American Journal of Management Science and Engineering
2017; 2(6): 199-204
http://www.sciencepublishinggroup.com/j/ajmse
doi: 10.11648/j.ajmse.20170206.16
ISSN: 2575-193X (Print); ISSN: 2575-1379 (Online)

Study of Propagation Properties of Rossby Waves in the Atmosphere and Relationship Between the Phase Velocity and the Group Velocity

Mohammed Anwar Batal, Sawsan Othman

Department of Physic, Aleppo University, Aleppo City, Syria

Email address:
mbatal2001@yahoo.com (M. A. Batal), sawsanothman89@gmail.com (S. Othman)

To cite this article:
Mohammed Anwar Batal, Sawsan Othman. Study of Propagation Properties of Ross by Waves in the Atmosphere and Relationship Between the Phase Velocity and the Group Velocity. *American Journal of Management Science and Engineering*. Vol. 2, No. 6, 2017, pp. 199-204. doi: 10.11648/j.ajmse.20170206.16

Received: March 30, 2017; **Accepted**: May 11, 2017; **Published**: January 30, 2018

Abstract: Using Rossby wave equations, the dispersion equation is developed. The wave normal diagram for Rossby waves on a beta plane is a circle in wave number (k, l) space whose center is displaced along the negative X axis, and whose radius is less than this displacement, which means that phase propagation is entirely westward. The phase velocity diagram is a circle whose center is displaced along the negative X axis, the group velocity diagram is an ellipse whose center is displaced westward and whose major and minor axes give the maximum all the directions group speeds as function of the frequency and parameter Q.

Keywords: Rossby Waves, Planetary Waves, Phase Velocity, Group Velocity, Barotropic Fluid, Baroclinic Fluid

1. Introduction

The wave type that is of most importance for large-scale meteorological processes is the Rossby wave, or planetary wave. In an inviscid barotropic fluid of constant depth (where the divergence of the horizontal velocity must vanish), the Rossby wave is an absolute vorticity-conserving motion that owes its existence to the variation of the Coriolis parameter with latitude, the so-called β-effect. More generally, in baroclinic atmosphere, the Rossby wave is a potential vorticity conserving motion that owes its existence to the isentropic gradient of potential vorticity [1]. Rossby waves play a central role in geophysical fluid dynamics and dynamical meteorology, particularly in the dynamics of quasi-geostrophic flow [2]. Their reflection properties at coast lines help to explain certain features of western boundary currents [3]. The anisotropic and dispersive propagation properties of Rossby waves (in particular the "backward" property in which phase and group velocities are opposite in the N-S direction) have been invoked, together with equatorial heating, to explain dipole-like formation of equatorial easterly jets accompanied, at higher latitudes, by westerly jets [4]. In addition, an inverse turbulent cascade together with Rossby waves can lead to the formation of zonal flows [2]. These ideas have been further developed recently [5] in models of an eddy-driven jet. Rossby wave propagation can be understood in a qualitative fashion by considering a closed chain of fluid parcels initially aligned along a circle of latitude [1].

The major type of planetary scale waves can be classified into several cases, based on energy source or horizontal and vertical structure. Based on energy source, of planetary scale waves can be divided into free and forced modes. The free modes are the normal oscillations in the atmosphere. They can be excited by small random forcing and are limited in amplitude by dissipation. The free planetary (rossby) modes are global extent and with periods of about 2, 5, 10 and 16 days [6, 7]. The forced Rossby modes can be excited by flow over topography or by land-sea heating contrasts. Based on horizontal structure classification, planetary waves can be divided into two, global modes and equatorial modes. The global modes are the planetary waves can propagate both meridionally and zonally. Rossby waves are the most important global modes. The equatorial modes are trapped in the equatorial wave-guide and propagate zonally along the equator. Kelvin and Rossby-gravity waves are the most important equatorial modes. In addition to the above two classification, planetary waves can be divided into external and internal modes from their vertical structure. External modes are vertically trapped modes and the energy density of such modes decays exponentially in the vertical. Internal modes are vertically propagating (phase surfaces tilt with height) modes and they can transfer momentum and energy vertically over many scale heights. The forced Rossby waves, Kelvin waves, Rossby-gravity waves and Gravity waves can propagate vertically and transfer energy and momentum between the lower and upper atmosphere under suitable background conditions [8].

Rossby waves play a vital role in the dynamics of the middle atmosphere. They primarily responsible for the asymmetries in the polar vortex, sudden warming in the stratosphere, forcing of the Quasi-biennial Oscillation (OBO), Semi-Annual Oscillation (SAO). Meridionally propagating Rossby waves carry angular momentum between the tropics and middle latitudes. Planetary waves can reach large amplitudes if they propagate upwards to the MLT region. Vertical propagation is determined by wave/mean flow interactions [9]. Most traveling waves are westward propagating, and so they can propagate through the eastward winds of the winter stratosphere, but not the westward winds of the summer stratosphere. The continental configuration and topography of the northern hemisphere produces more planetary wave activity than in the southern hemisphere. This planetary wave activity acts to cause sudden stratospheric warming and break down the polar vortex at the end of winter in the northern hemisphere. In contrast, the southern polar vortex is more stable and there has only been one major sudden stratospheric warming in the southern hemisphere, which occurred in 2002. [7]

2. The $\beta-$Plane

We saw in the derivation of the barotropic vorticity equation the potential importance of the fact that the Coriolis parameter varies with latitude, a consequence of spherical geometry. However, dealing with spherical geometry is (a little) more complicated than with planar geometry, so it is common to represent a strip of the sphere-limited in latitude but going all the way around the world in longitude-as a plane, as in Figure 1.

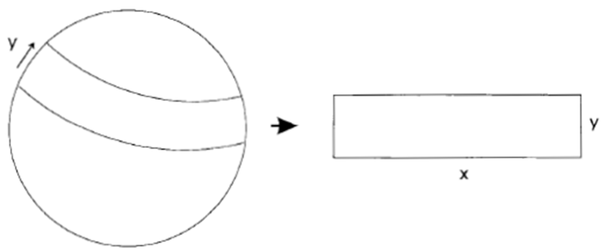

Figure 1. Shows the $\beta-$plane.

Let us consider a strip centered on longitude φ_0 , and define a y coordinate $y = a(\varphi - \varphi_0)$, and an x coordinate $x = a\lambda$, where λ Is longitude. Since $f = f(\varphi) = 2\Omega\sin\varphi$, in the (x, y) system it becomes $f = f(y)$. Assuming that the width of the strip is small enough, we can approximate $f(\varphi)$ as Taylor series about the central latitude:

$$f(\varphi) \approx f(\varphi_0) + (\varphi - \varphi_0)(\frac{df}{d\varphi})(\varphi_0) + ...$$

Where $f(\varphi_0) = 2\Omega\sin\varphi_0$, $df.d\varphi_0 = 2\Omega\cos\varphi_0$. Substituting for y , we get

$$f(y) = f_0 + \beta y$$

Where $f_0 = 2\Omega\sin\varphi_0$ and $\beta = \frac{2\Omega}{a}\cos\varphi_0$. For a latitude of $\frac{\pi}{4}, \beta = 1.617\times10^{-11} m^{-1}s^{-1}$. the sign of f changes from north to south hemisphere, β Is always positive (since f always increase northward)[1].

Figure 2 shows Rossby waves in the atmosphere.

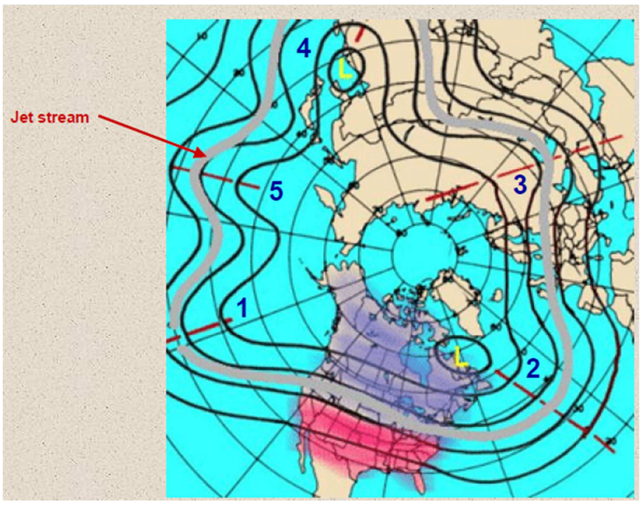

Figure 2. Shows Rossby waves in the atmosphere at 500 mb pressure.

3. Purpose of Study

Study of relationship between the phase velocity and the group velocity of Rossby waves in the atmosphere.

4. Results and Discussion

4.1. The Dispersion Equation

To get the dispersion equation for Rossby waves lets depend on the quasi-geostrophic equations:

$$D_g u_g - f_0 v_a - \beta y v_g = 0 \tag{1}$$

$$D_g v_g + f_0 u_a + \beta y u_g = 0 \tag{2}$$

$$\frac{\partial u_a}{\partial x}+\frac{\partial v_a}{\partial y}+\frac{\partial w_a}{\partial z}=0 \tag{3}$$

$$D_g(-\frac{g\rho'}{\rho_0})+N_\beta^2 w_a=0 \tag{4}$$

$$D_g=\frac{\partial}{\partial t}+u_g\frac{\partial}{\partial x}+v_g\frac{\partial}{\partial y}$$

Where:

Where $u_a=u-u_g$,$v_a=v-v_g$,$w_a=w$ are the difference between the true velocity and the geostrophic flow, N_β^2 is the square of the buoyancy frequency, ρ_0 is reference value of the background density, ρ' is perturbation in density, g is acceleration gravity, f_0 is Coriolis parameter.

Where $f_0=2\Omega\sin\varphi_0$

Geostrophic balance can be expressed by:

$$u\approx u_g=-\frac{\partial\psi}{\partial y} \text{ ‘ } v\approx v_g=\frac{\partial\psi}{\partial x}$$

Where $\psi=\dfrac{p'}{f_0\rho_0}$ is the geostrophic stream function, p' is the perturbation in pressure.

Hydrostatic balance can be written as:

$$\rho'=-\frac{f_0\rho_0}{g}\frac{\partial\psi}{\partial z} \tag{5}$$

Take $\dfrac{\partial}{\partial x}$(1), $\dfrac{\partial}{\partial y}$ (2) to obtain:

$$\frac{\partial}{\partial y}\left[D_g u_g-f_0v_a-\beta y v_g\right]=0 \tag{6}$$

$$\frac{\partial}{\partial x}\left[D_g v_g+f_0u_a+\beta y u_g\ \right]=0 \tag{7}$$

Subtraction (7) from (6) and using the mass-continuity equation (3) to obtain:

$$D_g\zeta=f_0\frac{\partial w_a}{\partial z}$$

$$\zeta=f_0+\beta y-\frac{\partial u_g}{\partial y}+\frac{\partial v_g}{\partial x}=f_0+\beta y+\frac{\partial_2\psi}{\partial x^2}+\frac{\partial_2\psi}{\partial y^2} \tag{8}$$

is the Z component of the absolute vorticity associated with the geostrophic flow. Using equations (4) and(5) the vertical velocity can be expressed as:

$$\begin{aligned} w_a&=D_g(\frac{g\rho'}{\rho_0N_\beta^2})\\ w_a&=-D_g(\frac{f_0}{N_\beta^2}\frac{\partial\psi}{\partial z})\end{aligned} \tag{9}$$

Substitution of this expression into the vorticity equation (8) and further careful manipulation lead finally to the quasi-geostrophic potential vorticity equation:

$$\begin{aligned} &D_gq=0\\ &q=\zeta+\frac{\partial}{\partial z}(\frac{f_0^2}{N_\beta^2}\frac{\partial\psi}{\partial z})\\ &q=f_0+\beta y+\frac{\partial_2\psi}{\partial x^2}+\frac{\partial_2\psi}{\partial y^2}+\frac{\partial}{\partial z}(\frac{f_0^2}{N_\beta^2}\frac{\partial\psi}{\partial z})\end{aligned} \tag{10}$$

Let us consider small-amplitude disturbances to a uniform zonal background flow $(u,0,0)$, where u is a constant; by this uniform flow corresponds to a geostrophic stream function $\psi=-uy$. For the total flow (the background plus a small disturbance):

$$\psi=-uy+\psi' \tag{11}$$

Substitute into the equation (10) and neglect terms that are quadratic in ψ' . The quasi-geostrophic potential vorticity equation for this flow is:

$$q=f_0+\beta y+\Gamma\psi' \tag{12}$$

Where Γ is the elliptic operator.

$$\Gamma=\frac{\partial_2}{\partial x^2}+\frac{\partial_2}{\partial y^2}+\frac{\partial}{\partial z}(\frac{f_0^2}{N_\beta^2}\frac{\partial}{\partial z})$$

In addition, the quasi-geostrophic potential vorticity equation linearizes to:

$$(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x})\Gamma\psi'+\beta\frac{\partial\psi'}{\partial x}=0 \tag{13}$$

Let us take N_β^2 to be constant and look for plane-wave solutions to this equation by substituting:

$$\psi'=\mathrm{Re}\,\hat\psi\exp\left[i(kx+ly+mz-\omega t)\right]$$

Where $\hat\psi$ is a complex amplitude, k,l,m are wavenumbers on X, Y, Z. We obtain the dispersion relation for Rossby waves:

$$\omega=ku-\frac{\beta k}{k^2+l^2+\dfrac{f_0^2m^2}{N_\beta^2}} \tag{14}$$

When $u=0$, the dispersion relation for Rossby waves [10]:

$$\omega = -\frac{\beta k}{k^2 + l^2 + \frac{f_0^2 m^2}{N_\beta^2}} \tag{15}$$

Or

$$l^2 + (k + \frac{\beta}{2\omega})^2 = \frac{\beta^2}{4\omega^2} - \frac{f_0^2 m^2}{N_\beta^2} \tag{16}$$

The latter wave normal form is a circle centred at $(-\frac{\beta}{2\omega}, 0)$ and radius given by $\sqrt{\frac{\beta^2}{4\omega^2} - \frac{f_0^2 m^2}{N_\beta^2}}$.

4.2. The Phase Velocity

The phase velocity $v_p = \frac{\omega}{K}$ can be written:

$$\bar{v}_{py}^2 + (\bar{v}_{px} + \frac{Q}{2})^2 = \frac{Q^2}{4}(1 - \frac{4\bar{\omega}^2}{Q}) \tag{17}$$

Where $Q = \frac{N_\beta^2}{f_0^2 m^2}$, $\bar{\omega} = \frac{\omega}{\sqrt{\beta}}$ and $\bar{v}_p = \frac{v_p}{\sqrt{\beta}}$

The phase velocity diagram is a circle of radius $\frac{Q}{2}\sqrt{(1 - \frac{4\bar{\omega}^2}{Q})}$ whose origin is displaced westward by $-\frac{Q}{2}$ units and therefore lies entirely in the regime of Westward propagation. It represented in Figure 3. The smallest value of the westward $\bar{v}_{py} = 0$:

$$(\bar{v}_{px})_{\min} = \frac{-Q}{2} + \frac{Q}{2}\sqrt{1 - \frac{4\bar{\omega}^2}{Q}} \tag{18}$$

Which approaches $(\bar{v}_{px})_{\min} \to -\bar{\omega}^2$ when $Q \to \infty$

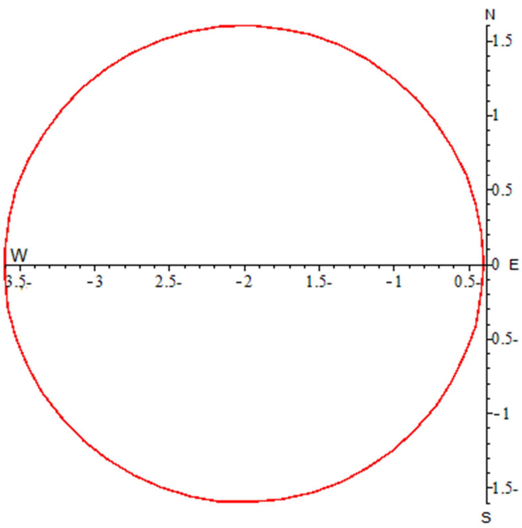

Figure 3. *Shows the phase velocity when* $Q = 4$, $\bar{\omega} = 0.6$.

4.3. The Group Velocity

The group velocity $v_g = \frac{\partial \omega}{\partial K}$ can be written

$$v_{gx} = \frac{\beta(k^2 - (l^2 + \frac{f_0^2 m^2}{N_\beta^2}))}{(k^2 + l^2 + \frac{f_0^2 m^2}{N_\beta^2})^2} \tag{19}$$

$$v_{gy} = \frac{2\beta k l}{(k^2 + l^2 + \frac{f_0^2 m^2}{N_\beta^2})^2} \tag{20}$$

by eliminating the denominator from equations (19) and (20) using the dispersion equation, so that in normalized form, equations (19) and (20) become

$$\bar{v}_{gx} = \bar{\omega}^2 (2 + \frac{1}{\bar{\omega} k}) \tag{21}$$

$$\bar{v}_{gy} = 2\bar{\omega}^2 (\frac{l}{k}) \tag{22}$$

It is now straightforward to eliminate k in favour of $\bar{v}_{gx}$ from equation (21), which on substitution into the square of equation (22) gives directly the group velocity curve in the form

$$\bar{v}_{gy}^2 = 4\bar{\omega}^4 \left[(1 - \frac{\bar{v}_{gx}}{\bar{\omega}^2}) - \frac{\bar{\omega}^2}{Q}(\frac{\bar{v}_{gx}}{\bar{\omega}^2} - 2)^2 \right] \tag{23}$$

This equation can be written

$$\frac{Q\bar{v}_{gy}^2}{4\bar{\omega}^2} + \left[\bar{v}_{gx} + \frac{Q}{2}(1 - \frac{4\bar{\omega}^2}{Q}) \right]^2 = \frac{Q^2}{4}(1 - \frac{4\bar{\omega}^2}{Q}) \tag{24}$$

The southward group velocity is simply a reflection of the northward group velocity in the x-axis. Equation (24) can be written

$$\frac{\bar{v}_{gy}^2}{d^2} + \left[\bar{v}_{gx} + \frac{Q}{2}(1 - \frac{4\bar{\omega}^2}{Q}) \right]^2 / b^2 = 1 \tag{25}$$

Where $b^2 = \frac{Q^2}{4}(1 - \frac{4\bar{\omega}^2}{Q})$, $d^2 = Q\bar{\omega}^2 (1 - \frac{4\bar{\omega}^2}{Q})$

Equation (25) can be written in the polar form

$$\bar{v}_g = \frac{p}{1 + e\cos\chi} \tag{26}$$

Where $p = \frac{d^2}{b} = 2\bar{\omega}^2 (1 - \frac{4\bar{\omega}^2}{Q})^{1/2}$, $e = \sqrt{1 - \frac{d^2}{b^2}} = (1 - \frac{4\bar{\omega}^2}{Q})^{1/2}$

The shift along the negative $\bar{v}_x$ axis

$$\frac{Q}{2}(1-\frac{4\bar{\omega}^2}{Q})=\frac{ep}{1-e^2} \tag{27}$$

In terms of $\bar{\omega}^2$ and Q, the ellipse (26) may be written

$$v_g=2\bar{\omega}^2 / \left[\left(1-\frac{4\bar{\omega}^2}{Q}\right)^{-1/2}+\cos\chi\right] \tag{28}$$

The group velocity diagram is a ellipse whose origin is displaced westward by $-\frac{Q}{2}(1-\frac{4\bar{\omega}^2}{Q})$ units along $\bar{v}_{gx}$. It represented in Figure 4.

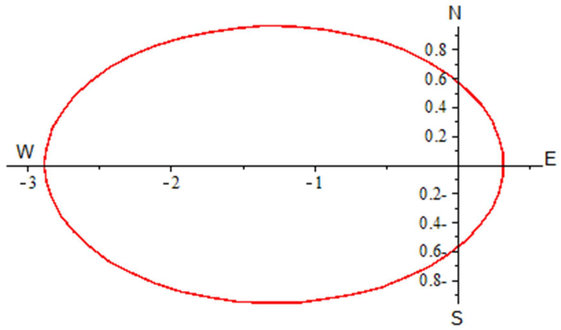

Figure 4. *Shows The group velocity when $Q=4$, $\bar{\omega}=0.6$.*

The maximum eastward and westward group speeds are given by

$$\bar{v}_{gx}^{\pm}=\frac{Q}{2}(1-\frac{4\bar{\omega}^2}{Q})^{1/2}\left[\pm1-(1-\frac{4\bar{\omega}^2}{Q})^{1/2}\right] \tag{29}$$

and the maximum northward (southward) group speed also follows as

$$\bar{v}_{gy}=\pm\bar{\omega}m^{1/2}(1-\frac{4\bar{\omega}^2}{m})^{1/2} \tag{30}$$

Examples of the behavior of external group velocities are shown in Figure 5.

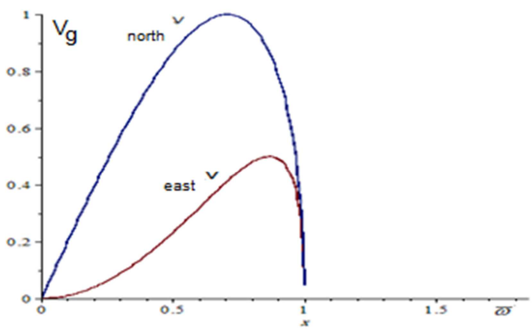

Figure 5. *Shows the maximum northward and eastward group Speeds as a function of frequency for the case $Q=4$.*

The ellipse collapses to the origin as $Q\to4\bar{\omega}^2$ at which the wave normal collapses to the point $k=-\frac{1}{2\bar{\omega}}$. In the limiting case in which $Q>>1$, which can prevail quite near the equator, Equation (23) tends to the parabola [11].

$$\bar{v}_{gy}=\pm2\bar{\omega}^2(1-\frac{\bar{v}_{gx}}{\bar{\omega}^2}) \tag{31}$$

Which represented in Figure 6.

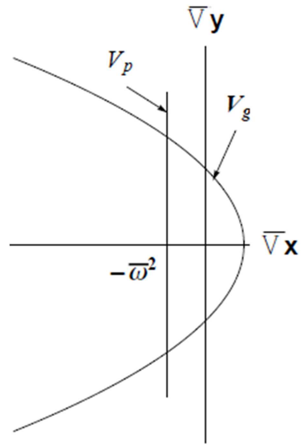

Figure 6. *Shows the group velocity (a parabola) and phase velocity (a line) Diagrams for the case $Q\to\infty$.*

It is of some interest to emphasize that Rossby waves are backward in the sense that the latitudinal components of their phase and group velocities are always in opposite directions. This property

can be invoked to describe the formation of a dipole pair of jets in the following way: Northward (away from the equator) wave energy flux is associated with southward, towards the equator, wave momentum flux; and the opposite in the case of southward directed energy flux, away from the equator, corresponds to northward (towards the equator) momentum flux. In other words, a pole ward energy flux from the equator is associated with an equator ward flux of momentum. Hence, Rossby wave dynamics implies that localized equatorial heating gives rise to equatorial easterly zonal jets. This "convergence" of equatorial momentum implies a deficit at higher latitudes such that a westerly jet must necessarily form there [12].

5. Conclusion

the phase speed is negative, so the phase of rossby waves always propagates westward. Since ω is a nonlinear function of k, Rossby waves are dispersive. The magnitude of the group velocity is, typically, greater for the westward propagating long waves than for the eastward propagating short waves. The latitudinal components for Rossby waves of their phase and group velocities are always in opposite direction.

References

[1] HOLTEN J., 2004-An introduction to dynamic meteorology. International geophysics series, 4th ed, New York, 553.

[2] VALLIS G. K., 2006- Atmospheric Oceanic Fluid Dynamics, Fundamentals and Large-scale Circulation, p. 745. (Cambridge University Press: Cambridge).

[3] PEDLOSKY J., 1987- Geophysical Fluid Dynamics, p. 710, (Springer-Verlag: New York).

[4] DIAMOND P. H, GURCAN D. D, HAHN, T. S, MIKI K and GARBET X., Momentum theorems and the structure of atmospheric jets and zonal flows in plasmas. Plasma Phys. Control. Fusion 2008, 50, 124018. doi:10.1088/0741-3335/50/12/124018.

[5] KIDSTON J. and VALLIS G. K., 2012- The relationship between the speed and the latitude of an eddy-driven jet in a stirred barotropic model. J. Atmos. Sci, 69, 3251–3263.

[6] BHAGAVATHIAMMAL G., 2012-Studies on mean winds and planetary waves in the middle atmosphere. *Equatorial geophysical research laboratory*, India, 8-11.

[7] DAVIS R., 2014-Wave Dynamics of the Middle Atmosphere. *University of Bath*, 26-29.

[8] RHINES P., 2009-Rossby waves. *J. Atmos. Sci,* 1-3.

[9] RHINES P., 2003- Rossby waves. In Encyclopedia of Atmospheric Sciences. *Academic Press: Oxford*, 22-30.

[10] ANDREWS D., 2010-An Introduction to Atmospheric Physics. Cambridge University Press, 2nd ed, New York, 249.

[11] MCKENZIE, J., 2014- The group velocity and radiation pattern of Rossby waves. *Geophysical and Astrophysical Fluid Dynamics.* 1-5.

[12] DIAMOND P. H, GURCAN D. D, HAHN T. S, MIKI K., and GARBET X., 2008- Momentum theorems and the structure of atmospheric jets and zonal flows in plasmas. Plasma Phys, Control. Fusion, 50,124018, doi:10.1088/0741-3335/50/12/124018.