\ell^{(i,j)}(\sigma_i)+\sum_{\alpha=1}^{n}{}' Q_{i\alpha}^{(j)}(\sigma_i)]d\sigma_i\}\tilde{g}_\ell(k_i\sigma_i), \quad i=2,3,.....,n\ \ j=1,2,3,.....,n \tag{25}$$

where the delta functions δ_{ji}, $i,j=1,2,3,....,n$, specify two independent forms of solutions for each of $f_\ell^{(1,j)}(x_1)$ and $g_\ell^{(i,j)}(\sigma_i)$, i =2, 3,…., n according to the channel considered. The functions $\tilde{f}_\ell(\eta)\ and\ \tilde{g}_\ell(\eta)$, $\eta=k_1x_1, or\ \eta=k_i\sigma_i\ \ i=2,2,3,.....,n$ are related to the Bessel functions of the first and second kinds, i.e. $j_\ell(\eta)$ and $y_\ell(\eta)$, respectively, by the relations $\tilde{f}_\ell(\eta)=\eta\, j_\ell(\eta)$ and $\tilde{g}_\ell(\eta)=-\eta\, y_\ell(\eta)$.

It is obvious from Eqs. (24, 25) that the solutions can be only found iteratively and the ν th iteration are calculated by

$$f_\ell^{(1,j,\nu)}(x_1)=\{\delta_{j1}+\frac{1}{k_1}\int_0^{X_1}\tilde{g}_\ell(k_1x_1)[2\mu_1U_{st}^{(1)}(x_1)f_\ell^{(1,j,\nu-1)}(x_1)+\sum_{\alpha=2}^{n}Q_{1\alpha}^{(j,\nu-1)}(x_1)]dx_1\}\tilde{f}_\ell(k_1x_1)+\{-\frac{1}{k_1}\int_0^{X_1}\tilde{f}_\ell(k_1x_1)[2\mu_1U_{st}^{(1)}(x_1)f_\ell^{(1,j,\nu-1)}(x_1)+\sum_{\alpha=2}^{n}Q_{1\alpha}^{(j,\nu-1)}(x_1)]dx_1\}\tilde{g}_\ell(k_1x_1),\qquad j=1,2,3,....,n;\ \ \nu\geq1. \tag{26}$$

$$g_\ell^{(i,j,\nu)}(\sigma_i)=\{\delta_{ji}+\frac{1}{k_i}\int_0^{\Sigma_i}\tilde{g}_\ell(k_i\sigma_i)[2\mu_iU_{st}^{(i)}(\sigma_i)g_\ell^{(i,j,\nu)}(\sigma_i)+\sum_{\alpha=1}^{n}Q_{i\alpha}^{(j,\nu)}(\sigma_i)]d\sigma_i\}\tilde{f}_\ell(k_i\sigma_i)+\{-\frac{1}{k_i}\int_0^{\Sigma_i}\tilde{f}_\ell(k_i\sigma_i)[2\mu_iU_{st}^{(i)}(\sigma_i)+\sum_{\alpha=1}^{n}Q_{i\alpha}^{(j,\nu)}(\sigma_i)]d\sigma_i\}\tilde{g}_\ell(k_i\sigma_i),\qquad i=2,3,....,n,\ \ j=1,2,3,....,n;\ \ \nu\geq0. \tag{27}$$

where X_1,Σ_i, $i=2,.....n$ specify the integration range away from the nucleus over which the integrals at equations (26, 27) are calculated using Simpson's expansions.

To find the starting value of $f_\ell^{(1,j,0)}(x_1)$, we consider the Taylor expansion of $U_{st}^{(1)}(x_1),\tilde{f}_\ell(k_1x_1)$ and $\tilde{g}_\ell(k_1x_1)$ around the origin (see ref. [8])

Equations (26, 27) can be abbreviated to

$$f_\ell^{(1,j,\nu)}(x_1)=a_1^{(j,\nu)}\tilde{f}_\ell(k_1x_1)+b_1^{(j,\nu)}\tilde{g}_\ell(k_1x_1),\qquad j=1,2,3,.....n;\ \ \nu>0 \tag{28}$$

$$g_\ell^{(i,j,\nu)}(\sigma_i)=a_i^{(j,\nu)}\tilde{f}_\ell(k_i\sigma_i)+b_i^{(j,\nu)}\tilde{g}_\ell(k_i\sigma_i),\qquad i=2,....,n,j=1,2,3,.....n;\ \ \nu>0 \tag{29}$$

The preceding coefficients of eqs (28, 29) are elements of the matrices a^υ and b^υ which are given by:

$$\left.\begin{aligned}(a^\upsilon)_{ij}&=\sqrt{2\mu m_i/k_i}\,a_i^{(j,\upsilon)}\\(b^\upsilon)_{ij}&=\sqrt{2\mu m_i/k_i}\,b_i^{(j,\upsilon)},\\ i,j&=1,2,....,n,\quad \nu>0\end{aligned}\right] \tag{30}$$

which are connected with the reactance matrix, R^υ, through the relation:

$$\{\mathrm{R}^\upsilon\}_{\beta\gamma}=\left\{b^\upsilon(a^\upsilon)^{-1}\right\}_{\beta\gamma},\qquad \beta,\gamma=1,2,3,.....,n\quad \nu>0 \tag{31}$$

The partial elastic and hydrogen formation, $H(n\ell)$, cross sections are determined (in πa_0^2):

$$\sigma_{\beta\gamma}^{(\ell,\nu)}=\frac{4(2\ell+1)}{k_1^2}\left|\mathrm{T}_{\beta\gamma}^{\nu}\right|^2,\qquad \beta,\gamma=1,2,3,.....,n\quad \nu>0 \tag{32}$$

where k_1 is the momentum of the incident protons, ν is the number of iterations and $\mathrm{T}_{\beta\gamma}^{\nu}$ is the elements of the $n\times n$ transition matrix T^ν which is given by:

$$\mathrm{T}^\nu=\mathrm{R}^\nu\left(\mathrm{I}-\tilde{i}\ \mathrm{R}^\nu\right)^{-1},\qquad \nu>0, \tag{33}$$

where R^ν is the reactance matrix and I is a *n x n* unit matrix and $\tilde{i}=\sqrt{-1}$.

Finally, the total cross sections (in πa_0^2 units) are expressed (in ν^{th} iteration) by:

$$\sigma_{ij}^{\nu}=\sum_{\ell=0}^{\infty}\sigma_{ij}^{(\ell,\nu)},\qquad i,j=1,2,3,.....,n\quad \nu>0 \tag{34}$$

3. Proton-Sodium Scattering

We are going to apply our multi-channels coupled static approximation (MCSA) in the case of *n*=4 (four channels coupled static approximation) to the inelastic scattering of protons by sodium atoms. Our problem can be sketched by

$$p+Na(3s)=\begin{bmatrix}p+Na(3s) & \text{Elastic channel (first channel)}\\ H(1s)+Na^+ & H(1s)\text{ formation channel (second channel)}\\ H(2s)+Na^+ & H(2s)\text{ formation channel (third channel)}\\ H(2p)+Na^+ & H(2p)\text{ formation channel (fourth channel)}\end{bmatrix} \tag{35}$$

and $\Phi_1(r_1)$ is the valence electron wavefunction of the target (sodium) atom which is calculated using (Clementi's tables [10]), and $\Phi_i(\rho_i)$, $i=2,3,4$ are the wavefunctions of the hydrogen formation which are given by:

$$\left.\begin{aligned}\Phi_2&=\frac{1}{\sqrt{\pi}}\exp(-\rho_2),\\ \Phi_3&=\frac{1}{\sqrt{32\pi}}(2-\rho_3)\exp(-\rho_3/2),\\ \Phi_4&=\frac{1}{\sqrt{32\pi}}\rho_4\cos\theta_{\rho_4,\sigma_4}\exp(-\rho_4/2)\end{aligned}\right] \tag{36}$$

4. Results and Discussion

We start our calculations of p-Na scattering by testing the variation of the static potentials $U_{st}^{(1)}(x_1)$ and $U_{st}^{(i)}(\sigma_i)$, $i=2,3,4$ of the considered channels with the increase of x_1, σ_i $(i=2,3,4)$. Values of x_1, σ_i $(i=2,3,4)$ have been chosen such that $x=\sigma=\frac{1}{16},\frac{2}{16},\frac{3}{16},.....,\frac{512}{16}$ where $h=\frac{1}{16}$ is the mesh size (or Simpson's step) employed for calculating integrals appearing in integral equations using Simpson's rule. Calculation of cross sections of proton-sodium ($p-Na$) scattering has been proceeded by investigating variation of elements of R^{ν} with increase of integration range (IR) and the number of iterations. We fix h at $\frac{1}{16}$ and have obtained all the results presented below with 512 mesh points (i.e. IR = 32 a_0). It is found that excellent convergence can be obtained with υ = 50, this demonstrates stability of our iterative method. Final calculations have been carried out for seven partial waves corresponding to $0 \le \ell \le 6$ at values of k_1^2 representing the kinetic energy region $(50 \le k_1^2 \le 1000$ keV).

Table 1. *Present total cross sections (in πa_0^2) of p-Na scattering with the compared results ([1], [2], [4] and [5]).*

k^2 keV.	Present H(1s) (MCSA)	Choudhury H(1s) [1] (IP)	Daniele H(1s) [2] (Eikonal)	Ferrante H(1s) [4] (OBK)	Present H(2s) (MCSA)	Choudhury H(2s) [1] (IP)	Tiwari H(2s) [5] (CPB)	Present H(2p) (MCSA)	ChoudhuryH (2p) [1] (IP)
50	4.95668E-03	5.28E-3			1.07737E-03	1.16E-3	5.269E-2	7.15149E-04	7.70E-4
100	9.21046E-04	1.00E-3	3.9834E-3	1.1749E-2	7.61705E-05	8.27E-5	4.912E-2	6.29074E-05	6.83E-5
150	3.09314E-04	3.35E-4			2.22521E-05	2.41E-5	1.827E-2	1.56965E-05	1.70E-5
200	1.26818E-04	1.34E-4	4.2107E-4	1.6803E-4	9.43894E-06	1.03E-5	5.206E-3	4.71947E-06	5.15E-6
250	8.64352E-05				6.77508E-06			3.07336E-06	
300	5.61886E-05				4.23864E-06			1.39275E-06	
350	2.81282E-05				1.44200E-06			6.98814E-07	
400	1.41235E-05				8.60700E-07			3.99149E-07	
450	6.31288E-06				5.48427E-07			1.37928E-07	
500	3.32235E-06	3.56E-5			2.71574E-07	2.91E-7	6.022E-6	7.58726E-08	8.40E-8
550	1.70526E-06				1.68063E-07			5.98207E-08	
600	8.91973E-07				8.83654E-08			3.89418E-08	
650	6.57777E-07				5.57767E-08			3.18592E-08	
700	5.59345E-07				3.35529E-08			1.51802E-08	
750	4.51836E-07				1.93474E-08			8.71499E-09	
800	2.80993E-07				1.45379E-08		1.029E-6	6.56233E-09	
850	1.81571E-07				8.78523E-09			5.17250E-09	
900	7.89709E-08				7.49131E-09			3.80237E-09	
950	6.50483E-08				5.90930E-09			1.95321E-09	
1000	5.01633E-08				3.85956E-09		1.105E-7	9.48699E-10	

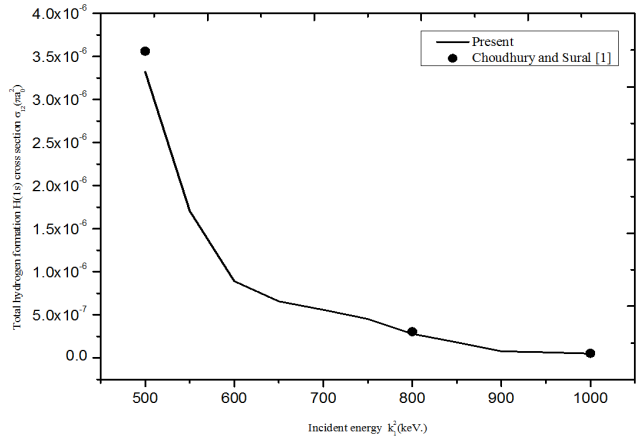

Figure 2. *Present total $H(1s)$ formation cross sections (σ_{12} in πa_0^2) of $p-Na$ scattering with those of Choudhury and Sural [1].*

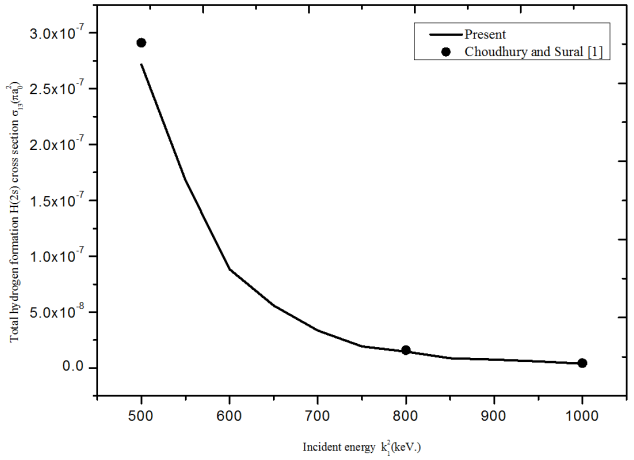

Figure 3. *Present total $H(2s)$ formation cross sections (σ_{13} in πa_0^2) of $p-Na$ scattering with compared theoretical Choudhury and Sural [1].*

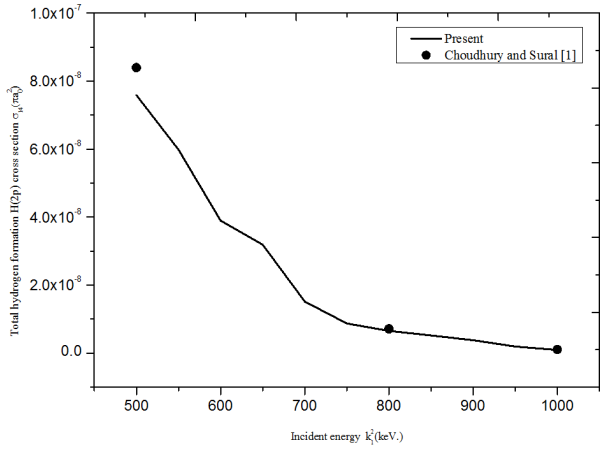

Figure 4. *Present total $H(2p)$ formation cross sections (σ_{14} in πa_0^2) of $p-Na$ scattering with those of Choudhury and Sural [1].*

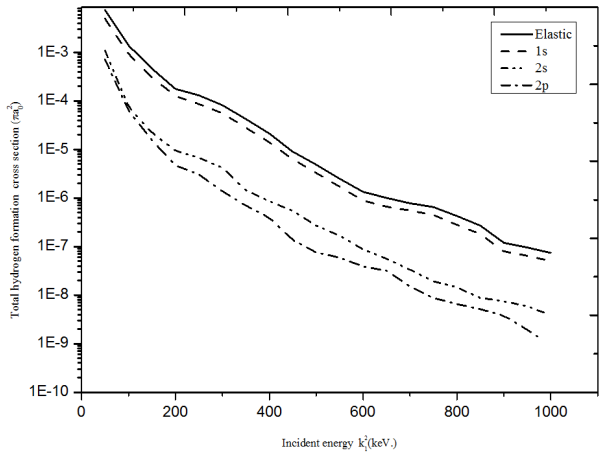

Figure 5. *Present total elastic and hydrogen formation ($H(1s)$, $H(2s)$ and $H(2p)$) cross sections (in πa_0^2) of $p-Na$ scattering.*

Table 1 shows the present total cross-sections of p-Na scattering with those of Choudhury and Sural of the wave formation of impulse approximation [1], Daniele et al. that obtained using eikonal-approximation [2], Ferrante et al. using Oppenheimer Brinkman Kramers (OBK) approximation [4], and Tiwari using the Coulomb-projected Born approximation [5] in the energy range (50-1000 keV). Our results and those of comparing results at the range of energy (500-1000 keV.) are also displayed in Figures (2-4). In Figure 5, we also show the present results of the total cross-sections of the four channels (elastic and the hydrogen formation (*H(*1s*), H(*2s*), H(*2p*)*) in the range of energy (50-1000 keV.). It is seen that the present values of the total cross-sections of the four channels have similar trends of the comparison results. Our values of the total cross-sections of the four channels decrease with the increase of the incident energies. The calculated total cross-sections σ_{12} of *H(*1s*)* are about (5.4%-7.9%) lower than the Choudhury and Sural results [1]. The total cross-sections σ_{13} of *H(*2s*)* are slightly about (6.7%-8.8%) lower than those of Choudhury and Sural [1]. Our results of the total cross-sections σ_{14} of *H(*2p*)* are about (7.9%-9.7%) lower than the available values of Choudhury and Sural [1]. We also noticed that the available compared results of Daniele et al. [2], Ferrante et al. [4], and Tiwari [5] are about (24.5%-99.9%) larger than our values. The present calculated total cross sections have the same trend of the compared results and give good agreement with the available previous results of Choudhury and Sural [1]. Our calculations also show that we have more hydrogen formation if we open more excited channels of hydrogen formation in the collision of protons with sodium atoms.

5. Conclusions

Proton-sodium inelastic scattering is studied for the first time using the multi-channels coupled static approximation, (MCSA), as a four channels problem (elastic, *H(*1s*), H(*2s*)* and *H(*2p*)*). Our interest is focused on the formation of ground, $H(1s)$, and excited hydrogen, $H(2s)$, and $H(2p)$ in $p-Na$ inelastic scattering. The difference between the four channels problem and the three or two channels problems (which are used by Elkilany ([6]-[9] is in improving the total cross sections of the considered channel by adding the effect of more kernels of the other three channels (in two channels problem, we have only one kernel and in three channels, we have two kernels).

References

[1] K. B. Choudhury and D. P. Sural, J. Phys. B25, 853(1992). http://moscow.sci-hub.bz/859dfec273be5a916394257287a6bb9b/10.1088%400953-4075%4025%404%40013.pdf

[2] R. Daniele, G. Ferrante and E. Fiordilino, IL Nuovo Cimento B54 185(1979). http://moscow.sci-hub.bz/2e26007f976503ff8796173e160124b9/10.1007%40bf02908234.pdf

[3] G. Ferrante and E. Fiordilino, IL Nuovo Cimento B57, 1(1980). http://link.springer.com/article/10.1007/BF02722397

[4] G. Ferrante, E. Fiordilino and M. Zarcone, IL Nuovo Cimento B52, 151(1979). http://moscow.sci-hub.bz/9b3307d16bed6bf30c7976e04f6506bd/10.1007%40bf02739030.pdf

[5] Y. N. Tiwari, Pramana-J. Phys. 70, 753(2008). http://moscow.sci-hub.bz/6b499b516c13d6dd2dcc6526d77168ef/10.1007%40s12043-008-0036-x.pdf

[6] S. A. Elkilany, Brz. J. Phys., 44(6), 629(2014). http://moscow.sci-hub.bz/d44be0b030251c8778983655d99a8c67/10.1007%40s13538-014-0268-9.pdf

[7] S. A. Elkilany, Chin. Phys. B24(3), 033402 (2015). http://iopscience.iop.org/article/10.1088/1674-1056/24/3/033402/pdf

[8] S. A. Elkilany, Can. J. Phys., 93, 1283 (2015). http://moscow.sci-hub.bz/d1749e635ef4c4de989d60d5a9a92fe7/10.1139%40cjp-2015-0116.pdf

[9] S. A. Elkilany, Pramana-J. Phys. 5, 78 (2016). http://cyber.sci-hub.bz/MTAuMTAwNy9zMTIwNDMtMDE2LTEyODIteQ==/10.1007%40s12043-016-1282-y.pdf

[10] E. Clementi, C. Roetti, Atomic Data Nuclear Data Tables, 14, 177 (1974). http://moscow.sci-hub.bz/28ccc17a68d26df8557aa44d6840c125/10.1016%40s0092-640x%2874%2980016-1.pdf