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Training Dynamic Neural Networks for Forecasting Naira/Dollar Exchange Returns Volatility in Nigeria

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Abstract: This paper examined the monthly volatility of Naira/Dollar exchange rates in Nigeria between the periods of January, 1995 to January, 2016. Forecasting volatility remains to be an important step to be taken in several decision makings involving financial market. Traditional GARH models were usually applied in forecasting volatility of a financial market. This study was aim at enhancing the performance of these models in volatility forecasting in which both the traditional GARCH and Dynamic Neural Networks were hybridized to develop the proposed models offorecasting the volatility of Inflation rate in Nigeria. The values of the volatility estimated by the best fitted GARH model are used as input to the Neural Network. The inputs of the first hybrid model also included past values of other related endogenous variables. The second hybrid model takes as inputs both series of the simulated data and the inputs of the first hybrid model. The forecasts obtained by each of those hybrid models have been compared with those of GARCH model in terms of the actual volatility. The computational results demonstrate that the second hybrid model provides better volatility forecasts.

Keywords: Volatility, ARCH Models, Dynamic Neural Networks, Monthly Standard Deviation

1. Introduction

Before the establishment of structural adjustment programme in Nigeria in 1986, there was a fixed exchange rate in the country which was maintained by the exchange control regulations that brought about significant distortions in the economy. This is because Nigeria depends seriously on imports from different countries as nearly all industries import their raw materials from foreign countries. Similarly, there were huge importation of finished goods with the adverse consequences for domestic production, balance of payments position and the nation's external reserves level [1]. The foreign exchange market in the fixed exchange rate period was characterized by high demand for foreign exchange which cannot be adequately met with the supply of foreign exchange by the Central Bank of Nigeria (CBN). The fixed exchange rate period was also characterized by sharp practices perpetrated by dealers and end-users of foreign exchange [1]. The inadequate supply of foreign exchange by the CBN promoted the parallel market for foreign exchange and created uncertainty in foreign exchange rates. The introduction of SAP in Nigeria in September 1986 which deregulated the foreign exchange market led to the introduction of market determined exchange rate, managed floating rate regime. The CBN usually intervene in foreign exchange market through its monetary policy actions and operations in the money market to influence the exchange rate movement in the desired direction such that it ensures the competitiveness of the domestic economy. This introduction of managed floating rate regime tends to increase the uncertainty in exchange rates, thus, increasing the volatility of exchange rate by the regime shifts. This made the exchange rate to be the most important asset price in the economy.

The volatility of time series as defined by econometricians refer to the 'conditional variance' of the data and the time-varying volatility typical of asset return which is otherwise known as 'conditional heteroscedasticity'. The conditional heteroscedasticity was a theory first introduced to economists by [2] in which conditional variance of a time series is considered as a function of past shocks; the autoregressive conditional heteroscedastic (ARCH) model. The model provided a precise approach of empirically investigating issues concerning the volatility of economic variables. An example is Friedman's hypothesis that higher inflation is more volatile [3]. Using data for the UK, Engle [2] found that the ARCH model supported Friedman's hypothesis. [4] Applied the ARCH model to US inflation and the converse results emerged, although [5] criticize this paper as they believe that Engle estimates a mis-specified model. The relationship between the level and variance of inflation has continued to interest applied econometricians [6].

Artificial Neural Network (ANN) gives a better way of investigating the dynamics of different problems pertaining to Economics and Finance [7]. The use of these techniques to economic application is growing fast [8, 9, 10, 11]. [12] Improved traditional GARCH models with the use of ANN in which they forecast the volatility of daily return of Istanbul Stock Exchange. [13] Used three hybridized time series with ANN models in order to forecast the volatility of Korea Composite Stock Price Index (KOSPI 200). Traditional GARCH models are mostly used in forecasting and simulation of a financial variable. The ability of model toforecast volatility very well indicates the power of that model because volatility is central to several decisions in financial markets. The aim of this study is to integrate dynamic neural networks in to traditional GARCH models in the forecast volatility of Naira/Dollar Exchange ratein Nigeria.

2. GARCH Models

The unpredictable manners in financial markets is known as the "volatility". Volatility has turn into a central theory in diverse areas of financial engineering, such as multi-period portfolio selection, risk management, and derivative pricing [7]. Volatility estimates is also being utilized by various asset pricing models as a simple risk measure. It is as well considered as an important contribution to the well-known Black–Sholes model for option pricing with its many branches. In statistics, volatility is commonly calculated by standard deviation or variance [14]. In recent times, different models which used stochastic volatility process and time series modeling were used in place of implied and historical volatility method. [2] Developed and used ARCH model to estimate volatility and his original work on ARCH was extended in GARCH [15], EGARCH [16] and GJR-GARCH [17]. Generalized Autoregressive Conditional Heteroscedasticity, GARCH (p, q) makes the current conditional variance to depend on the p past conditional variances and the q past squared innovations. The GARCH (p, q) model can be expressed as:

$$\varepsilon_t = \sigma_t z_t \quad (1)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (2)$$

$\omega, \alpha_i, \beta_j$ are non-negativeparameters to be estimated, z_t is anindependently and identically distributed (i.i.d.) random variables with zero mean and unit variance and ε_t is a serially uncorrelated sequence with zero mean and the conditional variance of σ_t^2 which may be nonstationary, the GARCH model reduces the number of parameters necessary when information in the lag(s) of the conditional variance in addition to the lagged ε_{t-i}^2 terms were considered, but was not able to account for asymmetric behavior of the returns. [16] establish Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) model in order to capture leverage effects of price change on conditional variance and which can be written as follows:

$$log\sigma_t^2 = \omega + \sum_{j=1}^{P} \alpha_i \left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right)\right| + \sum_{j=1}^{q} \beta_j log\sigma_{t-j}^2 + \sum_{k=1}^{r} \gamma_k \left(\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right) \quad (3)$$

The parameters α_i and β_j in equation (3) were not constrained in order to guarantee positive and zero conditional variances. GJR-GARCH is another model which used to measure the asymmetric features of the returns series and it is very similar to the Threshold GARCH (TGARCH), and Asymmetric GARCH (AGARCH) due to [18] and [19] respectively. The GJR (p, q) can be written as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{P} [\gamma_i d(\varepsilon_{t-i} < 0)\varepsilon_{t-i}^2] + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (4)$$

Where $\gamma_i (i = 1, \dots, P)$ are the asymmetric parameter and $d(.)$ is the indicator function defined such that $d((\varepsilon_{t-i} < 0) = 1$ if $\varepsilon_{t-i} < 0$ and $d((\varepsilon_{t-i} > 0) = 0$ if$\varepsilon_{t-i} > 0$

Equally, asymmetric ARCH known as APARCH (p, q) was introduced by [20] and is represented as:

$$\sigma_t^\delta = \omega + \sum_{j=1}^{P} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j log\sigma_{t-j}^\delta \quad (5)$$

where the asymmetric parameter $-1 < \gamma_i < 1 (i = 1 \dots, P)$, with non-negative δ transforms the conditional standard deviation and asymmetric absolute innovations.

3. Dynamic Neural Networks

Neural networks can be classified into dynamic and static categories. Static (feed forward) networks have no feedback elements and contain no delays; the output is calculated directly from the input through feed forward connections. In dynamic networks (such as Non-linear Auto-Regressive model with exogenous inputs (NARX), the output depends not only on the current input to the network, but also on the current or previous inputs, outputs, or states of the network.

Dynamic networks are generally more powerful than static networks (although somewhat more difficult to train). Because dynamic networks have memory, they can be trained to learn sequential or time-varying patterns. This has applications in such disparate areas as prediction in financial markets [21], channel equalization in communication systems [22], phase detection in power systems [23] and many more dynamic network applications in [24].

In this paper, Non-linear Auto-Regressive with Exogenous (NARX) inputs was employed. This model has a parametric component plus a nonlinear part, where the nonlinear part is approximated by a single hidden layer feed-forward ANN. The Non-linear Auto-Regressive with Exogenous (NARX) inputs is a recurrent dynamic network, with feedback connections enclosing several layers of the network. The NARX model is based on the linear ARX model, which is commonly used in time-series modeling.

The defining equation for the NARX model is as follow:

$$y(t) = f(y(t-1), y(t-2) \dots, y(t-n), u(t-1), u(t-2), \dots, u(t-n)) \quad (6)$$

Where the next value of the dependent output signal $y(t)$ is regressed on the previous values of the output signal and previous values of an independent (exogenous) input signals. The output is feed back to the input of the feed-forward neural network as part of the standard NARX architecture as depicted in Fig. 1. Because the true output is available during the training of the network, a series-parallel architecture can be created in which the true output is used instead of feeding back the estimated output, as shown in Fig. 2. This has two advantages. The first is that the import to the feed-forward network is more accurate. The second is that the resulting network has purely feed-forward architecture and static back propagation can be used for training.

Dynamic networks are trained in the same gradient-based algorithms that were used in back-propagation. Although they can be trained using the same gradient-based algorithms that are used for static networks, the performance of the algorithms on dynamic network can be quite different and the gradient must be computed in a more complex way [25]. A diagram of the resulting network is shown by Fig. 3, where a two-layer feed-forward network is used for the approximation:

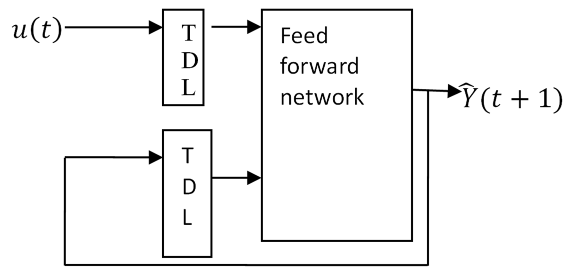

Fig. 1. *Parallel architecture of training a NARX model.*

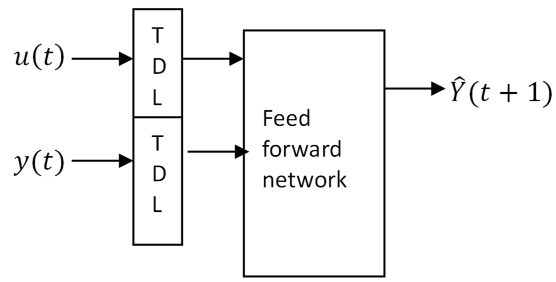

Fig. 2. *Series Parallel architecture of training a NARX model.*

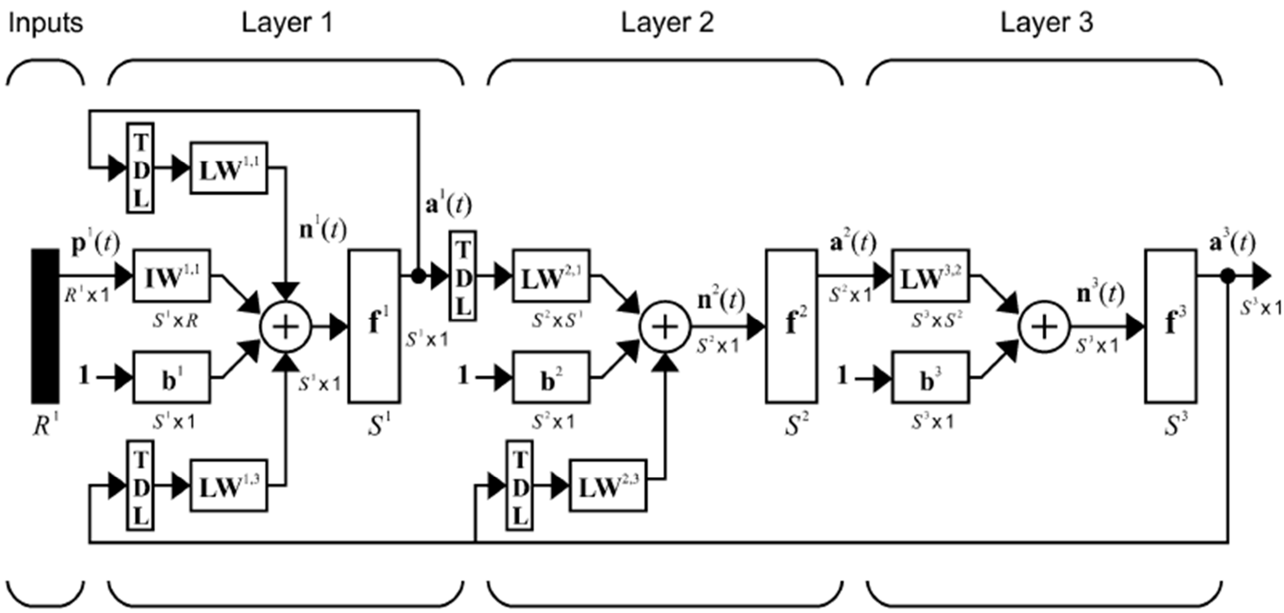

Fig. 3. *The architecture of nonlinear autoregressive network with exogenous inputs (NARX).*

- Set of weight matrices that come in to that layer (which connect from other layers or from external inputs), associated weight function rule used to combine the weight matrix with its input (normally standard multiplication, dotprod) and associated tapped delay line
- Bias vector
- Net input function rule that is used to combine the outputs of the various weight functions with the bias to produce the net input (normally a summing function, netprod)

- Transfer function

The network has inputs that are connected to special weights, called input weights and denoted by $IW_{i,j}$, where j denotes the number of the input vector that enters the weight and i denotes the number of the layer to which the weight is connected. The weights connecting one layer to another are called layer weights and are denoted by $LW_{i,j}$, where j denotes the number of the layer coming in to the weight and i denotes the number of the layer at the output of the weight. This type of network's weights has two different effects on the network output. The first is the direct effect, because a change in the weight causes an immediate change in the output at the current time step (this first effect can be computed using standard back propagation). The second is an indirect effect, because some of the inputs to the layer, such as $(t, 1)$, are also functions of the weights. To account for this indirect effect, the dynamic back propagation must be used to compute the gradients, which are more computationally intensive [25]. Expect dynamic back propagation to take more time to train, in part for this reason. In addition, the error surfaces for dynamic networks can be more complex than those for static networks. Training is more likely to be trapped in local minima. This suggests that that you might need to train the network several times to achieve an optimal result [26].

4. Artificial Neural Network-Based Methodology

The nonlinear autoregressive network with exogenous inputs is used to forecast the closing price returns of the Naira/Dollar exchange rate in the following.

It is assumed that r_t is the closing returns value at the moment of time t. For each time t, there is a vector $X_t = (X_{t(1)}, X_{t(2)}, \dots, X_{t(n)})^T$ whose entries are the values of the indicators significantly correlated to r_t, that is the correlation coefficient between $X_{t(i)}$ and r_t is greater than a certain threshold value, for $i = 1,2, \dots, n$.

The neural model used in this research is a dynamic network. The direct method was used to build the model of prediction of the returns closing value, which is described as follows.

$$\hat{r}_{(t+p)} = f_{ANN}(r_t^d, X_t^d) \tag{7}$$

$$r_t^d = \{r_t, r_{t-1}, r_{t-2}, \dots r_{t-d}\} \tag{8}$$

$$X_t^d = \{X_t, X_{t-1}, X_{t-2}, \dots, X_{t-d}\} \tag{9}$$

Where $\hat{r}_{(t+p)}$ is the forecasted value of the returns for the prediction period p and d is the delay expressing the number of pairs (X_k, r_k), $k = t, t-1, \dots, t-d$ used as inputs of the neural model.

The dynamic neural networks consider the delay has significant influence on the training set and prediction process.

5. Performance Comparison

In addition, four measures are used to evaluate the performance of models in forecasting volatility as follows: mean forecast error (MFE), root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). These measures are defined as:

$$RMSE = \left(n^{-1} \sum_{t=1}^{n} (\sigma_t - \hat{\sigma}_t)^2\right)^{1/2}, \tag{10}$$

$$MAE = n^{-1} \sum_{i=1}^{n} |\sigma_t - \hat{\sigma}_t|, \tag{11}$$

$$MAPE = n^{-1} \sum_{i=1}^{n} \left|\frac{\sigma_t - \hat{\sigma}_t}{\hat{\sigma}_t}\right| * 100, \tag{12}$$

$$MFE = n^{-1} \sum_{i=1}^{n} (\sigma_t - \hat{\sigma}_t), \tag{13}$$

6. Nigerian Naira/US Dollar Exchange Rate Returns Characteristics

This study used Monthly prices of Naira/US Dollar Exchange rate in Nigeria over a period of January 1995 to February, 2016. All the data were gathered from the Central Bank of Nigeria through their website www.cbn.gov.ng. The logarithmic returns of the series were also calculated.

Figure 4 shows the monthly exchange rate price and its logarithmic returns. Informally, it suggests that the exchange is trending or non stationary and it shows that the returns concentrate around zero.

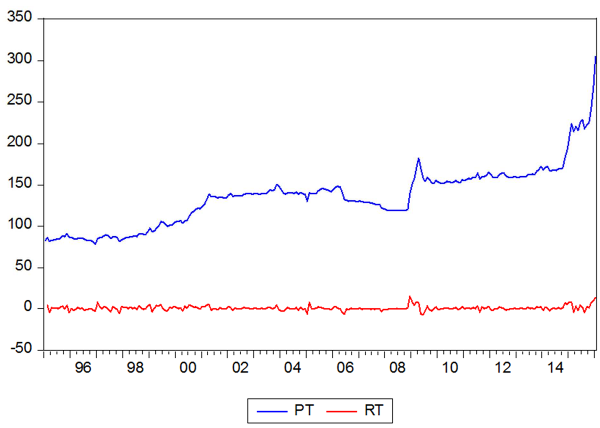

Fig. 4. *Plot of Monthly Exchange rate and its logarithmic returns in Nigeria from 1995-2016.*

Table 1. *Data description and preliminary statistics of the Naira/US Dollar Exchange rate reruns in Nigeria.*

Mean	**0.51885**
Standard Deviation	2.802000
Skewness	1.138862
Kurtosis	9.117109
Jarque-Bera	299.4011 (0.000)*
Observation	253
$Q^2(15)^a$	27.083 (0.022)*
ARCH test (15)[b]	23.453 (0.0012)*

[a] is the Ljung-Box Q test for the 15[th] order serial correlation of the squared returns

[b] Engle's ARCH test also examines for autocorrelation of the squared returns.

There are significant price fluctuations in the markets as suggested by positive standard deviation. The positive skewness indicates that there is high probability of losses in the market. The excess value of kurtosis suggests that the market is volatile with high probability of extreme occurrences. Moreover, the rejection of Jarque-Bera test of normality shows that the returns deviate from normal distribution significantly and exhibit leptokurtic. The Ljung Box statistic for squared return and Engle ARCH test prove the exhibition of ARCH effects in the returns series. Therefore, it is appropriate to apply GARCH, EGARCH and GJR models.

7. Computational Results

Table 2. *GARCH models.*

GARCH models	AIC	BIC	Adjusted R^2	S. E. Regression
GARCH (3, 1)	-7.152931	-7.250971	0.982946	0.047060
EGARCH (1, 2)	-5.3211222	-5.419162	0.981433	0.004911
GJR (1, 1)	-5.666157	-5.750191	0.982706	0.047391

The next step is to use the results of GARCH-type models to developthe proposed models for forecasting volatility of the Exchange rate returns in Nigeria. As the first step, GARCH, GJR-GARCH and EGARCH models with various combinations of (p, q) parameters ranging from (1, 1) to (3, 3) were calibrated on historical return data and best fitted ones were chosen from each group based on some certain performance measures as shown in table 2. It also evident from the table that returns series can be best forecasted by GARCH (3, 1) and it is therefore considered as the preferred model. So the proposed model is henceforth based on the preferred GARCH model.

In this study, two hybrid models are proposed for forecasting conditional volatilities for exchange in Nigeria and in each of these models, a preferred GARCH model is identified upon which the hybrid model is built and used to forecast volatility and this serves as an input and target to the dynamic network. In the first hybrid model, some endogenous variables (Price of the exchange rate, the squared price, returns and squared returns) were used as signals in addition to the actual forecasted volatility from the preferred GARCH. Similarly, the second hybrid model, in addition to these variables also used the simulated synthetic series from the preferred GARCH model as signals to the network. The inputs to first hybrid model consist of the forecasted volatility from GARCH (3, 1) which is an input and the target value, the returns series and its squares, price of the exchange rate and its squares. The first hybrid model has five inputs variables and is therefore depicted in figure 5.

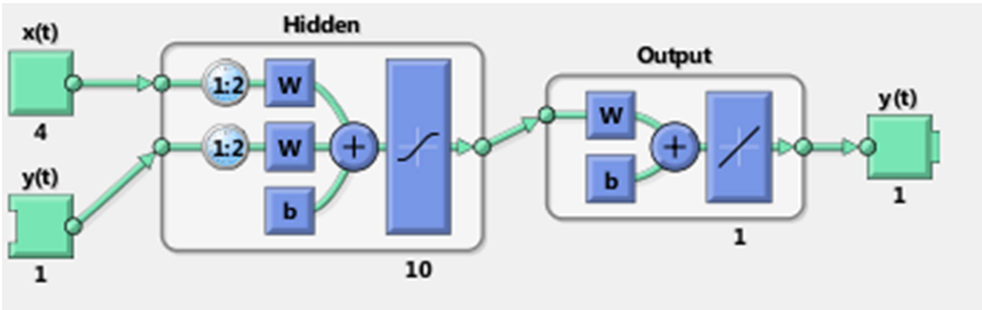

Fig. 5. *Trained Hybrid model I series-parallel network design with five inputs and ten hidden neurons.*

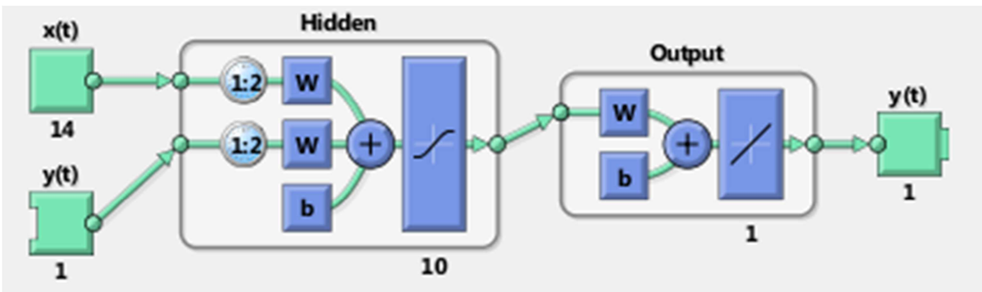

Fig. 6. *Trained Hybrid model II series-parallel network with fourteen inputs and ten hidden neurons.*

In order to keep the properties of the best fitted GARCH model while enhancing it with an ANN model, we will have to somehow introduce the autocorrelation structure of data (captured by GARCH model) to the network. Otherwise, the hybrid model could not recognize the underlying autocorrelation from a single set of estimated time series. We therefore generated ten more synthetic series from estimated preferred GARCH model and add to the previous five inputs of the hybrid model which is depicted in Fig. 6. It is therefore expected that this hybrid model captured more information about the preferred GARCH model.

Table 3. *Neural networks Design for the for Hybrid models.*

Inputs	Hidden neurons	R^2	MSE
5	20	0.99431	0.0587697
15	10	1	0.0011540

In training the hybrid models, the number of hidden neurons was varied and their performances compared in terms of their mean square errors. During the training, in each of these numbers of neurons, the network was trained severally with different weights initialization and update and best ones selected. The result as shown table 3 indicates that the networks trained with 20 hidden neurons and 10 hidden neurons have the smallest minimum square error for hybrid model I and Hybrid model 11 respectively.

Similarly, it is also indicated in figure 7, that with the use of synthetic series in Hybrid model II, the network learnt better to the extent that it perfectly relate the network outputs with target values.

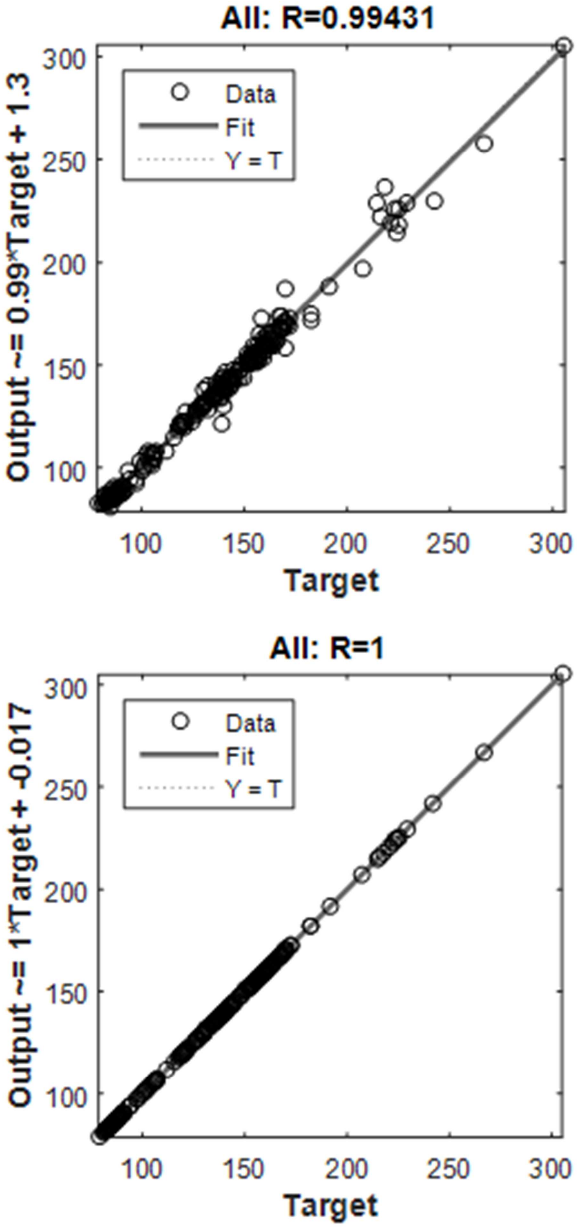

Fig. 7. *The regression coefficient and data fitting for Hybrid model I and II.*

To examine the fitness of these models, each of them has been used to forecast the volatilities for 5 and 10 months ahead and the results reported in table 4 and 5 respectively.

Table 4. *Hybrid models performance to volatility forecasting for 5 months ahead.*

Measures	GARCH (3, 1)	Hybrid I	Hybrid II
RMSE	0.08717	0.072763	0.045804
MAE	0.08606	0.06420	0.02382
MAPE	0.138304	0.100438	0.037381
MFE	0.08606	0.0194	0.01778

Table 5. *Hybrid models performance to volatility forecasting for 10 months ahead.*

Measures	GARCH (3, 1)	Hybrid I	Hybrid II
RMSE	0.064668	0.047273	0.042838
MAE	0.05134	0.03261	0.02876
MAPE	0.082417	0.05368	0.046871
MFE	0.0029	0.00249	0.001084

The computational results show that both hybrid models outperform GARCH model. Hybrid model II demonstrates better ability to forecasting volatility of the real market return with respect to all four fitness measures. That might not be unconnected to the inclusion of simulated series as extra inputs to hybrid model II

8. Conclusion

This paper examines how application of dynamic neural networks can be used to enhance the performance of the existing GARCH models in volatility forecasting of the Nigerian financial market. This is because the conventional GARCH models are generally known to perform better in relatively stable markets and could not capture violent volatilities and fluctuations. Therefore, it is recommended that these models be combined with other models when applied to violent markets [7].

In this paper, Non-linear Auto-Regressive model with exogenous inputs (NARX) was used in the problem of modeling and forecasting volatility of monthly Naira/Dollar exchange rate in Nigeria and in which three types of traditional GARCH models were fitted for forecasting the volatility. The results of these models were evaluated using some performances measures. The results also indicate that GARCH (3, 1) fits the data best. To enhance the forecasting power of the selected model, two hybrid models have been built using Dynamic Neural Networks. The inputs to the proposed hybrid models include the volatility estimates obtained by the fitted GARCH model as well as other endogenous variables. Furthermore, the second hybrid model takes simulated volatility series as extra inputs. Such inputs have been intended to characterize the statistical properties of the volatility series when fed into Dynamic Neural Networks. The computational results on Naira/Dollar Exchange rate demonstrate that the second hybrid model, using simulated volatility series, provides better volatility forecasts. This model significantly improves the forecasts over the ones obtained by the best GARCH model.

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