

# Improving the BER Performance of M-FSK in a Noisy Multipath Rayleigh, and Rician Fading Channels Using Reed-Solomon Forward Error Correction Method

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**Abstract:** This paper focuses on the application of Reed-Solomon Error Correction Code evaluation technique on the Bit-Error-Rate performance of M-ary Frequency Shift Keying (M-FSK) in noisy multipath Rayleigh and Rician fading channel. The detection and correction of the received information errors from the multipath fading channel was done using the Forward Error Correction (FEC) technique. The message is encoded and decoded using the error-correcting Reed-Solomon Codes were employed to encode and decode the message. The entire model based design, modelling and simulation was done using MATLAB/SIMULINK. It was observed that the BER performance of M-FSK with RS codes improved greatly compared to M-FSK system without error correction. Decrease of BER performance was observed as the length of codeword symbols ( $n$ ) increase at constant code rate and error correcting capability. The redundancy increases as code rate decreases leading to an improved Bit-Error-Rate performance. The improvement of the BER of M-FSK by using Reed-Solomon error correction code was achieved.

**Keywords:** MFSK, Reed-Solomon Code, Bit Error Rate, Multipath, Rayleigh, Rician

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## 1. Introduction

Modern mobile and wireless communication systems adopt digital modulation scheme as an essential module for signal processing instead of previously used analog modulation. Digital modulation is very effective in dealing with noise and robust to channel impairments. The choice of which digital modulation scheme or technique to employ is largely dependent on cost and efficiency in providing high data rates and very low bit error rates (BER) in the midst of prevailing challenges such as fading [4, 5]. The major disadvantage is its high bandwidth requirement and poor BER performance when compared to other digital modulation techniques like BPSK and QPSK. Therefore, it is meaningful to adopt ways of improving the performance of a wireless mobile communication system with the deployment of M-ary (MFSK) modulation technique. The objective of this paper is to evaluate the BER performance of M-ary

Frequency Shift Keying (FSK) modulation scheme in a channel characterized by noise, Rayleigh fading (Non Line of Sight(LOS) fading), and Rician fading (LOS fading) using the Reed-Solomon Forward Error Correction method.

[1] In their paper, It was experimental that Binary input RS Encoder-Decoder provides an improved BER when compared to that of Integer input RS Encoder-Decoder, especially for the preferred smaller values of  $M$ . [2] In their paper, BER performance of M-FSK was tested under multi-channel environments AWGN, Rayleigh fading and Rician fading channels with coherent and non-coherent demodulation and different sizes of modulation constellation. Convolutional code with hard and soft decision, extended Golay code and Reed-Solomon code, were used to improve the performance of the system under AWGN channel.

[3] In their paper, observed that the BER performance improves as the length of codeword symbols ( $n$ ) increase at constant code rate  $R_c$  and error correcting capability increases.

## 2. Theoretical Background

The FSK modulation for a signal can be expressed generally as,

$$S_m(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_m t + \phi), 0 \leq t \leq T, \{m = 1, 2, \dots, M\} \quad (1)$$

$$TR_b = \frac{K}{T} \text{ bits/sec} \quad (2)$$

and the corresponding information rate can be expressed as,

$$R_m = R_c R_b \text{ bits/sec} \quad (3)$$

where  $R_c$  (code rate) =  $\frac{K}{N}$ .

$R_b$  = bit rate

### 2.1. Reed-Solomon Codes

Basically, Reed-Solomon [1] codes are non-binary systematic cyclic linear block codes. They are cyclic because each valid code produces another valid code when it is circularly shifted. They are linear because a new code word with the same length can be generated by adding any two valid code words. As the RS encoder processes each block of message symbols, represented as a sequence of m-bits with m as any positive integer which is greater than 2, these codes are referred as Block codes.

The RS (N, K, t) code parameters can be represented as follows [3].

Codeword symbols:  $N = 2m - 1$

Information symbols:  $k = 1, 3, \dots, n - 2$

Code minimum distance:  $d_{min} = n - k + 1$  and

The error-correction capability symbols:  $t = \frac{(d_{min}-1)}{2} = \frac{(n-k)}{2}$

### 2.2. BER Mathematical Calculation of M-FSK System

M- FSK with Coherent Detection in AWGN channel [5-8].

$$P_s = 1 - \int_{-\alpha}^{\alpha} \left[ Q \left( -q - \sqrt{\frac{2kE_b}{N_o}} \right) \right]^{M-1} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{q^2}{2} \right) dq \quad (4)$$

$$P_b = \frac{2^{k-1}}{2^{k-1}} P_s \quad (5)$$

Where

$P_s$ : Symbol error rate (SER)

$P_b$ : Bit error rate (BER)

M: Size of modulation constellation

K: Number of bits per symbol  $\rightarrow k = \log_2 M$

$\frac{E_b}{N_o}$ : Energy per bit-to-noise power-spectral-density ratio.

• M-FSK with Non-coherent Detection in AWGN channel [5].

$$P_s = \sum_{m=1}^{M-1} (-1)^{m+1} \binom{M-1}{m} \frac{1}{m+1} \exp \left[ -\frac{m}{m+1} \frac{kE_b}{N_o} \right] \quad (6)$$

$$P_b = \frac{1}{2} \frac{M}{M-1} P_s \quad (7)$$

$$P_{MFSK} = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} \exp \left[ \left\{ -\frac{1}{2} (y - \sqrt{2\lambda})^2 \right\} \right] [1 - Q(y)]^{M-1} dy \quad (8)$$

MFSK shows in contrast to the other modulation schemes, the error probability decreases as M increases.

• Reed-Solomon code with  $N = Q - 1 = 2^q [5]$

$$P_b = \frac{1}{q} \frac{1}{N} \sum_{m=t+1}^N m \binom{N}{m} (P_s)^m (1 - P_s)^{N-m} \quad (9)$$

Otherwise,

$$\text{If } \frac{\log_2 Q}{\log_2 M} = \frac{q}{k} = h$$

Where  $h$  is an integer

$$P_s = 1 - (1 - s)^h \quad (10)$$

Where  $s$  is the symbol error rate (SER) in uncoded AWGN channel.

• M-FSK in rayleigh fading channel [5]

$$P_s = P_b = \frac{1}{2^L} \left( 1 - \sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}} \right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \frac{1}{2^k} \left( 1 + \sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}} \right)^k \quad (11)$$

Where

L: diversity branch

$M_{\gamma l}$  Moment generating functions for each diversity branch

For Rayleigh fading:

$$M_{\gamma l}(s) = \frac{1}{1 - s\gamma l}$$

• M-FSK in rician fading channel [5]

$$P_s = \sum_{r=0}^{M-1} \frac{(-1)^{r+1} e^{-\frac{LK\bar{\gamma}_r}{1+\bar{\gamma}_r}}}{(r(1+\bar{\gamma}_r)+1)^L} \binom{M-1}{r} \sum_{n=0}^{L-1} \frac{\Gamma(L+n)}{\Gamma(L)} \left[ \frac{1+\bar{\gamma}_r}{r+1+\bar{\gamma}_r} \right]^n \quad (12)$$

$$P_b = \frac{1}{2} \frac{M}{M-1} P_s \quad (13)$$

$$\bar{\gamma}_r = \frac{1}{1+K} \bar{\gamma} \quad (14)$$

$$\beta_{nr} = \sum_{i=n-(L-1)}^n \frac{\beta_{i(r-1)}}{(n-i)!} I_{[0,(r-1)(L-1)]}(i) \quad (15)$$

$$\beta_{00} = \beta_{r0} = 1$$

$$\beta_{n1} = \frac{1}{n!}$$

$$\beta_{1r} = r$$

And  $I_{[a,b]}(i) = 1$  if  $a \leq i \leq b$  and 0 otherwise.

Where K is the ratio of energy in the specular component to the energy in the diffuse component (linear scale).

For identically-distributed diversity branches:

$$M_{\gamma l}(s) = M_{\gamma} \text{ for all } l.$$

### 3. Model-Based Design Steps

The model used in this paper was designed with Simulink [9-11].

- Select Random integer Generator block from the Channels sub library of the Communications.
- Select Communications system toolbox→Error Detection and correction → Block →Integer-input RS encoder, and decoder blocks
- Select DSP system Toolbox→Signal Management→Buffers→Buffer, and Unbuffer blocks.
- Select Communications system

toolbox→Modulation→Digital Modulation→FM→M-FSK Modulator Baseband and Demodulator Baseband blocks. Fig 4: Configuration parameters for M-FSK Modulator and Demodulator blocks

- Select Communications system toolbox→Modulation→Interleaving→Block →Matrix Interleaver and Deinterleaver blocks.
- Select Communications system toolbox→Comm Sinks→Error Rate Calculation Block.
- Select Sinks Sub library →Display Block
- Select Commonly Used Blocks sub library→Gain block.

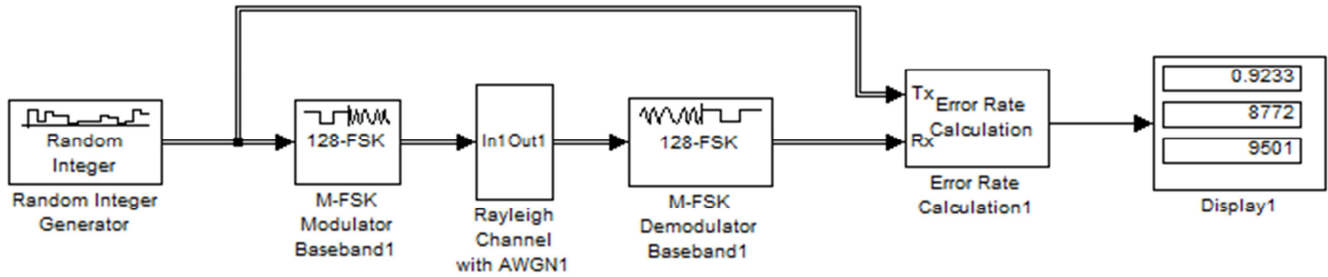


Fig. 1. Simulink Model of M-FSK system in Multipath Rayleigh Fading Channel without RS code.

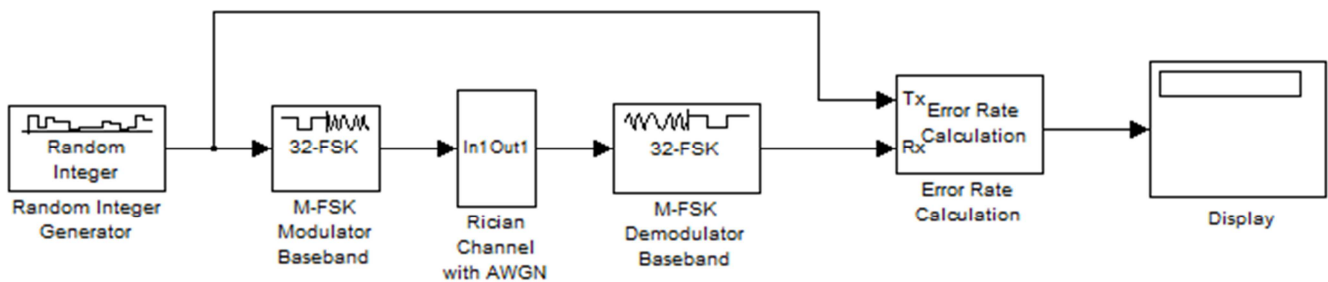


Fig. 2. Simulink Model of M-FSK system in Rician Fading Channel without RS code.

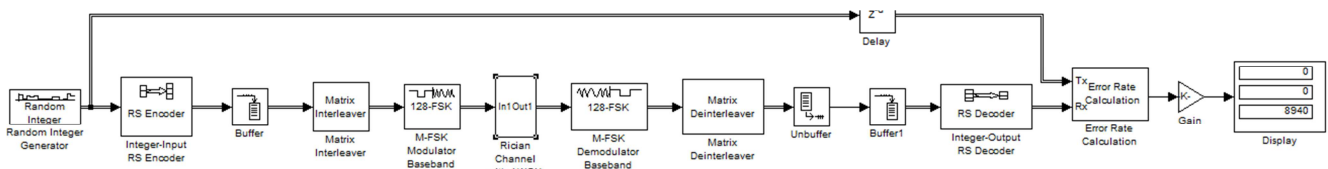
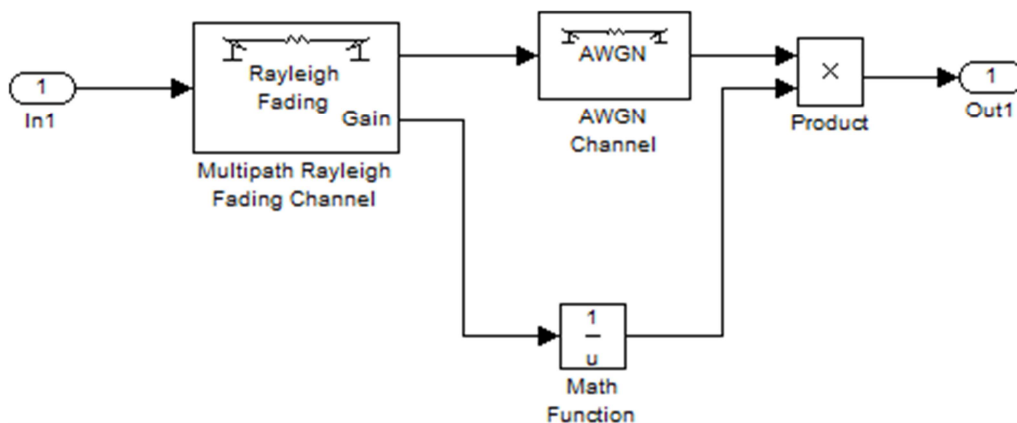


Fig. 3. Simulink Model of M-FSK in Multipath Rician Fading channel with AWGN.

The Simulink model for the Rayleigh and Rician channels with AWGN are shown in Fig. 4 below.



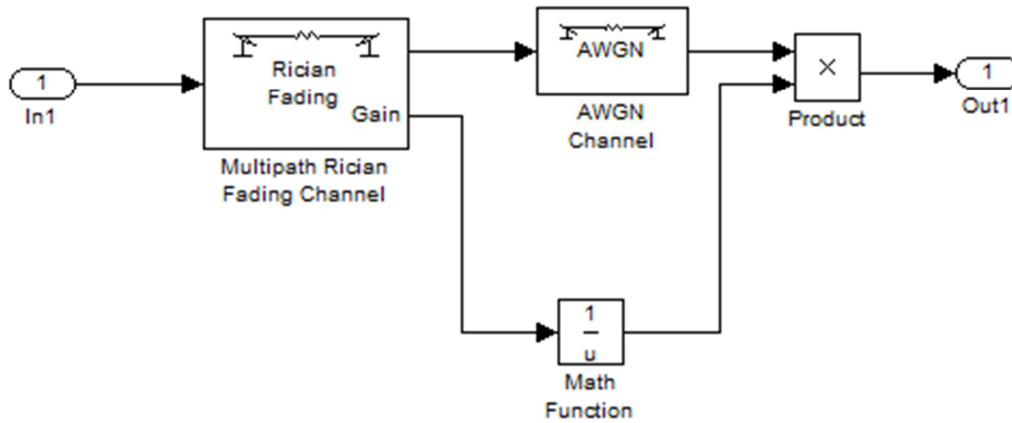


Fig. 4. Simulink model for the Multipath Rayleigh and Rician channels with AWGN.

## 4. Simulation Results and Discussion

Simulation time: 25000 Matlab seconds

Table 1. BER performance of Uncoded M-FSK system.

MFSK UNCODED				
$E_s = E_b/N_0 + E_b/N_0 * \log_{10}(M)$				
5db	10db	15db	20db	
0.7813	0.6757	0.6757	0.6757	
0.8400	0.8403	0.8197	0.8197	
0.9434	0.9009	0.9009	0	

Table 2. BER performance for Multi Rayleigh channel with RS code (n,k,t).

Rayleigh Channel					
$E_s/N_0 = E_b/N_0 + E_b/N_0 * \log_{10}(M * k/n)$					
M	Code-word	$E_b/N_0=5db$	10db	15db	20db
32	(31,15)	0.2531	0.006262	0	0
64	(63,47)	0.1364	0.008187	0	0
128	(127,111)	0.06072	0.01192	0	0

Table 3. BER performance for Rayleigh channel with different code-word.

Rayleigh Channel					
$E_s/N_0 = E_b/N_0 + E_b/N_0 * \log_{10}(M * k/n)$					
M	Code-word	$E_b/N_0=5db$	10db	15db	20db
32	(31,19)	0.1645	0.005749	0	0
64	(63,51)	0.04814	0.008525	0	0
128	(127,115)	0.04108	0.007121	$8.987e^{-05}$	0

Table 4. BER performance for Rayleigh channel with different code-word.

M	Code-word	$E_b/N_0=5db$	10db	15db	20db	Code rate=k/n
32	(31,15)	0.2531	0.006262	0	0	0.4839
64	(63,31)	0.2762	0.0007498	0	0	0.4762
128	(127,61)	0.2897	0	0	0	0.4803
16	(15,7)	0.2159	0.009917	$6.428e^{-5}$	0	0.4920

Table 5. Multipath Rician channel (no code-word).

Multipath Rician channel (no code word)				
$E_s = E_b/N_0 + E_b/N_0 * \log_{10}(M)$				
M	$E_b/N_0=5db$	10db	15db	20db
16	0.9317	0.9375	0.9434	0.9434
32	0.9677	0.9804	0.9868	0.9868
64	0.9804	0.9804	0.9804	0.9804

**Table 6.** Multipath Rician channel (code-word).

Multipath Rician channel						
M	Code word	5db	10db	15db	20db	Code rate=k/n
32	(31,15)	0.1931	$4.209e^{-05}$	0	0	0.4839
16	(15,7)	0.1587	$8.57e^{-05}$	0	0	0.4927
64	(63,31)	0.26221	0	0	0	0.4762

**Table 7.** Multipath Rician channel with different Code Rate(CR).

M-ary number						
	32	64	128			
BIT ERROR RATE						
Es/No = Eb/No + Eb/No * log10 (M * k/n)	(31,15,8)	(31,19,6)	(63,47,8)	(63,51,6)	(127,111,6)	(127,115,8)
	CR=0.48	0.61	0.75	0.81	0.87	0.91
5db	0.2531	0.1645	0.1364	0.08814	0.06072	0.04108
10db	0.006262	0.005749	0.008187	0.008525	0.01192	0.007121
15db	0	0	0	0	0	$8.987e^{-05}$
20db	0	0	0	0	0	0

*Case 1:* R-S FEC M-FSK system vs Uncoded M-FSK system

Table 1 shows the BER performance of M-FSK without forward error correction. Table 2 shows the BER performance of M-FSK with Reed-Solomon Forward Error Correction method under multipath Rayleigh Fading channel (the nature of the channel is shown in Fig 4). Case 1 compares BER recorded in table 1 against the BER recorded in table 2 under the same channel. Case 1 also compares the BER performance of M-FSK with FEC under multipath Rician fading channel against the BER performance of M-FSK without FEC. The results show that the BER greatly improved for different values of different values of Energy per symbol to Noise power spectral density ( $E_s/N_0$ ) because of the Reed-Solomon Error correction method. The poor BER performance of M-FSK without error correction is greatly improved when used with Reed-Solomon forward error correction method. It is observed from the simulation results that the BER performance of the M-FSK system with Reed-Solomon FEC is much better under multipath Rician fading channel than under multipath Rayleigh fading channel.

*Case 2:* behaviour as the Function of Code length

The results showed in Table 4 and Table 6 show the performance of M-FSK as a function of code length at constant code rate. As shown in Tables 3 and 4, the value of the code rate is held nearly constant (at 0.48). It is observed that the BER improves as the codeword symbols (N) decreases from 127 to 16 for different values of Energy per symbol to Noise power spectral density ( $E_s/N_0$ ). It is observed from the simulation results that at constant code rate, the BER performance of the M-FSK system with Reed-Solomon FEC is much better under multipath Rician fading channel than under multipath Rayleigh fading channel.

*Case 3:* Performance as the Function of Redundancy

In the results shown in Table 7 with RS coding, the RS code sets used have identical codeword symbols (N) whereas the number of information symbols (K) decreases, that apparently resulted in different error correction capability (t)

6 and 8 respectively. Also, the code rate ( $R_c = k/n$ ) is increased from 0.48 to 0.91. It is observed that the BER performance increases as the error correction capability (t) decreases from 8 to 6 at a constant codeword symbol (N).

## 5. Conclusion

In this paper, the improvement of Bit-Error-Rate (BER) performance of M-FSK in a noisy multipath Rayleigh and Rician fading channel using Reed-Solomon forward error correction method has been achieved. BER improvement can be obtained if RS coding is properly chosen. The following deductions are made:

- BER performance of the M-FSK system with Reed-Solomon FEC is much better under multipath Rician fading channel than under multipath Rayleigh fading channel
- At constant code rate, the BER performance of the M-FSK system with Reed-Solomon FEC is much better under multipath Rician fading channel than under multipath Rayleigh
- The BER performance increases as the error correction capability (t) decreases from 8 to 6 at a constant codeword symbol (N) (meaning an increase in the code rate of the RS code from 0.48 to 0.91).

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