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Study of radiation in spherical media using moment method

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Abstract: A moment technique is presented to improve the performance of the discrete ordinates method when solving the radiation problems in spherical media. In this approach the angular derivative term of the discretized 1-D radiative transfer equation is derived from an expansion of the radiative intensity on the basis of angular moments. The set of resulting differential equations, obtained by the application of the S_N method associated to moment method, is numerically solved using the boundary value problem with the finite difference algorithm. Results are presented for the different independent parameters. Numerical results obtained using the moment approximation compare well with the benchmark approximate solutions. Moreover, the new technique can easily be applied to higher-order S_N calculations.

Keywords: RTE, Spherical Medium, Angular Derivative Term, DOM, Moment Method

1. Introduction

In practical engineering problems, radiative transfer in participating media appears in many applications such as combusting systems, furnaces and reactor nuclear theory. Many of these systems can be considered as spherical enclosure. Therefore, it is desirable to have an accurate and reliable model for solving the radiative transfer equation for this geometry which contain, absorbing, emitting and scattering medium. A number of studies interested in resolving the radiative transfer equation in such media have been conducted. These works included various numerical techniques: integral transformation techniques [1,2], spherical harmonics method [3], Galerkin method [4-5] and others [6-7].

The discrete ordinates method has also been used in solving the radiative transfer equation in spherical media[8].This method enjoys great popularity owing to its accuracy and to its compatibility with other techniques; Sghaier et al [9] and Trabelsi et al [10] developed a discrete ordinates method associated with the finite Legendre transform. Recently, Aouled-Dlala et al [11] used the finite chebyshev transform to treat the angular derivative term of the discretized one-dimensional radiative transfer equation. Kim et al[12]used the combined finite volume and discrete ordinates method to investigate radiative heat transfer between two concentric spheres and Li et al [13] developed a Chebyshev collocation spectral method for coupled radiation and conduction in a concentric spherical participating medium. Very recently, Mishra et al [14] used a lattice Boltzmann and modified discrete ordinates method to study radiative transport in a spherical medium with and without conduction.

In order to apply the discrete ordinates method, it is necessary to approximate the angular derivative term appearing in the radiative transfer equation in spherical coordinates. This term is generally approximated by a classical finite differencing scheme [15].

In this work, we introduce a new approach to evaluate the angular derivative term using an angular moment technique. This yields a quasi-analytical expression of the discrete angular derivative term. The obtained higher moments of the radiative intensity are expressed in term of the incident radiation, the net radiative heat flux and the radiation pressure using the generalized eddington approximation [16]. We therefore adopt the discrete ordinates method to study the radiation transfer in such media. In this paper, the considered medium is a hollow sphere. The boundaries are maintained at different but uniform temperatures and are considered to be opaque, gray diffusely emitting and diffusely reflecting. The obtained results are compared with those given by the standard discrete ordinates method and with those available in the literature. The mathematical formulation is given for gray media but its extension to any absorption coefficient based non gray model is straightforward.

2. Analysis

The radiative transfer equation through an absorbing, emitting and isotropically scattering spherical shell medium is

$$\begin{aligned}&\frac{\mu}{r^2}\frac{\partial}{\partial r}\left[r^2 I(r,\mu)\right]+\frac{1}{r}\frac{\partial}{\partial\mu}\left[\left(1-\mu^2\right)I(r,\mu)\right]+\beta I(r,\mu)\\&=\chi I_b\left[T(r)\right]+\frac{\sigma}{2}\int_{-1}^{+1}I(r,\mu')d\mu',\quad \mathrm{R}_1<r<R_2 \text{ and } -1\le\mu\le+1.\end{aligned} \tag{1}$$

The above equation is subject to the following boundary conditions

$$I\left(R_1,\mu\right)=\varepsilon_1 I_{b,1}+2\left(1-\varepsilon_1\right)\int_0^1 I\left(R_1,-\mu'\right)\left|\mu'\right|d\mu',\qquad \mu>0, \tag{2a}$$

$$I\left(R_2,\mu\right)=\varepsilon_2 I_{b,1}+2\left(1-\varepsilon_2\right)\int_0^1 I\left(R_2,\mu'\right)\left|\mu'\right|d\mu',\qquad \mu<0, \tag{2b}$$

In Eqs.(1), (2a) and (2b), r is the space radial variable, μ is the cosine of the angle between the direction s of the radiation intensity $I(r,\mu)$ and the positive r-axis. χ, σ and β are the absorption, scattering and extinction coefficients, respectively, which are related by

$$\beta=\chi+\sigma.$$

The blackbody radiation is related to the temperature $T(r)$ in the medium through

$$I_b=\frac{n^2\bar{\sigma}T^4(r)}{\pi},$$

where n denotes the refractive index and $\bar{\sigma}$ the Stefan Boltzmann constant. In the boundary conditions given by Eqs (2a) and (2b), ε is the isotropic emissivity of the opaque boundaries. The subscripts 1 and 2 refer to the boundaries at $r=R_1$ and $r=R_2$ respectively. The geometry and coordinates for the hollow sphere are shown in Fig. 1.

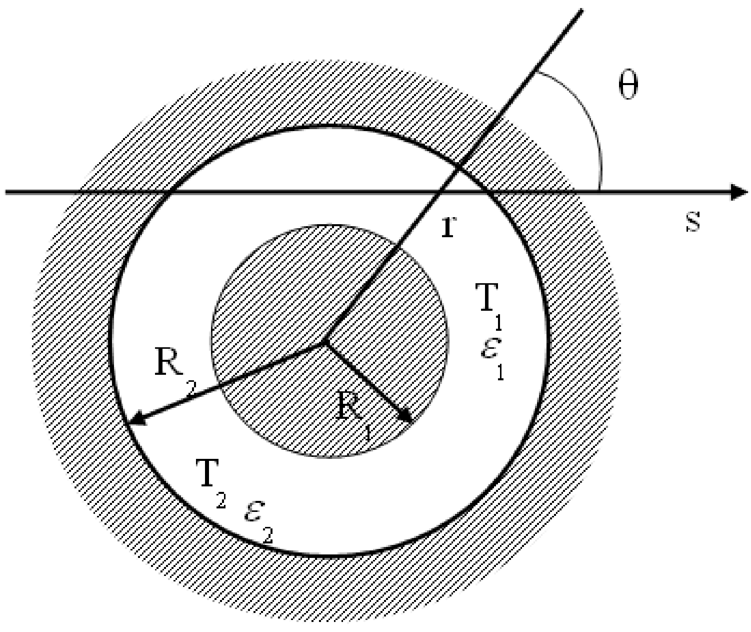

Fig 1. *Hollow sphere geometry and notations.*

2.1. Differencing Scheme

The discrete form of the radiative transfer equation is obtained by evaluating Eq. (1) at each of the discrete directions and replacing the integral by numerical quadrature to give

$$\frac{\mu_m}{r^2}\frac{\partial}{\partial r}\left[r^2 I_m\right]+\frac{1}{r}\frac{\partial}{\partial \mu}\left[\left(1-\mu^2\right)I\right]_{\mu=\mu_m}+\beta I_m=\chi I_b\left(T(r)\right)+\frac{\sigma}{2}\sum_{m'=1}^{M} w_{m'} I_{m'} \tag{3}$$

The discrete ordinate representation of the boundary conditions, Eqs. (2a) and (2b) is given by

$$I_m(R_1)=\varepsilon_1 I_{b,1}+2(1-\varepsilon_1)\sum_{m'=1\,,\ \mu_{m'}<0}^{M} w_{m'} I_{m'}\left|\mu_{m'}\right| \qquad \mu_m>0 \tag{4a}$$

$$I_m(R_2)=\varepsilon_2 I_{b,2}+2(1-\varepsilon_2)\sum_{m'=1\,,\ \mu_{m'}>0}^{M} w_{m'} I_{m'}\mu_{m'} \qquad \mu_m<0 \tag{4b}$$

where subscripts m and m' refer to discrete directions, M is the total number of these directions. If a standard difference scheme is used, the discrete form of the term involving the angular derivative term is written

$$\frac{\partial}{\partial \mu}\left[\left(1-\mu^2\right)I\right]_{\mu=\mu_m}\approx\frac{\alpha_{m+1/2}I_{m+1/2}-\alpha_{m-1/2}I_{m-1/2}}{w_m}, \tag{5}$$

where $I_{m+1/2}$ and $I_{m-1/2}$ are the angular intensities in the directions m+1/2 and m-1/2. The constants $\alpha_{m+1/2}$ and $\alpha_{m-1/2}$ only depend on the differencing scheme and therefore they may be determined by examining the case of an isotropic intensity field as described in Ref. [15]. As far as the curved geometries are concerned, the differencing scheme introduces two angular variables; $I_{m+1/2}$ and $I_{m-1/2}$. They must be determined at each space position r. For this purpose, the standard diamond difference approximation is used to relate the $I_{m+1/2}$ and $I_{m-1/2}$, namely

$$I_m=\frac{1}{2}\left[I_{m+1/2}+I_{m-1/2}\right].$$

In the calculation of the average angular intensity I_m, we need the starter intensity $I_{1/2}$. For spherical geometry, it is obtained from the solution of the transfer equation in slab geometry with starter direction cosine $\mu=-1$. We develop in what follows an alternative technique based on angular moment equations. .The results from both approaches will be compared.

2.2. Moment Method

We develop a new approach to approximate the angular derivative term:

$$D(r,\mu)=\frac{\partial}{\partial \mu}\left[(1-\mu^2)I(r,\mu)\right] \tag{6}$$

We start by defining the k-th order moment of $D(r,\mu)$

$$D^k(r)=\int_{-1}^{+1}D(r,\mu)\mu^k d\mu \quad k\geq 0, \tag{7}$$

The application of the angular moment technique to the angular derivative term, denoted $D(r,\mu)$ yields

$$\int_{-1}^{+1}D(r,\mu)\mu^k d\mu=k\int_{-1}^{+1}I\left(r,\mu\right)\mu^{k+1}d\mu-k\int_{-1}^{+1}I\left(r,\mu\right)\mu^{k-1}d\mu. \tag{8}$$

The moments of the radiative intensity represent a generalized incident radiation, which for the case n=0, n=1 and n=2 reduces the usual definition of the incident radiation, radiative flux and the radiation pressure. The obtained higher moments of the radiative intensity are then expressed in term of the incident radiation, the net radiative flux and the radiation pressure using the generalized eddington approximation (16).

In order to obtain the angular derivative term, the integrals over direction are replaced by the discrete form

$$\int_{-1}^{+1}f(r,\mu)d\mu\cong\sum_{m=1}^{M}w_m f(r,\mu_m), \tag{9}$$

where the w_m are the quadrature weights associated with the directions μ_m. Thus Eq. (8) is approximated by

$$\sum_{m=1}^{M}w_m D_m\mu_m^k=k\sum_{m=1}^{M}w_m I_m\mu_m^{k+1}-k\sum_{m=1}^{M}w_m I_m\mu_m^{k-1}. \tag{10}$$

Now, the angular derivative terms D_m are obtained from Eq. (12) written for k=1, M-1. Thus, this system is closed by using the obvious relation

$$\sum_{m=1}^{M} w_m D_m \cong 0 . \tag{13}$$

Thus the unknowns D_m are the solution of a linear algebraic system: **A d = b**, with the matrix A given by

$$\mathbf{A}=\begin{bmatrix} 1 & 1 & . & . & 1 \\ \mu_1^1 & \mu_2^1 & . & . & \mu_M^1 \\ . & . & . & . & . \\ . & . & . & . & . \\ \mu_1^{M-1} & \mu_2^{M-1} & . & . & \mu_M^{M-1} \end{bmatrix} \tag{14}$$

and the vector d and b given by

$$\text{d}=\begin{bmatrix} w_1 D_1 \\ w_2 D_2 \\ . \\ . \\ w_M D_M \end{bmatrix}, \qquad \text{b}=\begin{bmatrix} 0 \\ . \\ . \\ . \\ (M-1)\sum_{m=1}^{M} w_m I_m \mu_m^M - (M-1)\sum_{m=1}^{M} w_m I_m \mu_m^{M-2} \end{bmatrix} \tag{15}$$

The angular derivative terms D_m are then given by

$$D_m = \frac{1}{w_m}\sum_{j=1}^{M} (A^{-1})_{mk} B_k , \tag{16}$$

where

$$B_k = k\sum_{m'=1}^{M} w_{m'} I_{m'} \mu_{m'}^{k+1} - k\sum_{m'=1}^{M} w_{m'} I_{m'} \mu_{m'}^{k-1} , k \geq 1 , \tag{17}$$

and $\left(A^{-1}\right)_{mk}$ is the set of components $\left(A^{-1}\right)$, A^{-1} the inverse of the matrix which is given by $A = \left(\mu_m^k\right)_{\substack{0\le k\le M-1 \\ 1\le m\le M}}$. It is called the transpose Vandermonde matrix.

The new discrete ordinates representation of Eq. (1), for a finite number of discrete ordinates may be written

$$\frac{\mu_m}{r}\frac{\partial}{\partial r}(rI_m) + \frac{1}{r}\frac{1}{w_m}\sum_{j=1}^{M}(A^{-1})_{mk} B_k + \beta I_m = \chi I_b\left(T\left(r\right)\right) + \frac{\sigma}{2}\sum_{m'=1}^{M} w_{m'} I_{m'}, \quad \text{m} = 1,\text{M} \tag{18}$$

and the discrete ordinates representation of the boundary conditions is given by Eqs.(4a, 4b)

Once the directional intensities I_m are known, the radiative heat flux $q_r\left(r\right)$ and the incident radiation energy $G\left(r\right)$ in the medium are determined from their definitions as

$$q_r(r) = 2\pi\int_{-1}^{+1} I\left(r,\mu\right)\mu d\mu = 2\pi\sum_{m=1}^{M} \mu_m w_m I_m , \tag{19a}$$

$$G\left(r\right) = 2\pi\int_{-1}^{+1} I\left(r,\mu\right) d\mu = 2\pi\sum_{m=1}^{M} w_m I_m . \tag{19b}$$

Equations (18) and (4a, 4b) provide the complete mathematical formulation for the radiation problem in a dimensional spherical medium. A numerical technique, namely the boundary value problem with finite difference (BVPFD) [17] is used to solve this problem. The new technique called MOM-DOM with eight directions (N=8) is adopted. The weights and quadrature points are those of corresponding Gaussian quadratures.

3. Results and Discussion

In table1, we give the numerical values of $r^2\ q^*(r)$ for a radiation problem between two concentric spheres with diffusely emitting and reflecting boundaries subject to pure radiative transfer. Results obtained from the new technique denoted by MOM-DOM have been compared with that of kim et al [12], Jia et al [7], Sghaier et al [9], standard discrete ordinates method, denoted by DOM [11] and those of Mishara et al [14].For the standard discrete ordinates method, the coefficient $\alpha_{m+1/2}$ and $\alpha_{m-1/2}$ are calculated following the procedure as described by Modest [16]. We present results for $r_1^* = 0.5, \tau_2 = 1$ and for various combinations of the boundary emissivities ε_1 and ε_2 and for different values of outer sphere temperature θ_2. MOM results show an excellent comparison.

In table 2, we present the numerical values of moment of order 1($\int_{-1}^{+1} I(r,\mu)ud\mu$) for a purely radiation problem with black boundaries, inner sphere temperature θ_1=1 and outer sphere temperature θ_2=0, and for different optical thickness of inner and outer sphere $\tau_1 = \beta R_1$ and $\tau_2 = \beta R_2$ respectively. Results are compared with those of Viskanta et al, Jia et al and Abulwafa et al. The MOM results compare well with those available in literature [4, 5, 6].

An increase in optical thickness $\tau_2 = \beta R_2$, the medium becomes more denser. The net radiative heat flux decreases in the medium as shown in Fig.2.

For the optical thickness $\tau_2 = 2, \theta_2 = 0.5$ and for black boundaries, the effects of the ratio R_1/R_2 on the radiative heat flux is shown in Fig.3. With an increase in the ratio R_1/R_2, the medium tends to planar one and the radiative heat flux becomes constant.

Fig.4 shows the effect of the outer sphere temperature θ_2 on the radiative heat flux distribution in the medium. When the outer sphere becomes hot, the net radiatif heat flux changes sign and becomes negative.

In Fig.5, we study the effect of emissivity of the inner and outer sphere on the radiative heat flux distribution in the medium. For this study, $\tau_2 = 1, \theta_2 = 0.5$ and R_1/R_2=0.5. It 'is observed when the hot inner sphere is more reflecting, the net radiative heat flux in the medium becomes less.

4. Conclusion

An analysis of a radiation problem in one dimensional absorbing, emitting and isotropically scattering hollow spherical medium is investigated. The angular derivative term appearing in this geometry is approximated by making use of a new approach called MOM-DOM approximation. This leads to an accurate expression for the angular derivative term. The set of differential equations is solved using the boundary value problem with finite difference algorithm. The accuracy of the new technique has been verified by comparison with benchmark approximate solutions.

Table 1. *Values of $r^2q(r)$ for various combinations of ε_1 and ε_2 ε_2 with $r_1^* = 0.5$, Θ_2 =0.5 and $\tau_2 = 1$*

ε_1	ε_2	Θ_2	$r^{*2} q_r^*$ **DOM[11]**	$r^{*2} q_r^*$ **FLT[9]**	$r^{*2} q_r^*$ **MOM-DOM**	$r^{*2} q_r^*$ **Galerkin[4] Method**	$r^{*2} q_r^*$ **FVM[12]**	$r^{*2} q_r^*$ **MDOM[14]**
1	1	0.5	0.21733	0.21733	0.21030	0.21038	0.20827	0.20820
0.5	1	0.5	0.11312	0.11312	0.11160	0.1108	0.11027	0.11022
1	0.5	0.5	0.17281	0.17281	0.17127	0.1718	0.17038	0.17000
0.5	0.5	0.5	0.09977	0.09977	0.09966	0.0991	0.09866	0.09849
1	1	2	- 3.47938	- 3.47938	-3.36480	- 3.36557	-3.33625	-3.37597
0.5	1	2	-1.79980	-1.79980	-1.7623	-1.77357	-1.76643	-1.78700
1	0.5	2	-2.8160	-2.8160	-2.77656	- 2.74880	-2.72926	-2.75610
0.5	0.5	2	-1.61580	-1.61580	-1.59463	-1.58604	-1.58034	-1.59690

Table 2. *The net radiative heat flux with transparent boundaries, isotropic incidence at the inner surface and ω=1.0.*

τ_1	τ_2	Ref.[4]	Ref.[5]	Ref.[6]	Present work
0.5	1	0.11221	0.11220	0.11220	0.11215
0.9	1	0.38305	0.38310	0.38308	0.39110
0.95	1	0.43658	0.43660	0.43660	0.44680
1	10	__[a]	__[a]	0.00343	0.00339
5	10	__[a]	__[a]	0.04793	0.04784
9	10	__[a]	__[a]	0.24609	0.24685

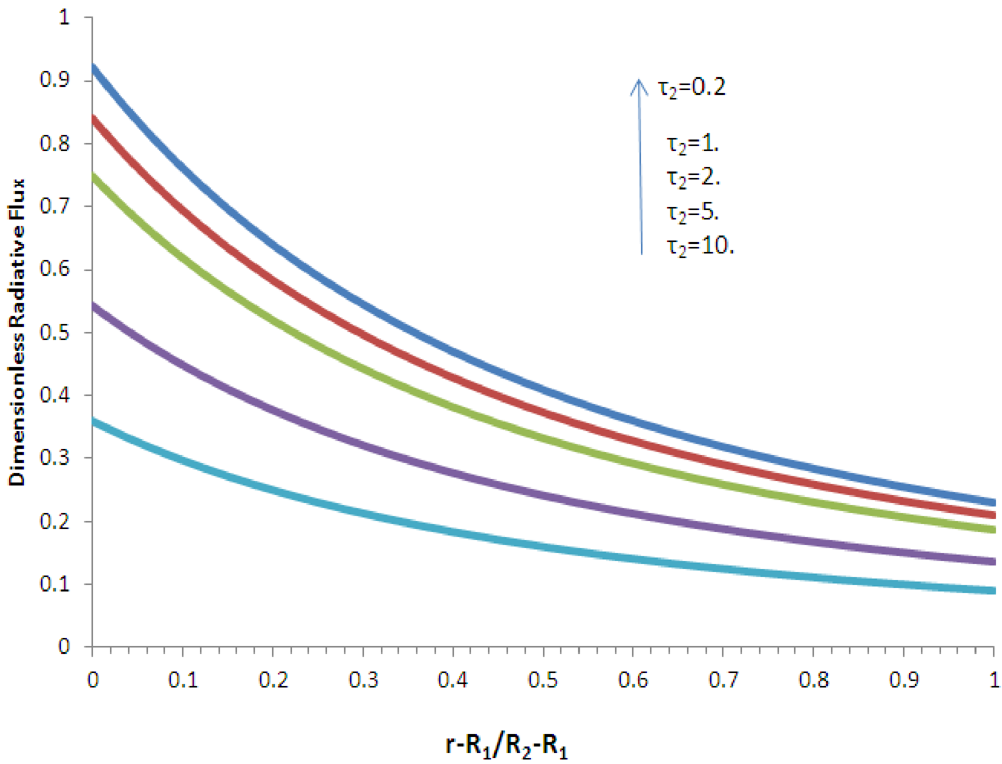

Fig 2. *Effect of the optical thickness τ_2 on the dimensionless radiative flux with $\varepsilon_1=\varepsilon_2=1$, $\theta_2=0.5$ and $R_1/R_2=0.5$.*

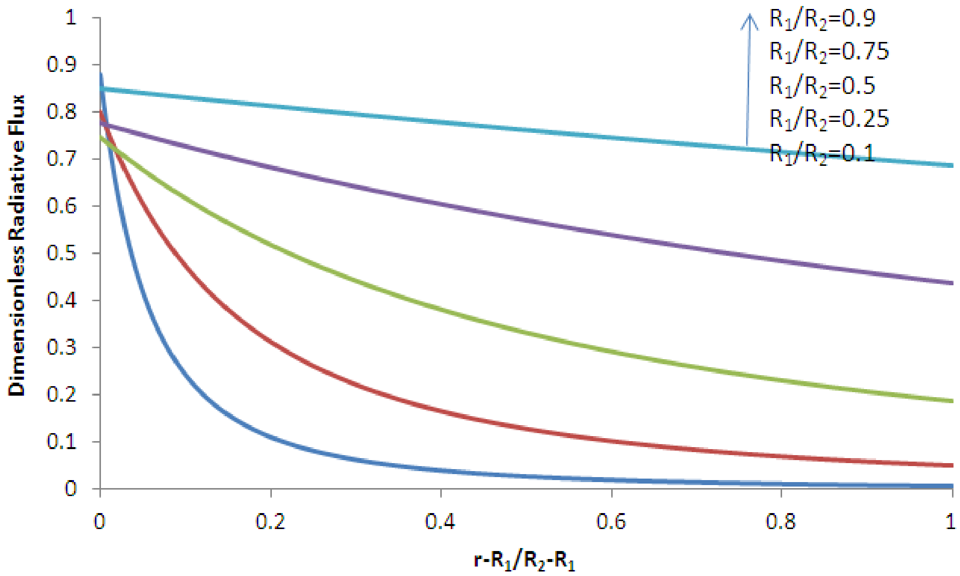

Fig 3. *Effect of the ratio R1/R2 on dimensionless radiative flux with ε1=ε2=1, θ2=0.5 and τ2=2.0.*

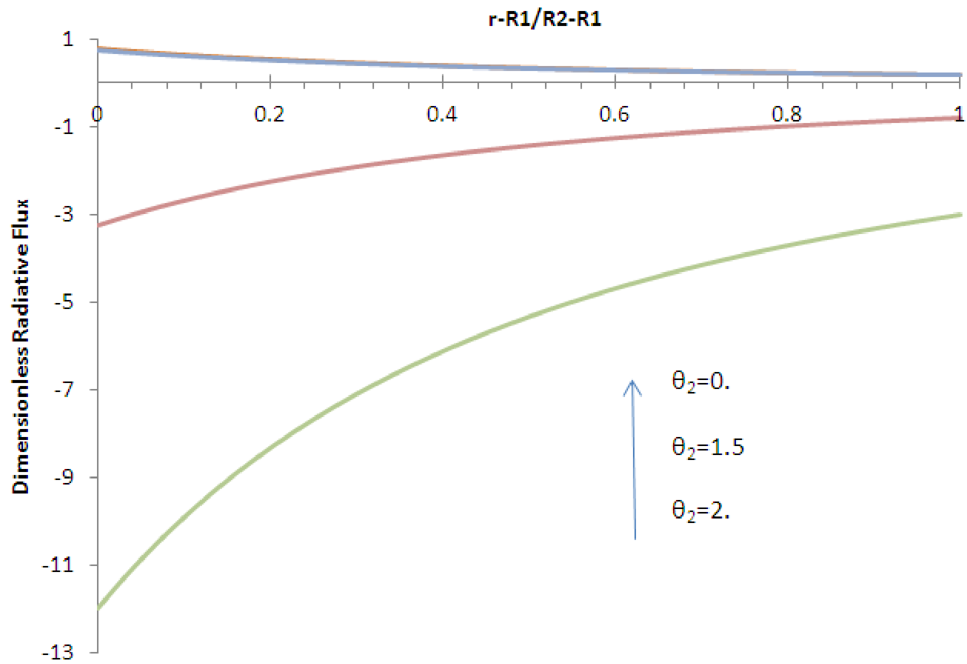

Fig 4. *Effect of the boundary temperature θ 2 on the dimensionless radiative flux with ε1=ε2=1, τ2=2 and R1/R2=0.5.*

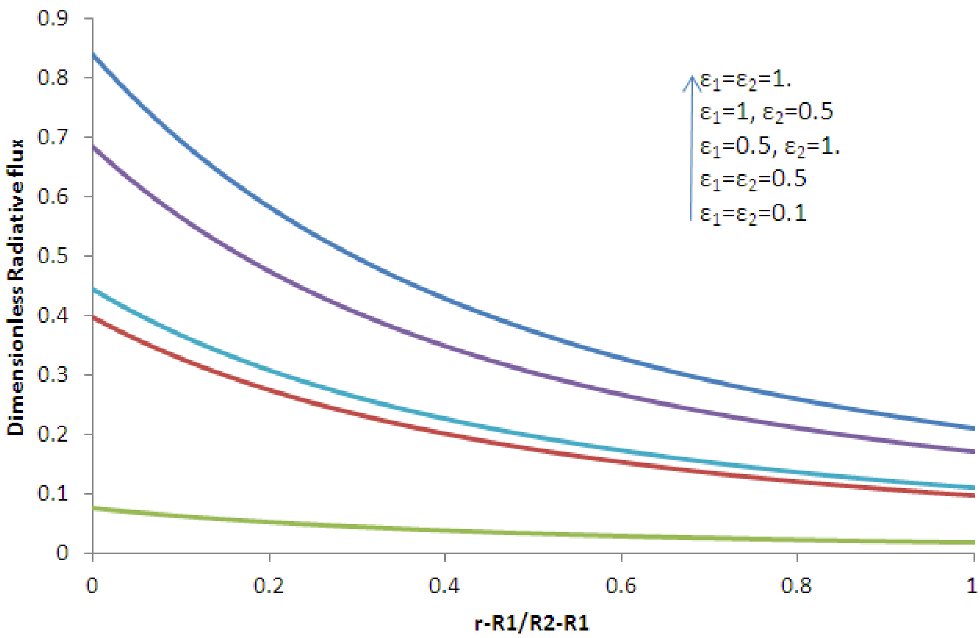

Fig 5. *Effect of the surfaces emissivity on the dimensionless radiative flux with θ2=0.5, τ2=1 and R1/R2=0.5.*

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