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Quantum Phases and Entanglement in an Optically Active Solution of Amino Acids

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Abstract: In optical active medium (OPM) the physics behind the rotation of plane of polarization of incident plane polarized light has been studied from the view point of transfer of energy and angular momentum and quantum entanglement. The absorbed energy of the polarized light in the optical active medium induces the mechanical rotation of the chiral molecule. Quantum mechanically the molecule acquires the quantum phase due to passage of the polarized light. As the chiral molecule has fixed helicity, the phase is helicity dependent or spin angular momentum (SAM) phase. The rotation of plane of polarization is due to equivalence between Optical and mechanical torque in the optically active medium. Polarized light has its OAM dependence on intensity of light. The loss of intensity or reduction of OAM is proportional to the concentration of the optical active medium. This indicates a transfer of angular momentum occur between light and chiral molecule. Moreover, in this work we first focused on the quantum correlation of polarized photon and chiral molecules which is realized by the form of a singlet state through quantum entanglement. This theoretical knowledge has been reflected experimentally to find the comparative study of absorbed intensity and geometric phase of six essential and five non-essential amino acids.

Keywords: Optical Activity, Chirality, Geometric Phase, Entanglement

1. Introduction

Optical activity is the phenomenon by which the light matter interaction [1] is manifested through the rotation of the plane of linear polarization as light passes through chiral media. It pervades several fields of sciences and mostly an important role in the pharmaceutical industry [2]. In optical active medium (OPM) the rotation of plane of polarization due to transfer of energy could be fruitfully studied in terms of angular momenta of photons [3]. Polarized light with fixed intensity has particular orbital angular momentum (OAM). Spin to orbital conversion of angular momentum is very important where variation of SAM occurring from medium's birefringence gives rise to the appearance of OAM, arising from medium's inhomogeneity. In rotationally symmetric geometries, such as "q-plates" (liquid crystal) only spin to orbital conversion [4] (STOC) process happens instead of no net development of angular momentum of light. In case of optical active solution, the rotation of polarized state on Poincare sphere indicate the change of angular momentum from one point to another. The actual nature of angular momentum transfer in optically active medium will be sorted out here.

With the increase of concentration of the OPM solution, intensity of transmitted light decreases with increase of absorbed energy. Considering six essential and five non-essential amino acids we will investigate experimentally the energy absorbed at different concentrations through angle of optical rotation. The optical rotation of polarized photon is due to the striking with the chiral molecule in the optical active solution. This results the mechanical torque acquired by the molecule. The quantum manifestation of this mechanical torque implies that the chiral molecule will acquire a quantum phase identified as Berry phase [5]. This Berry phase had been pointed out as the natural Aharanov-Bohm phase in OPM acting as micro-solenoids [6].

Quantum entanglement [7, 8] and quantum phases are the basic tools for quantum information processing. Quantum communication involving geometric/Berry phase plays a key role identifying Geometric quantum computation [9]. In

quantum optical rotatory dispersion, the entangled photons are used to study optical activity as a function of wavelength [10]. They used multi-wavelength polarization entangled photon pairs to measure the optical activity and optical rotatory dispersion acquired by solutions of chiral molecules. There could be a single particle entanglement associated with macroscopic behavior. In view of recent experimental works by Kumar et al. [11], quantum entanglement involves in the biomolecule inside one living or between other neighboring livings samples. Entangled optical photons and entanglement between the polarizations of qubits (photon or fermion) are now widely used for solving fundamental problems in Quantum Physics [12].

Hence, we will study here both from theoretical and experimental point of view, the variation of absorbed intensity and geometric phase (Berry phase) of some OPM samples (Amino acids) for different concentrations. Moreover, the optical rotation of the plane of polarization due to interaction between polarized photon and chiral molecule (amino acids) has been realized through entanglement where Berry phase plays the key role. This present work would develop new approach in the area of optical activity both from classical and quantum aspect. In the next few sections we will discuss about the energy absorbed, angular momentum, then geometric phase and entanglement in the OPM.

2. Optical Active Medium (OPM) and Energy Absorbed

The optical activity in a medium is caused by the chiral behavior of individual molecules which constraint to move electrons along helical path under the influence of the incident electric field. E. Condon [13] pointed out that, linearly polarized light (which is a superposition of two circularly polarized light) if passed through such medium, a difference in velocity of propagation of wave takes place, which further causes the difference in their refractive indices due to which the plane of polarization is rotated by an angle θ as

$$\theta = \frac{\pi(n_L - n_R)l}{\lambda} = S\, l\, c_n \tag{1}$$

where S is the specific rotation and c_n is the concentration of the optically active medium and n_L and n_R are the refractive indices for two respective chirality's (L &R).

This rotation of plane of polarization of incident light is due to transfer of energy in terms of angular momentum [14]. As polarized light is rotated in a chiral medium, the angular momentum is found to be related to the concentration of the corresponding optically active medium. Classically, observed rotation in a chiral medium [15] depends on the number of molecules which are interacted by the polarized light, hence increase in concentration results many number of rotations. The optical rotation in the optical active medium of particular concentration could be represented in terms of difference of angular momentum for two chirality's

$$\theta = \frac{f c_n V^2}{r}\left(\frac{1}{\mathcal{L}_L} - \frac{1}{\mathcal{L}_R}\right) \tag{2}$$

where we assumed that the solution of fixed concentration C_n is kept in a glass tube of volume $V(\pi r^2 l)$ of length l, radius r and $\mathcal{L}$'s for the angular momentum for left and right chiral molecules respectively in presence of polarized light of the frequency f.

An anisotropic medium exhibiting circular anisotropy is analogous to a rotation operator. The optical active medium could be represented by Jones matrix R_c whose action with plane polarized light [16] is responsible for breaking into LCP and RCP to propagate with respective phases φ_L and φ_R

$$R_c = \begin{bmatrix} e^{i\varphi_L} & 0 \\ 0 & e^{i\varphi_R} \end{bmatrix} \tag{3}$$

By introducing the differential phase shift $\varphi = \varphi_L - \varphi_R = \Delta n_c k_o e$, and $\theta = \varphi/2$, the matrix R_c changes to R in course of change of basis vector of linear states [X, Y] to circular states [L, R]. The basis between linear states [X, Y] and the basis of the circular states [L, R] indicate the change of emerging state of polarization V' from V as V'=RV, where $\psi = (\varphi_L + \varphi_R)/2$

$$R = e^{i\psi}\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \tag{4}$$

It is thus seen that ignoring the phase factor $e^{i\psi}$, the optical active medium behaves as rotation matrix (eq. 4) of angle θ (eq. 2). It could be mentioned recalling the previous study [17] that this rotator only rotates the plane of polarization of linearly polarized light by securing a geometric phase in turn where no phase appear for circularly polarized light instead.

With the variation of concentration of the optically active solution, this optical rotation of plane of polarization changes indicating that the rotation matrix

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{5}$$

change the incident linearly polarized light.

The Jones matrix of analyzer becomes

$$A_\alpha = \begin{bmatrix} cos^2\alpha & sin\alpha cos\alpha \\ sin\alpha cos\alpha & sin^2\alpha \end{bmatrix} \tag{6}$$

If the polarized light state, $X = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ from polarizer is made to pass consecutively through optical active medium (R) and analyzer A_α, the corresponding emerging ray X' from the device will be J'=A_α R X. The emergent ray becomes using eqn. (5) & (6) as

$$X' = \cos(\theta-\alpha)\begin{bmatrix} cos\alpha \\ sin\alpha \end{bmatrix} \tag{7}$$

If I_0 be the initial intensity of the light source, the optical rotation of the plane of polarization results the change of the intensity. According to Malus law [18], the intensity of the emergent light from analyzer will be

$$I = I_0 \cos^2(\theta-\alpha) \tag{8}$$

depending on the final optical rotation by an angle ϑ.

This absorbed energy is balanced by the rotatory power by $(\theta-\alpha)= \vartheta =\rho d$ where ρ being the rotatory power of the medium.

For monochromatic light (fixed wavelength), the intensity is constant over a particular concentration. With respect to several concentrations of the optical medium this angle of rotation, ϑ, of the plane of polarization could be measured by the digital polarimeter (here CDP001). The absorbed intensity for a particular concentration of the optical medium will be

$$I'=I_0 – I=I_0\text{-}I_0 \cos 2\,\vartheta =I_0\,(1\text{-}\cos\vartheta)(1+\cos\vartheta) \qquad (9)$$

Theoretically extinction of light takes place for $\cos(\theta-\alpha)=0$ which implies $\theta=\alpha\mp\pi/2$. In fact, the rotation of analyzer (or of polarizer) control the transmitted/absorbed intensity in a periodic (π) way. For a polychromatic light source, the rotatory power ρ, depends on the wavelength as proportional to λ^{-2}. The wavelengths λ_k will not be absorbed or extinguished following the relations

$$\theta_k=\alpha +\pi/2+k\pi \text{ or } \rho\,(\lambda_k)=(\alpha +\pi/2+k\pi)\,/d \qquad (10)$$

where k is natural numbers. In this paper we are using only monochromatic light without rotating the polarizers etc in digital polarimeter CDP001 to study the light absorption with respect to optical rotation at different concentrations.

3. Angular Momentum States in OPM

Photon is a two-state system. To describe polarization state in the orthogonal basis formed by the linear polarization states in the horizontal and vertical component of linearly polarized light can be written in terms of LCP and RCP

$$|H> = \tfrac{1}{\sqrt{2}}(|L> + |R>) \text{ and } |V> = \tfrac{1}{\sqrt{2}}(|L> - |R>) \quad (11)$$

Here the circularly polarized states $|L>$ and $|R>$ represent left and right rotation of electric vector identifying the two states of Spin angular momentum (SAM) of polarized light as

$$\left|L>= \tfrac{1}{\sqrt{2}}\binom{1}{i}, |R>= \tfrac{1}{\sqrt{2}}\binom{1}{-i}\right. \qquad (12)$$

A horizontal polarized light |H> which is a combination of circular states [L, R] is rendred by rotation of polarization due to action of optically active medium having circular anisotropy (eq. 5)

$$|H'>=R|H\geq (e^{-i\theta}\left|L> +e^{i\theta}\right|R \geq (\,|+> + |\text{-}>) \qquad (13)$$

This transfer of energy from light to chiral molecule is a quantum process. It could be realized through the interaction of linearly polarized light with the chiral molecules. The optical active medium has fixed chiral molecules (L or R) with fixed direction of helicity. This helicity of massless particle makes the space-time structure anistropic [&]. With the interaction of polarized light, the direction of helicity is shifted by an angle equivalent to optical rotation. The torque

responsible for optical rotation of plane of polarization of incident linearly polarized light is balanced by the mechanical rotation of the molecule in optical active medium. In fact, this implies that the molecules will acquire a quantum mechanical phase by the impulse of incident photon which is equivalent to natural Berry phase as long as the polarized light passes through the solution.

In quantum information field, the interest of OAM degree of freedom of light mainly arises from the possibility of using its higher-dimensionality for encoding a large amount of information in single photons [19]. The chiral medium absorbs light intensity differently for LCP and RCP. The linearly polarized incident light becomes elliptical due to unequal absorption of amplitude. Reduction of intensity and wave front indicate loss of OAM that is transferred to SAM of molecules too. In quantum regime, the optical rotation of the plane of polarization will develop a phase that varies with the physical or dynamical parameters of the medium. Rotation of plane polarization results the change of helicity. With the increase of concentration more energy is absorbed. As this intensities and wave-front are representing the Orbital angular momentum (OAM) of light, the decrease of intensities due to absorption implies the reduction of OAM of Laser light. It is OAM to SAM conversion process (OTSC), where the chiral molecules gain SAM through helicity dependent Berry Phase. In the quantum aspect absorbed intensity has its origin in SAM dependent geometric phase. This idea has been verified through our study with essential amino acids.

4. Geometric Phase and Entanglement

Quantum theory describes a physical system in terms of state vectors in a linear space. If two systems are combined then they have correlations between observations and the state is called entangled.

Entangled photons are used to study optical activity for multi (two) wavelengths light source to study the mean and difference of optical rotation of an optical active medium. Rotation of single photon in optical active medium has expressed as the form $U(\alpha(C, \lambda))=\exp[-i\Lambda\alpha\,(C, \lambda)]$ where Λ is the helicity of the light and $\alpha(C, \lambda)$ is the angle of rotation of single photon for C being concentration of optical medium. With this consideration the authors [10] mentioned the expectation values of entangled biphoton states by $\tfrac{1}{4}(1\mp\cos\theta)$ where θ depends on α, C and λ_1, λ_2.

We realized the role of helicity in every polarized quanta through the appearance of Berry phase from the view point of quantization procedure [20]. The chiral anomaly that arises from short distance singularity will have full influence on this topological phase of Berry. In fact, inclusion of helicity through quantization extend the sphere and apart from conventional spherical polar coordinates θ and φ, the extra parameter χ designate the helicity by an operator. The chiral molecule will acquire a helicity dependent quantum phase by $e^{2i\pi\mu}$ where $\mu=\mp1/2$ are the two respective values of helicity for fermi field in the anisotropic space-time. This helicity dependent topological phase visualizes the physics behind

the rotation about the quantization axis in the anisotropic medium where the helicity which is depicted by an angle χ shifts to $\chi+\delta\chi$ in anisotropic space-time.

In general the physical manifestation of this quantum phase is the solid angle subtended by structure of quantized spin ½ particle which is equivalent to two component spinor. For the specific case, a spinor having quantum numbers l=1/2, |m|=1/2 and |μ|=1/2, can be written in use of quantum gates and an arbitrary super- position of elementary qubits |0> and |1>

$$|\uparrow, t>=[\cos\vartheta/2|0>+\sin\vartheta/2|1>e^{-i\varphi}]e^{i/2(\varphi-\chi)} \qquad (14)$$

Over a closed path, a spinor acquire the solid angle (neglecting the overall phase) as geometric phase [21] about the quantization axis with the variation of all the three own parameters ϑ, φ and χ.

$$\gamma_{\uparrow}= \oint L_{eff}dt=i \int A (\lambda)d\lambda=i \oint <\uparrow|\nabla| \uparrow>d\lambda$$

$$=i(\oint d\chi-\cos\vartheta \oint d\Phi)$$

$$==\pi(1-\cos\vartheta)=\pi -\frac{1}{2}(e^{i\vartheta} + e^{-i\vartheta}) \qquad (15)$$

It has been pointed out earlier, [21] that the chiral molecule (left or right) acquires the respective geometric phases by $\gamma_L=\pi(1-\cos\vartheta)$ or $\gamma_R=\pi(1+\cos\vartheta)$ which are being solid angle visualize the classical manifestation of quantum phase.

This leads us to rewrite eq. 7 as quantum state $|\Psi >$ at a particular instant t that is connected with the primary state $|\Psi_0>$as

$$|\Psi >=\cos \vartheta |\Psi_0>=1/ 2\pi (\gamma_R-\gamma_L)|\Psi_0> \qquad (16)$$

indicating optical rotation originated from the difference of geometric phases for two chiralities left and right.

In the quantum mechanical correlation of two spinors, here the polarized photon and chiral molecule, influence of Berry phase on an entangled state will be linked up with that of local observations of spins [21].

The qubit rotation of chiral molecules could be identified as in presence of polarized light

$$|\uparrow (\theta) >=1/2\pi(e^{i\vartheta} + e^{-i\vartheta})| \uparrow> \qquad (17)$$

$$| \downarrow (\theta) >= -1/2\pi(e^{i\vartheta} + e^{-i\vartheta})| \downarrow>$$

The shift of spin axis of the molecules is the outcome of mechanical torque balanced by the optical torque of the traversing polarized light through the optical active medium. The geometric phase gained by the polarized light due to the optical rotation ϑ will be of Pancharatnam [22]

$$< X|X' >=\cos(\theta-\alpha)=\cos\vartheta=\frac{1}{2}(e^{i\vartheta} + e^{-i\vartheta}) \qquad (18)$$

It is realized that if there are n number of molecules arranged in an array and light emerging from one strike another molecule then the final emergent light will be

$$|H'_n >=R(n\vartheta)|H >=(R>e^{in\vartheta} + |L > e^{-in\vartheta}) \qquad (19)$$

A linearly polarized light when entered into an optically active solution with specific concentration, the molecules get energy to rotate. These molecules are quantized having fixed helicity due to their own chirality and handedness either Levo or dextro of solution. Incident polarized light act as an external force to change their helicity by a small angle. This angle is the angle of optical rotation. Every molecule interacted by the polarized light acquires the Berry's geometric phase.

As the |H>, horizentaly polarized light vector is rotated by an optical angle θ, the transformation results the correlation un-affected and giving rise another singlet state.

$$\begin{pmatrix} |+> \\ |-> \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} |R> \\ |L> \end{pmatrix}$$

If two systems are combined then they have correlations between observations and the state is called entangled. An entangled state between a pair of two-level systems is a *singlet* state when the total spin=0 and the state is of the form.

Here we intuitively realized the physics behind the concentration dependent optical rotation. In general the incident polarized photon qubit at jth state and the chiral molecule at the ith state having joint rotation by angle ϑ are tied by the entangled singlet state of the following form

$$|\Psi_{singlet} > = \frac{1}{\sqrt{2}}(|\uparrow (\vartheta) >_1|+>_2 - |\downarrow (\vartheta) >_1|\rightarrow>_2)$$

$$=\frac{1}{\sqrt{2}}(|\uparrow (\vartheta) >_1 e^{i\vartheta}|R >_2 - |\downarrow (\vartheta) >_1 e^{-i\vartheta}|L >_2)$$

$$=\frac{1}{2\sqrt{2}}(|\uparrow>_1 e^{2i\vartheta}|R >_2 + |\downarrow>_1 e^{-2i\vartheta}|L >_2) \qquad (20)$$

This implies the non-appearance of topological phase of the chiral molecule where in the anisotropic optical medium the circular basis state [R, L] acquire the external OAM by the phase proportional to the optical rotation and the concentration of the medium. Our previous study on birefringent medium ref. [23] help to point out that the concurrence, *the measure of entanglement from one chiral molecule to other becomes cosϑ or cos2ϑ or so on which can be visualized through this geometric phase of Berry. The gradual change of polarized states from [|R>,|L>] to [|+>,|->] then to [| +,ϑ >_j |−,ϑ >] depending on the optical rotation through interaction with chiral molecules indicate that the shape of wave-front or the OAM of the incident light reduces without changing the SAM because the helical structure of polarized light decays though the handedness remains same.* Hence in optical active medium orbital angular momentum only changes with change of concentration.

5. Experiment with Amino Acids

Molecules having mirror image with respect to each other are termed as enantiomers. They usually referred to L, (plane of polarization clockwise/left rotation) and D (anticlockwise/right rotation of plane of polarization). It is an important mystery that only L-amino acids are in

biosphere to involve directly in the evolution of life [24]. Mixtures of equal amount of L & D called racemic mixture. Usually in biological samples one is active. It is to be noted that L & D forms have different biochemical and physiological properties. Due to the chirality of sunlight and the chirality of nuclear radiation, L amino acids are the more stable enantiomers and are favoured for abiogenesis [25].

Objects that are not identical with mirror image are chiral. Excepts in some bacteria, the amino acids that found in several proteins are Levo. D-type amino acids are toxic and used in medicines as antibiotics. The physicochemical properties of a protein are determined by the analogous properties of the amino acids in it. Proteins are formed from many amino acids through peptide bonds. If a solution of a fibrous protein flows through a narrow tube, the elongated molecules become oriented parallel to the direction of the flow, and the solution thus becomes *birefringent / optically active*. It splits a light ray into two components that travel at different velocities and are polarized at right angles to each other.

With the knowledge of L-Amino acids essential for growth of life, we have studied experimentally the optical activities of some Levo (L) amino acids only. Eight different concentrations starting from 0.25% to 2% have prepared from every six following essential amino acids *L-Lysine, L-Leucine, L-Valine, L-Phenylalanine, L-Histidine, L-Metheonine*. Maintaining the room temperature nearly at 18C, we at first measured for every concentration the optical rotation and the specific rotation by digital polarimeter CDP001. At first, we have measured the intensity of the Infrared laser light source using a current detector and current meter.

Incident intensity in terms of current=0.11 mA,

Source voltage=5V

Incident power, (I_0)=(0.11x5) mW/sec=0.55 mW per second.

In addition, we have taken five following non-essential amino acids *L-Alanine, L-Arganine, L-Serine, L-Glutamine* to study the absorbed intensity and geometric phase with respect to the above eight concentrations**.**

The respective intensities absorbed $(I_0 - I_0 \cos^2 \vartheta)$ and geometric phases $\pi(1-\cos\vartheta)$ are also evaluated by finding out $\cos\vartheta$. These practical findings are illustrated [26] through graphs to have a comparative study between these amino acids for different concentrations. These graphs figure 1 and figure 2 are drawn for eight different concentrations from 0.25% to 2% for six essential amino acids to compare absorbed intensities and geometric phase respectively. Except L-Histidine all the other five essential amino acids (L-Lysine, L-Leucine, L-Valine, L-Phenylalanine, L-Metheonine,) show similar nature of graph for respective geometric phase and absorbed energy. L-Histidine shows maximum and L-Valine has minimum absorption of energy at 2% concentration. Figure 3 is more prominent to represent the 3D variation of three entities absorption of light intensities and geometric phase for eight concentrations of six essential amino acids. The estimated values of energy absorption and geometric phase are obtained in decreasing order from L-Histidine, L-Leucine, L-Phenylalanine, L-Methionine, L-Lysine and L-Valine gradually. The histogram variation in Figure 4 represent same result in different manner for absorbed energy.
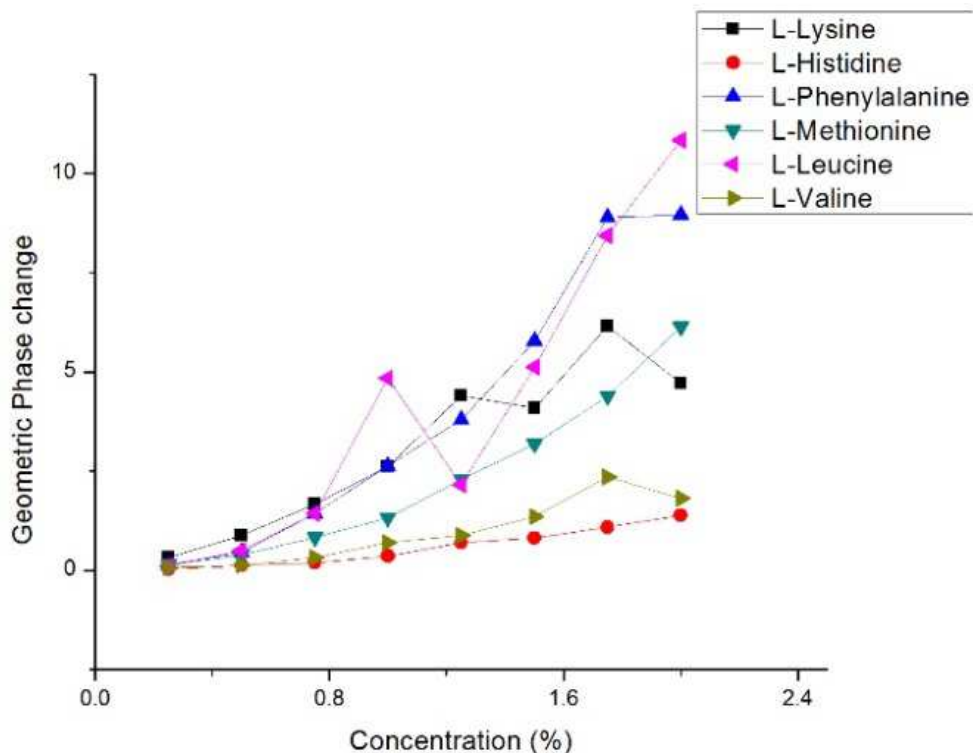

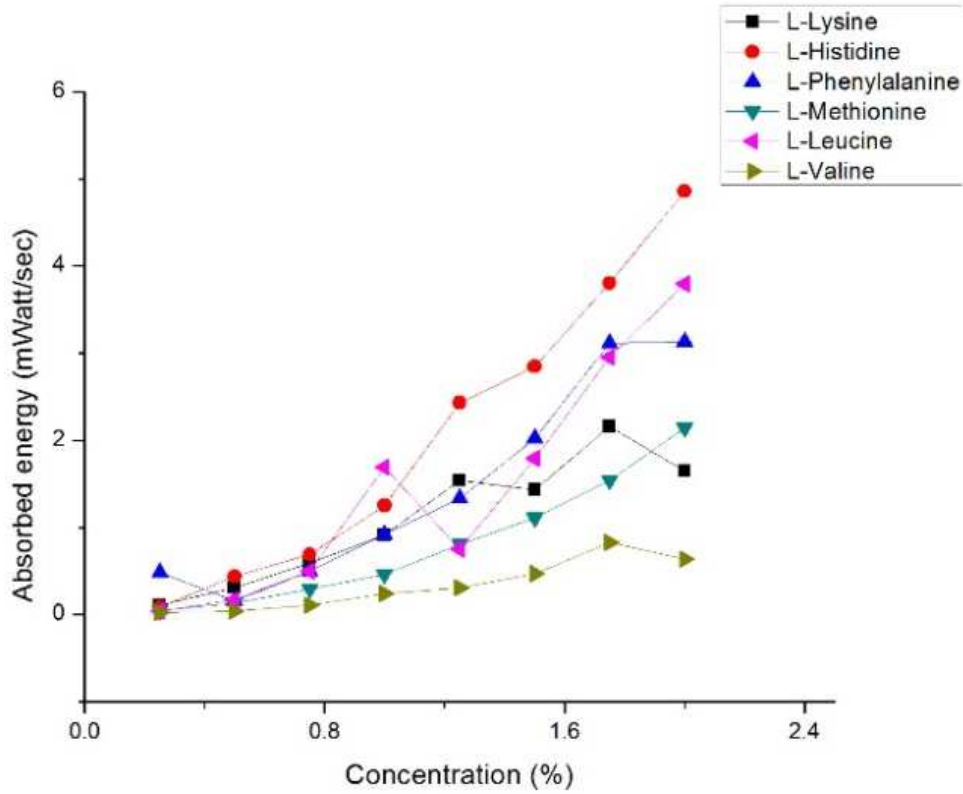*Figure 1. Variation of geometric phase with concentration of six essential amino acids.*

Figure 2. *Variation of absorbed energy with concentration of six essential amino acids.*
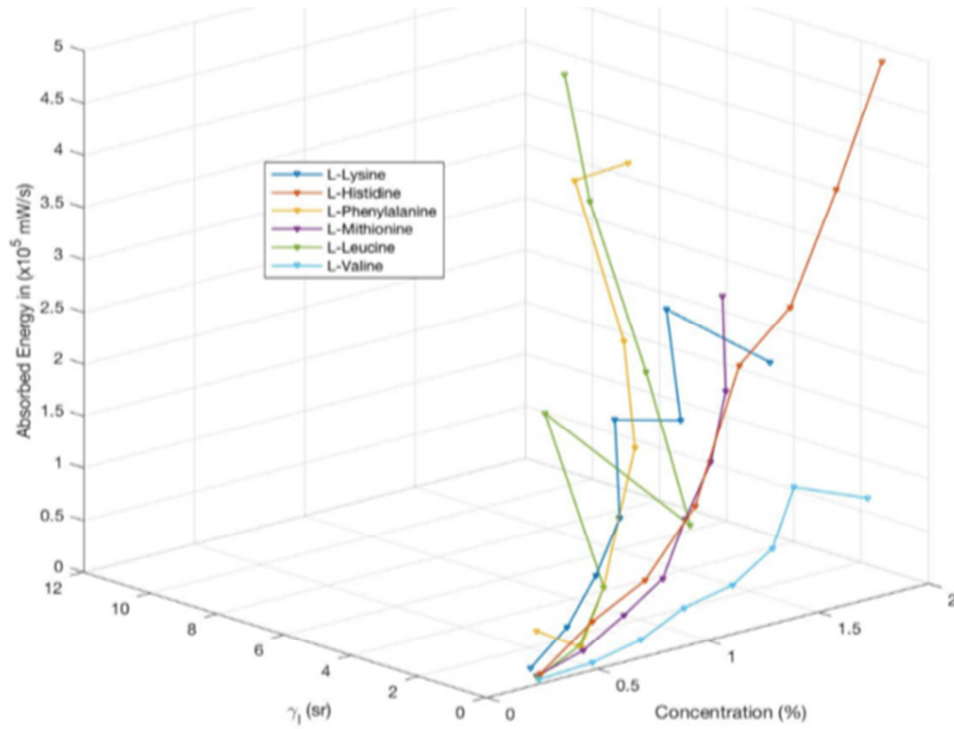

Figure 3. *Variation of absorbed energy with γ_l and concentration of six essential amino acids.*

Further with five non-essential amino acids, *L-Arganine, L-Glutamine, L-Serine, L-Alanine and L-Proline* we have repeated the experimental study for which figure 5 and figure 6 are drawn. It is seen in both the graphs that for *L-Proline* rise of absorbed energy and geometric phase is very high for which a different graph has to draw with respect to other four amino acids. We want to mention that measure of entanglement is as directly proportional to Berry phase, its graphical analysis was not carried out.
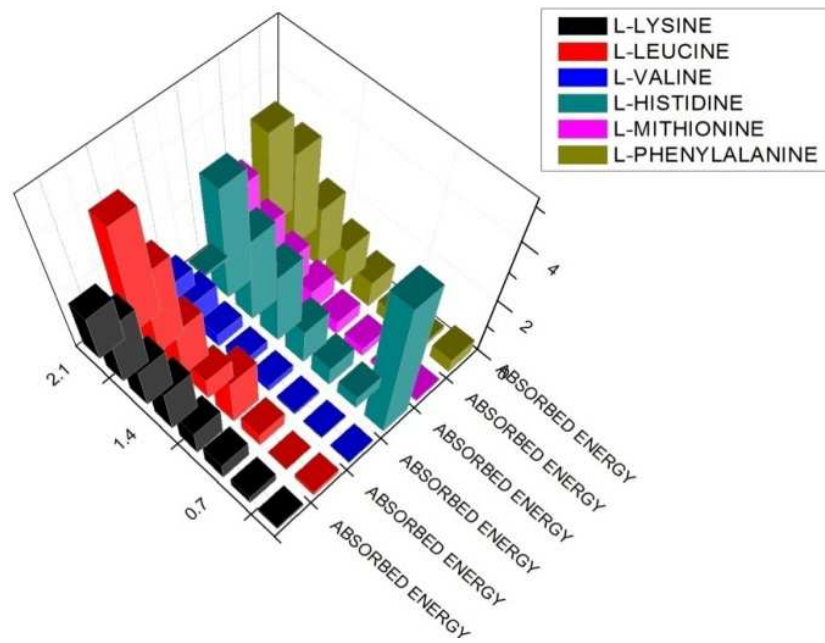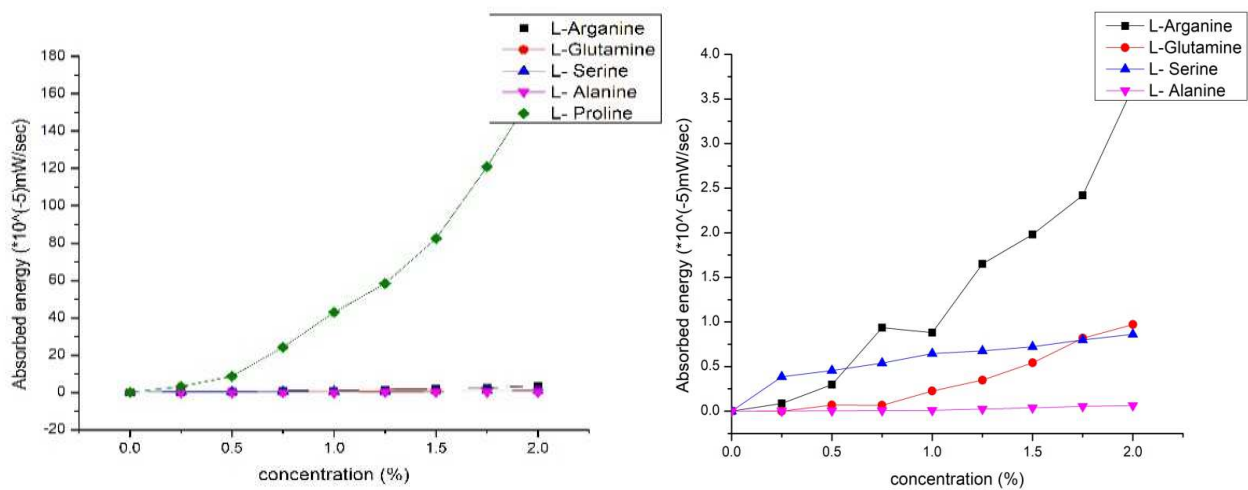
Figure 4. *Histogram of absorbed energy.*

Figure 5. *Variation of absorbed energy of five/four non- essential amino acids.*
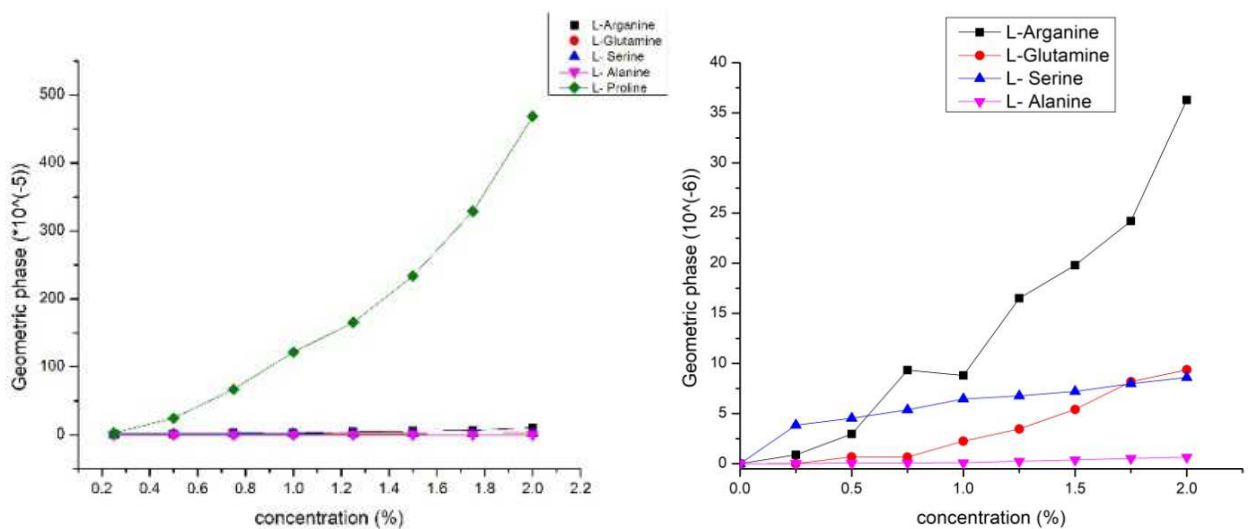

Figure 6. *Variation of geometric phase with concentrations of five/four non-essential amino acids.*

6. Conclusion

The rotation of plane of polarization of polarized light by chiral molecule is caused due to the equivalence between the optical and mechanical toque. With the increase of concentration more energy is absorbed due to the more interaction with polarized photon with chiral molecules. This implies the decrease of intensity or OAM of Laser light. It is a OTSC (OAM to SAM conversion) process, where the chiral molecules gain SAM through helicity dependent Berry Phase. The absorbed intensity has its origin in SAM dependent geometric phase whose classical counterpart is a solid angle both being dependent on angle of optical rotation. This idea has been verified experimentally by digital polarimeter with some essential and non-essential amino acids where similar graphs for entanglement are not repeated here. It could be mentioned that the spectroscopic study of optical activity in molecules have been reported before [26].

Comparative graphical analysis with some amino acids are studied to understand the variation of absorbed intensity and geometric phase with respect to eight different concentrations. This tied behavior of the two entities photon and fermion implies the entanglement forming a singlet state. In every interaction with chiral molecules in OPM, the incident states.

[|R>, |L>] changes from one singlet to other as from [|+>, |->] to $[| +, \vartheta >_j |-, \vartheta >]$. Using quantum memories for single photon entanglement with single molecule of amino acids, the quantum processing in forming further protein could be studied in future. This work could reopen new avenue involving Geometric quantum computation. With emphasis on angular momentum of optical active biological samples, quantum memory [27, 28] will be of further ample interest to work. In future this work will be in support of stability of DNA, and investigation of molecular logic devices in biological systems. There remains more scope in future to work with biological samples in connection with drugs or disorders specially with amino acids in Viruses.

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