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Thick shell evolution in Reissner–Nordström spacetime

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Abstract: The new extension formalism, developed by Khakshournia and Mansouri, is used to analyze the dynamics of a general shell of matter with an arbitrary finite thickness immersed in a curved space time. Within this new formulation the equations of motion of a spherically symmetric thick shell immersed in Reissner–Nordström (RN) space time is obtained.

Keywords: General Relativity, Thick Shell, Cosmology, Gravitation

1. Introduction

One of the persisting problems in the context of spherically symmetric gravitational collapse within the framework of general relativity is the occurrence of singularities. These singularities come in two varieties referred to shell crossing and shell focusing singularities. On the other hand, the shell crossing singularities arise from the piling up of concentric matter shells at radii outside the center thus forming a caustic surface. This phenomenon is best known in the case of dust that is described by the Tolman-Bondi metric. Frauendiener and Klein [1] study the simplest conceivable case of shell crossing namely the crossing of (a finite number) so called ”dust shells” in an otherwise empty universe following [2,3], and proved that dust shells cannot be obtained as limiting cases of extended dust regions.

The matching of two arbitrary spacetimes along a given hypersurface plays an important role in general relativity, with a rich plethora of applications, such as the dynamics of thin matter shells [2], construction of cosmological models, [4], collapse of bounded bodies [5], and wormholes [6]. The junction conditions for arbitrary surfaces and the equation for a shell have been well understood since the work of [7-13]. The gravitational collapse of a massive matter cloud within the framework of general relativity was investigated for the first time by the classical works of [14]. Also, Einstein and Straus [15] determined the metric of space time near a star embedded in an expanding universe without a cosmological constant within general relativity.

In the Sen-Lanczos-Darmois-Israel formalism [16], thin shells are regarded as idealized zero thickness objects, with a δ-function singularity in their energy-momentum and Einstein tensors. Eid and Langer [17] found that, the spherical dust N-shell model with an appropriate initial condition imitates the FRW universe very well. Therefore, shell models are available to study the Einstein-Straus vacuole and the Oppenheimer-Snyder model (ball of dust). Eid and Langer [18] found that, the motion of thick shell can be represented by an appropriate thin shells. Also, Comer and Katz [19] studied the thick Einstein shells and their mechanical stability. Widrow [20] used the Einstein-scalar equations for a static thick domain wall with planar symmetry. He then took the zero-thickness limit of his solution and showed that the orthogonal components of the energy-momentum tensor would vanish in that limit. Garfinkle and Gregory [21] presented a modification of the Israel thin shell equations to treat the evolution of thick domain walls in vacuum.

There are a large number of papers in last year’s dealing with different properties of thick walls and branes. Most of these papers use a solution of Einstein-scalar field equation in n-dimensional space time [22]. A completely different method based on the gluing of a thick wall, considered as a regular manifold, to two different manifolds on both sides of it, was suggested in [23]. The idea behind this suggestion is to understand the dynamics of a localized matter distribution of any kind confined within two principally different space times or matter phases. Such a matching of three different manifolds appears to have many applications in astrophysics, early universe, and string cosmology.

Our approach is similar to that used by MK [24]. A similar approach has been used in [19] to the special case of a spherically symmetric thick shell along with an application to a more restricted junction condition in a two-fluid model of oscillations of a neutron star [25]. The aim of this paper is to generalize their formalism and give the basic dynamical equations governing a thick shell immersed in RN.

The paper is organized as follows. In section 2 I give a brief introduction to MK junction condition formalism yielding a generic equation to study the evolution of thick shells in curved space time. In section 3, apply this formalism to obtain the equations of motion of a spherical thick shell in RN. A general conclusion is given in Section 4.

2. The Junction Formalism

The two boundary limits of the thick shell are called Σ_j with j =1,2. The core of the thick shell is denoted by $\Sigma_\circ$. For any quantity F let F_0 denote $F|_{\Sigma_\circ}$. Square bracket [F] indicates the jump of any quantity F across Σ_j. Greek indices refer to 4-dimensional indices, Latin letters refer to 3-dimensional indices on Σ. The geometrical units, G = c = 1 are used throughout the paper. Consider a thick shell with two boundaries (hypersurfaces) Σ_1 and Σ_2 dividing the space-time manifold M into three regions: two of them outside the shell, M_- and M_+, while M_s within the thick shell itself such that $\partial M_- \cap \partial M_s = \Sigma_1$ and $\partial M_s \cap \partial M_+ = \Sigma_2$. The suffix '+' denotes a quantity evaluated just outside the shell and '-' just inside the shell.

According to Darmois conditions, the two surface boundaries Σ_1 and Σ_2, which separate the manifold M_s to two distinct manifolds M_- and M_+ respectively, is a nonsingular timelike hypersurfaces, which is equivalent to the continuity of the intrinsic metric h_{ab} and the extrinsic curvature tensor K_{ab} of Σ_j across the corresponding hypersurfaces. Therefore,

$$[h_{ab}]^{\Sigma_j} = 0, \qquad (j=1,2)$$

$$[K_{ab}]^{\Sigma_j} = 0 \qquad (j=1,2)$$

Both conditions should be satisfied if Σ is a boundary surface. But in case of a thin shell, both conditions are not satisfied. In fact, the matter content of the shell should lead to a jump in the extrinsic curvature $[K_{ab}] \neq 0$. Now, the jump of the extrinsic curvature tensor K_{ab} on Σ_1 and Σ_2, is

$$K_{ab}\Big|^{+}_{\Sigma_2} - K_{ab}\Big|^{-}_{\Sigma_1} + K_{ab}\Big|^{s}_{\Sigma_1} - K_{ab}\Big|^{s}_{\Sigma_2} = 0 \tag{1}$$

where the superscripts - (+) and s mean that the extrinsic curvature tensor K_{ab} of Σ_1 (Σ_2) is evaluated in the regions M_- (M_+) and M_s, respectively. Introduce a Gaussian normal coordinate system (n, ξ_0^a) in the neighborhood of the core of the thick shell denoted by $\Sigma_\circ$, corresponding to $n = 0, \xi_0^a$ are the intrinsic coordinates of $\Sigma_\circ$, and n is the proper length along the geodesics orthogonal to $\Sigma_\circ$. Let us now expand the extrinsic curvature tensors in a Taylor series around $\Sigma_\circ$ situated at n =0:

$$K_{ab}\Big|_{\Sigma_j} = K_{ab}\Big|_{\Sigma_\circ} + \varepsilon_j \delta \frac{\partial K_{ab}}{\partial n}\Big|_{\Sigma_\circ} + O(\delta^2) \tag{2}$$

where 2δ denotes the proper thickness of the shell, and $\varepsilon_1 = -1, \varepsilon_1 = +1$. The above expansion is justified if the proper thickness of the shell is small relative to the radius of curvature. The derivative of the extrinsic curvature is now given by

$$\frac{\partial K_{ab}}{\partial n} = K_{ac}K^c_b - R_{\mu\sigma\nu\lambda} e^\mu_a e^\nu_b n^\sigma n^\lambda \tag{3}$$

where n^μ is the normal vector field, and $e^\mu_a = \partial x^\mu / \partial \xi^a_0$ are the three basis vectors tangent to $\Sigma_\circ$. Substituting (2, 3) into (1) to get:

$$K_{ab}\Big|^{+}_{\Sigma_\circ} - K_{ab}\Big|^{-}_{\Sigma_\circ} + \delta\Big((K_{ac}K^c_b - R_{\mu\sigma\nu\lambda}e^\mu_a e^\nu_b n^\sigma n^\lambda)\Big|^{-}_{\Sigma_\circ} + (K_{ac}K^c_b - R_{\mu\sigma\nu\lambda}e^\mu_a e^\nu_b n^\sigma n^\lambda)\Big|^{+}_{\Sigma_\circ} - 2(K_{ac}K^c_b - R_{\mu\sigma\nu\lambda}e^\mu_a e^\nu_b n^\sigma n^\lambda)\Big|^{s}_{\Sigma_\circ}\Big) = 0 \tag{4}$$

This is the basic equation for the dynamics of a thick shell in a curved space time, written up to the first order in δ. Equation (4), or the corresponding exact one (1), is to be considered as the generalization of the Israel's thin shell condition [7] to the thick case.

Then, apply this formalism to particular example of a spherically symmetric thick shell in RN, in which, δ is independent of angular coordinates. In general δ may be a function of time, but for simplicity here, take it to be constant in time.

3. Motion of a Spherical Thick Shell in RN

Now, derive the basic exact equations underlying the dynamics and expand it in powers of the thickness, and then calculate the peculiar velocity and its limit on the collapse of the shell following by a comparison to the thin shell limit already known.

3.1. The Thick Shell Solution

Consider a spherically symmetric thick shell immersed in Reissner–Nordström (RN). The space time exterior to the shell is RN, and the interior is taken to be the Minkowski flat spacetime:

$$ds_O^2 = -f dt_+^2 + f^{-1} dr_+^2 + r_+^2 d\Omega^2, \tag{5}$$

$$ds_i^2 = -dt_-^2 + dr_-^2 + r_-^2 d\Omega^2, \tag{6}$$

where $f = 1 - \frac{2m}{r_+} + \frac{e^2}{r_+^2}$, m being the mass of the thick shell, e is the charge and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$, is the standard metric on the unit sphere. In the synchronous comoving coordinates (t, r, θ, φ), the Lemaitre- Tolman-Bondi (LTB) metric, within the shell takes the following form:

$$ds_s{}^2 = -dt^2 + \frac{R'^2}{1+2E(r)}dr^2 + R^2(r,t)d\Omega^2, \tag{7}$$

where the prime denotes the differentiation with respect to r and $E(r)$ is an arbitrary real function such that $E(r) \succ -(1/2)$ and $R' \succ 0$ (because of the no shell crossing condition). The corresponding Einstein field equations, turn out to be,

$$R_{,t}^2(t,r) = 2E(r) + \frac{2M(r)}{R}, \quad \rho(t,r) = \frac{M'(r)}{R^2 R'}$$

where $\rho(t,r)$ is the energy density of the dust fluid in $M(r)$, and M_s is another arbitrary real smooth function interpreted as the mass. The angular component of (4) is given by:

$$K_{\theta\theta}\Big|_{\Sigma_\circ}^{O} - K_{\theta\theta}\Big|_{\Sigma_\circ}^{i} + \delta\Big(K_{\theta\theta}K_\theta^\theta\Big|_{\Sigma_\circ}^{i} + (K_{\theta\theta}K_\theta^\theta - R_{\theta\sigma\theta\lambda}n^\sigma n^\lambda)\Big|_{\Sigma_\circ}^{O} - 2(K_{\theta\theta}K_\theta^\theta - R_{\theta\sigma\theta\lambda}n^\sigma n^\lambda)\Big|_{\Sigma_\circ}^{s}\Big) = 0 \tag{8}$$

The corresponding time component of (4) is

$$K_{\tau\tau}\Big|_{\Sigma_\circ}^{O} - K_{\tau\tau}\Big|_{\Sigma_\circ}^{i} + \delta\Big(K_{\tau\tau}K_\tau^\tau\Big|_{\Sigma_\circ}^{i} + (K_{\tau\tau}K_\tau^\tau - R_{\mu\sigma\nu\lambda}u^\mu u^\nu n^\sigma n^\lambda)\Big|_{\Sigma_\circ}^{O} - 2(K_{\tau\tau}K_\tau^\tau - R_{\mu\sigma\nu\lambda}u^\mu u^\nu n^\sigma n^\lambda)\Big|_{\Sigma_\circ}^{s}\Big) = 0 \tag{9}$$

where u^μ is the four velocity tangent to $\Sigma_\circ$. The four velocity u^μ and the normal vector n^μ on the spherical thick shell's core $\Sigma_\circ$ for the RN and LTB space times, respectively, are

$$u^\mu\Big|_{\Sigma_\circ}^{O} = (\dot{t}_+, \dot{R}, 0, 0)\Big|_{\Sigma_\circ}, \quad n^\mu\Big|_{\Sigma_\circ}^{O} = (f^{-1}\dot{R}, f\dot{t}_+, 0, 0)\Big|_{\Sigma_\circ}, \tag{10}$$

$$u^\mu\Big|_{\Sigma_\circ}^{s} = (\dot{t}, \dot{r}, 0, 0)\Big|_{\Sigma_\circ}, \quad n^\mu\Big|_{\Sigma_\circ}^{s} = (v_\circ \dot{t}, \frac{\sqrt{1+2E}}{R'}\dot{t}, 0, 0)\Big|_{\Sigma_\circ}, \tag{11}$$

where the dot denotes the derivative with respect to the proper time $\tau_\circ$ on $\Sigma_\circ$, and $v_\circ$ is the peculiar velocity of $\Sigma_\circ$ relative to the LTB geometry defined as

$$v_\circ = \frac{R'}{\sqrt{1+2E(r)}}\frac{dr}{dt}\Big|_{\Sigma_\circ}, \tag{12}$$

and is related to the Lorentz factor in the LTB geometry as

$$\dot{t}\Big|_{\Sigma_\circ} = \frac{1}{\sqrt{1-v_\circ^2}}. \tag{13}$$

Note that, the peculiar velocity is valid for a thin shell, in which $dr/dt = 0$. Relevant components of the Riemannian curvature tensor for the RN space time are

$$R_{\theta r r\theta} = \frac{1}{f}\Big(\frac{m}{r_+} - \frac{e^2}{r_+^2}\Big), \quad R_{trrt} = \frac{2m}{r_+^3} - \frac{3e^2}{r_+^4}, \quad R_{\theta t t\theta} = f\Big(-\frac{m}{r_+} + \frac{e^2}{r_+^2}\Big) \tag{14}$$

and for the LTB spacetime are

$$R_{\theta r r\theta} = \frac{-RR'}{(1+2E)}(R_{,t}R'_{,t} - E'),$$

$$R_{trrt} = \frac{R'}{(1+2E)}R'_{,tt}, \quad R_{\theta t t\theta} = RR_{,tt} \tag{15}$$

The components of the extrinsic curvature tensor on $\Sigma_\circ$ evaluated with respect to the relevant regions are given by

$$K_\theta^\theta\Big|_{\Sigma_\circ}^{i} = \frac{1}{R_\circ}\sqrt{1+\dot{R}_\circ^2},$$

$$K_\theta^\theta\Big|_{\Sigma_\circ}^{s} = \frac{1}{R_\circ}\sqrt{1+\dot{R}_\circ^2 - \frac{2M_\circ}{R_\circ}},$$

$$K_\theta^\theta\Big|_{\Sigma_\circ}^{O} = \frac{1}{R_\circ}\sqrt{f+\dot{R}_\circ^2}, \tag{16}$$

and

$$K_\tau^\tau\Big|_{\Sigma_\circ}^{i} = \frac{\ddot{R}_\circ}{\sqrt{1+\dot{R}_\circ^2}},$$

$$K_\tau^\tau\Big|_{\Sigma_\circ}^{s} = \frac{\ddot{R}_\circ + \frac{M_\circ}{R_\circ^2} + R_\circ\frac{\rho_\circ v_\circ^2}{1-v_\circ^2}}{\sqrt{1+\dot{R}_\circ^2 - \frac{2M_\circ}{R_\circ}}},$$

$$K_\tau^\tau\Big|_{\Sigma_\circ}^{O} = \frac{\ddot{R}_\circ + \frac{1}{2}f'}{\sqrt{f+\dot{R}_\circ^2}}. \tag{17}$$

Substituting into (8) and (9), after some manipulations, two independent equations written up to the first order of $\delta/R_\circ$ are:

$$\alpha - \beta = \frac{8\pi\delta\rho_\circ R_\circ}{1-v_\circ^2} - \delta\Big(\frac{m-2M_\circ}{R_\circ^2} + \frac{e^2}{R_\circ^3}\Big), \tag{18}$$

and

$$
\begin{aligned}
(\alpha-\beta)\ddot{R}_\circ = & -\alpha\left(\frac{m}{R_\circ^2}-\frac{e^2}{R_\circ^3}\right)-\alpha\beta\delta\Bigg[\frac{\ddot{R}_\circ^2}{1+\dot{R}_\circ^2}+\frac{(\ddot{R}_\circ+\frac{m}{R_\circ^2}-\frac{e^2}{R_\circ^3})^2}{f+\dot{R}_\circ^2} \\
& -\frac{2(\ddot{R}_\circ+\frac{M_\circ}{R_\circ^2}+4\pi R_\circ\frac{\rho_\circ v_\circ^2}{1-v_\circ^2})^2}{1+\dot{R}_\circ^2-\frac{2M_\circ}{R_\circ}}-\frac{2(m-2M_\circ)}{R_\circ^3}+\frac{3e^2}{R_\circ^4}-8\pi\rho_\circ\Bigg],
\end{aligned}
\tag{19}
$$

where,

$$
\alpha=\sqrt{1+\dot{R}_0^2}, \quad \beta=\sqrt{f+\dot{R}_0^2}\ . \tag{20}
$$

The angular component of the extrinsic curvature of $\Sigma_\circ$ in the Gaussian normal coordinate, using (11), is

$$
K_\theta^\theta=\frac{1}{R}\left(\frac{\partial R}{\partial t}\frac{\partial t}{\partial n}+\frac{\partial R}{\partial r}\frac{\partial r}{\partial n}\right)\Big|_{\Sigma_\circ}=\frac{\dot{t}}{R}\left(vR_{,t}+\sqrt{1+2E}\right)\Big|_{\Sigma_\circ}, \tag{21}
$$

On the other hand one can write

$$
\dot{R}=\dot{t}\left(R_{,t}+v\sqrt{1+2E}\right)\Big|_{\Sigma_\circ}. \tag{22}
$$

Eliminating $\dot{t}$ from (21) and (22) and using (16), to get:

$$
v_\circ=\frac{\beta\sqrt{2E_\circ+\frac{2M_\circ}{R_0}}-\dot{R}_\circ\sqrt{1+2E_\circ}}{\dot{R}_\circ\sqrt{2E_\circ+\frac{2M_\circ}{R_0}}-\beta\sqrt{1+2E_\circ}}, \tag{23}
$$

which is valid only for the shells of finite thickness. Equations (18) and (19) together with (23) are the thick shell equations of motion written up to the first order in terms of the shell's proper thickness. Given the initial data: $R_\circ(0), \dot{R}_\circ(0), E_\circ, M_\circ$ and δ, one can in principle solve this set of differential equations to determine the time evolutions of the proper radius $R_\circ(\tau_\circ)$ and the mass density $\rho_\circ$ of the thick shell's core.

3.2. Thin Shell Limit of a Thick Shell

The limit of the proper thickness as well as the peculiar velocity on the dynamics of the shell can be investigated from (18). The relation between the surface energy density of the thin shell and the energy density of the thick shell can be evaluated on $\Sigma_\circ$ up to the first order in δ:

$$
\sigma=\int_{-\delta}^{+\delta}\rho(r,\tau)\,dn\cong 2\delta\rho_\circ+O(\delta^2). \tag{24}
$$

Now, assuming $v_\circ \prec 1$, then equation (18) can be written as:

$$
\alpha-\beta=4\pi G R_0\tilde{\sigma} \tag{25}
$$

where $\tilde{\sigma}$ is called the effective surface density, defined by

$$
\tilde{\sigma}=\sigma\left(1+v_\circ^2-\delta\left(\frac{m-2M_\circ}{4\pi\sigma R_\circ^3}+\frac{e^2}{4\pi\sigma R_\circ^4}\right)\right). \tag{26}
$$

Therefore, equation (25) has the well-known form of the dynamics of a spherical thin shell with the effective surface energy density $\tilde{\sigma}$ (Israel 1966a). The zero thickness limit of the shell is therefore defined by $\delta\to 0$ and $v_\circ\to 0$. Taking this limit, (25) reduces to the familiar equation of motion for the thin shell in RN [7]. Solving (25, 18) for $\dot{R}^2$, to get

$$
\begin{aligned}
\dot{R}_\circ^2 = & -1+\frac{m}{R_\circ}+4\pi^2\tilde{\sigma}^2R_\circ^2+\frac{m^2}{16\pi^2\tilde{\sigma}^2R_\circ^4}+ \\
& \frac{e^2}{2R_\circ^2}\left(-1-\frac{m}{8\pi^2\tilde{\sigma}^2R_\circ^3}+\frac{e^2}{32\pi^2\tilde{\sigma}^2R_\circ^4}\right)
\end{aligned}
\tag{27}
$$

Therefore, $\dot{R}_\circ^2$ depend on $\tilde{\sigma}$ for a given shell radius $R_\circ$. In the limit that the charge goes to zero, one recovers the Schwarzschild metric. In the limit that 2m/r and e go to zero, the metric becomes the Minkowski metric. Taking into account $R_\circ \succ 2m$, which means roughly that the radius of the shell's core is greater than its Schwarzschild radius, for the zeros of $\dot{R}_\circ^2$:

$$
\tilde{\sigma}_\pm^2=\frac{1}{8\pi^2R_\circ^2}\left(1-\frac{m}{R_\circ}+\frac{e^2}{2R_\circ^2}\pm\sqrt{1-\frac{2m}{R_\circ}+\frac{e^2}{R_\circ^2}}\right) \tag{28}
$$

For a comoving $\Sigma_\circ$ ($v_\circ=0$), having $\tilde{\sigma}\prec\sigma$ (where $m\succ 2M_\circ$), the thickness in the first order leads to a faster collapse of the thick shell relative to a thin shell in RN. While for a $\Sigma_\circ$ with a small peculiar velocity with respect to the LTB background, the effective energy density $\tilde{\sigma}$ tends to increase leading to a slowdown of the collapse velocity of the shell relative to a comoving one.

4. Conclusion

The dynamics of a thick shell embedded in curved space times has been studied by imposing the Darmois junction conditions. The equation of motion for the shell, up to the first order of the proper thickness is obtained. In fact the equations of motion could be written in a form similar to the thin shell cases with an effective surface density. In the case of a shell immersed in RN, it turns out that the effect of the proper thickness δ is to speed up the collapse of the comoving spherical thick shell, while the first order peculiar velocity correction tends to decrease the collapse velocity.

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