

# A role of the conservation laws in evolutionary processes and generation of physical structures

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**Abstract:** It is well known that the equations of conservation laws for energy, linear momentum, angular momentum, and mass are the equations of mechanics and physics of continuous media that describe material systems such as the thermodynamical, gas-dynamical and cosmological systems. And the field-theory equations, which are used for description of physical fields, are based on the conservation laws that one commonly relates with conservative quantities or objects. It is shown that to conservation laws for physical fields are assigned the closed exterior forms, which follow from the equations of conservation laws for material systems. The process of realization such closed exterior form describes the occurrence of observable formations in material systems (such as waves) and the generation of physical structures, the examples of which are physical structures that form physical fields.

**Keywords:** Two Types of Conservation Laws, the Equations of Material Systems, Evolutionary Relation, Skew-Symmetric Forms, the Field-Theory Equation

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## 1. Introduction

In the process of science development the concept of "conservation laws" in thermodynamics, physics and mechanics assumed different sense.

In thermodynamics the conservation laws are associated with the principles of thermodynamics. The first principle of thermodynamics that relates to the energy conservation law, can be written as [1]

$$de + dw = \delta q \quad (1)$$

where  $de$  is the energy variation,  $dw$  is the work done by the system,  $\delta q$  is the heat delivered to the system. (Here the ambiguity is also observed; this relation includes not only energy, but the mechanical work as well. This ambiguity of the first thermodynamical principle and the peculiarity of the second thermodynamical principle will be explained below.)

In mechanics and physics of continuous media the concept of "conservation laws" relates to the conservation laws for energy, linear momentum, angular momentum, and mass that establish the balance between the variations of physical quantities and external actions. These are conservation laws that are described by differential equations.

(From here on they will be referred to as the balance conservation laws.)

In areas of physics related to the field theory and in the theoretical mechanics "the conservation laws" are those according to which the conservative physical quantities or objects exist. These are conservation laws that are described by closed exterior form. (Below they will be referred to as "exact" ones.) The Noether theorems can serve as an example of such conservation laws formulation, which, under some conditions, can be written as

$$d\omega = 0 \quad (2)$$

(It is of interest to call attention to the fact that formulae (1, 2) have the form of relations in skew-symmetric differential forms).

Thus, the concept of "the conservation laws" is connected with exact conservation laws, balance conservation laws and the principles of thermodynamics.

It turns out that the exact conservation laws are connected with the balance conservation laws. Such a connection is at the basis of evolutionary processes.

The role in evolutionary processes of the balance conservation laws is demonstrated in section 1 on the basis of the analysis of the balance conservation law equations. From the balance conservation law equations it follows the evolu-

tionary relation for functionals that specify the material system state (such as the wave function, entropy, action functional and so on). Such an evolutionary relation possesses the properties that enable one to disclose the mechanism of evolutionary processes.

Firstly, the evolutionary relation proves to be *nonidentical*. The nonidentity is due to the inconsistency of the conservation law equations, and this is a consequence of the noncommutativity of the conservation laws.

It is impossible to obtain the state functional from the nonidentical relation. This means that the material system state is nonequilibrium.

Furthermore, the evolutionary relation proves to be a selfvarying one. Such a property of the evolutionary relation points to the fact that the nonequilibrium state of the system can vary (under the action of internal force).

Below it will be shown that the evolutionary relation connects the state functional and the evolutionary skew-symmetric form, which depends on external actions and the characteristics of material system. From the properties of skew-symmetric forms it follows that, under *degenerate* transformation, the closed exterior form can be realized from the evolutionary skew-symmetric form (which is unclosed).

In this case from the evolutionary relation it can be obtained the identical relation from which one can get the state functional, and this will point out to the transition of material system into the locally-equilibrium state. Such a transition is accompanied by an occurrence of some observable formations such as waves, turbulent pulsations and so on.

The closed exterior form obtained is defined only on a certain structure. On this structure the exact conservation law is satisfied, since the closed exterior form (with the differential being equal to zero) describes a conservative quantity. As it will be shown in the next section, the physical structures, which made up physical fields, are just such structures.

In section 2 it is shown that the connection the exact conservation laws with the closed exterior forms discloses the foundations of the field-theory equations and their connection with the equations of conservation law for material systems.

The results of present paper have been obtained due to using the skew-symmetric differential forms. Beside the exterior forms [2], the skew-symmetric forms, which are derived from differential equations, are used [3, 4]. Mathematical apparatus of such differential forms, which possess the evolutionary properties, includes nontraditional elements like nonidentical relations and degenerate transformations and this enables one to describe evolutionary processes and generation of various structures.

## 2. Properties and Peculiarities of Balance Conservation Laws

The balance conservation laws are conservation laws of

energy, linear momentum, angular momentum, and mass. They establish the balance between the variation of a physical quantity and the corresponding external action. These are the conservation laws for material systems (material media). [The material system is a variety of elements which have internal structure and interact to one another. Examples of elements that made up a material system are fluid particles, cosmic objects, electrons, protons, atoms and others. As examples of material systems it may be the thermodynamical, gas dynamical, cosmic systems, systems of elementary particles and others.]

The balance conservation laws are described by differential (or integral) equations [5, 6]. (The Euler and Navier-Stokes equations for gas-dynamical system are examples of such a set of equations [5].)

### 2.1. Analysis of the Equations of the Balance Conservation Laws

In mechanics and physics of material systems the equations of balance conservation laws are used for description of physical quantities, which specify the behavior of material systems. But the balance conservation laws not only define the variation of physical quantities. Their role is much wider. They, as it will be shown, control evolutionary processes in material systems that are accompanied by the origin of physical structures.

It appears that, even without knowledge of concrete form of these equations, with the help of skew-symmetric differential forms one can see the specific features of these equations that elucidate the properties of balance conservation laws and their role in evolutionary processes.

The solutions to equations of material system sought are usually functions which relate to such physical quantities like the particle velocity (of elements), temperature or energy, pressure and density. Since these functions relate to one material system, it has to exist a connection between them. This connection is described by state functional that specifies the material system state. The action functional, entropy, the Poincaré vector, Einstein tensor, wave function and others can be regarded as examples of such functional [7].

In the accompanying frame of reference, which is tied to the manifold made up by the trajectories of particles (elements of material system), the balance conservation law equations for the unknown functions convert into the equations for state functionals. The study of such equations just allow to disclose the properties and specific features of conservation laws.

#### 2.1.1 Evolutionary Relation

The functional properties and specific features of differential equations or sets of equations depend on whether or not the derivatives of differential equations or the equations in the sets of differential equations are consistent.

The equations are consistent if they can be contracted into identical relations for the differentials, i.e. for closed forms.

Let us now analyze the consistency of the equations that describe the conservation laws for energy and linear momentum.

In the accompanying frame of reference, the equation for energy is written in the form

$$\frac{\partial \Psi}{\partial \xi^1} = A_1 \quad (3)$$

Here  $\xi^1$  are the coordinates along the trajectory,  $\Psi$  is the functional of the state,  $A_1$  is the quantity that depends on specific features of the material system and external (with respect to the local domain) energy actions onto the system [5, 7]. [Thus, accounting for the fact that the total derivative with respect to time is that along the trajectory, the energy equation expressed in terms of the action functional  $S$  has following form:  $DS/Dt = L$ , where  $\Psi = S$  and  $A_1 = L$  is the Lagrange function. The energy equation for ideal gas can be presented in the form:  $Ds/Dt = 0$ , where  $s$  is entropy [5].]

Similarly, in the accompanying frame of reference the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial \Psi}{\partial \xi^v} = A_v, \quad v = 2, \dots \quad (4)$$

where  $\xi^v$  are the coordinates along the direction normal to the trajectory,  $A_v$  are the quantities that depend on the specific features of material system and external force actions.

Equations (3) and (4) can be convoluted into the relation

$$d\Psi = A_\mu d\xi^\mu, \quad \mu = 1, v \quad (5)$$

Relation (5) can be written as

$$d\Psi = \omega \quad (6)$$

Here  $\omega = A_\mu d\xi^\mu$  is a skew-symmetric differential form of the first degree.

Since the balance conservation laws are evolutionary ones, skew-symmetric differential form  $\omega$  and the relation obtained is also an evolutionary relation.

Relation (6) was obtained from the balance conservation law equations for energy and linear momentum. In this relation the form  $\omega$  is that of the first degree. If the balance conservation law equation for angular momentum be added to the equations for energy and linear momentum, this form in the evolutionary relation will be a form of the second degree. And in combination with the equation of the balance conservation law for mass this form will be a form of degree 3.

Thus, in the general case, the evolutionary relation can be written as

$$d\Psi = \omega^p \quad (7)$$

where the form degree  $P$  takes the values  $p = 0, 1, 2, 3$ . (The evolutionary relation for  $p = 0$  is similar to that in the differential forms, and it was obtained from the equations of energy and time.)

[In the case of the Euler and Navier-Stokes equations a concrete form of relation (6) and its properties were considered in papers [8, 9]. In this case the functional is the entropy  $s$ . In paper [3] (see, Appendix 3) and in [10] relations (7) when  $p = 2$  were considered for electromagnetic field.]

### 2.1.2. Nonidentity of Evolutionary Relation. Inconsistency of the Balance Conservation Law Equations

Evolutionary relation obtained from the equation of the balance conservation laws possesses some peculiarity. This relation proves to be nonidentical since the skew-symmetric differential form in the right-hand side of this relation is not a closed form, and, hence, this form can not be a differential like the left-hand side.

Let us analyse the relation (6).

The form  $\omega = A_\mu d\xi^\mu$  is not a close form since its differential is nonzero. The differential  $d\omega$  can be written as  $d\omega = K_{\alpha\beta} d\xi^\alpha d\xi^\beta$ , where  $K_{\alpha\beta} = \partial A_\beta / \partial \xi^\alpha - \partial A_\alpha / \partial \xi^\beta$  are the components of the differential form commutator built of the mixed derivatives (here the term connected with the nonintegrability of the manifold has not yet been taken into account). The coefficients  $A_\mu$  of the form  $\omega$  can be obtained from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator of the form  $\omega$  constructed from the derivatives of such coefficients is nonzero. The differential of the form  $\omega$  is nonzero as well. Thus, the form  $\omega$  proves to be unclosed and cannot be a differential. This means that the evolutionary relation cannot be an identical one. In the left-hand side of this relation it stands a differential, whereas in the right-hand side it stands an unclosed form that is not a differential. (Nonidentical relation was analyzed in paper J.L.Synge "Tensorial Methods in Dynamics" (1936). And yet it was allowed a possibility to use the sign of equality in nonidentical relation.) [The skew-symmetric form in evolutionary relation is defined on the manifold made up by trajectories of the material system elements. Such a manifold is a deforming manifold. The commutator of skew-symmetric form defined on such manifold includes an additional term connected with the differential of basis. This term specifies the manifold deformation and hence is nonzero. Both terms of the commutator (obtained by differentiating the basis and the form coefficients) have a different nature and, therefore, cannot compensate one another. This fact once more emphasize that the evolutionary form commutator, and, hence, its differential, are nonzero. That is,

the evolutionary form remains to be unclosed.]

Hence, without the knowledge of a particular expression for the form  $\Omega$ , one can argue that for actual processes the evolutionary relation proves to be nonidentical.

The nonidentity of the evolutionary relation means that the balance conservation law equations turn out to be inconsistent (thus, if from the energy equation one obtains the derivative of  $\Psi$  in the direction along the trajectory and from the momentum equation he finds the derivative of  $\Psi$  in the direction normal to the trajectory and next he calculate their mixed derivatives, from the condition that the commutator of the form  $\Omega$  is nonzero it follows that the mixed derivatives appear to be noncommutative).

[The first principle of thermodynamics is the example of nonidentical evolutionary relation (see, (1)). On the one hand, this relation involves energy and thus relates to the energy conservation law. And on the other hand, it involves the force components and thus is connected with the conservation law for linear momentum. This relation, as well as the evolutionary relation, combines two conservation laws: the balance law of energy conservation and that of linear momentum conservation. As well as evolutionary relation, this relation is nonidentical.]

**2.2. Noncommutativity of the Balance Conservation Laws. Nonequilibrium State of Material System**

Inconsistency of the balance conservation law equations points to the fact that the balance conservation laws are noncommutative [9].

Noncommutativity of the balance conservation laws reflects the state of material system.

Since the evolutionary relation is nonidentical, then from this relation one cannot get the differential of the state functional  $d\Psi$ . This means that the functional  $\Psi$  is not a state function. And this points to the fact that the material system is in nonequilibrium state. It is evident that the internal force producing such nonequilibrium state is described by the evolutionary form commutator. (If the evolutionary form commutator be zero, the evolutionary relation would be identical, and this would point to the equilibrium state, i.e. the absence of internal forces.) Everything that makes contribution to the commutator of the form  $\omega^p$  leads to emergence of internal force.

Nonidentical evolutionary relation also describes how the state of material system changes. This is due to that the evolutionary nonidentical relation is a selfvarying one. [This relation includes two objects one of which appears to be unmeasurable. The variation of any object of the relation in any process leads to the variation of another object and, in turn, the variation of the latter leads to the variation of the former. Since one of the objects is an unmeasurable quantity, the other cannot be compared with the first one, and hence, the process of mutual variation cannot terminate. This process is governed by the evolutionary form commutator, that is, by interaction between the commutator made up by derivatives of the form itself and by metric form commutator of deforming manifold made up by trajectories

of elements of material system.]

The process of selfvariation of the evolutionary relation points to a change of the material system state. But the material system state remains nonequilibrium in this process because the internal forces do not vanish due to the evolutionary form commutator remains to be nonzero.

Such selfvariation of the material system state proceeds under the action of internal (rather than external) forces. That will go on even in the absence of external forces. Here it should be noted that in a real physical process the internal forces can be increasing, and this can lead to development of instability in the material system. [For thermodynamic system this fact was firstly pointed out by Prigogine [11, 12]. "The excess entropy" in his papers is analogous to the commutator of the nonintegrable form for thermodynamic system. "Production of excess entropy" leads to the development of instability.]

**2.3. Transition of the Material System into a Locally - Equilibrium State. Origination of Observable Formations**

During selfvariation of evolutionary relation the conditions when an inexact (closed on pseudostructure) exterior form is obtained from evolutionary form can be realized. This leads to the fact that from nonidentical evolutionary relation it will be obtained an identical relation, from which one can get the state functional, and this will point to the transition of material system from nonequilibrium state to locally-equilibrium state.

Here there is a certain specific point. The transition from unclosed evolutionary form (with nonzero differential) to closed exterior form (with vanishing differential) is possible only as degenerate transformation, namely, a transformation that does not conserve the differential.

The degenerate transformation can take place under additional conditions.

The conditions of degenerate transformation are closure conditions of dual form that define the integral structure (the pseudostructure), on which the closed exterior form realizes.

[The conditions of degenerate transformation are reduced to vanishing of such functional expressions as determinants, Jacobians, Poisson's brackets, residues and others. They are connected with the symmetries, which can be due to the degrees of freedom (for example, the translational, rotational and oscillatory degrees of freedom of material system).]

If the conditions of degenerate transformation are realized, it will take place the transition

$$d\omega^p \neq 0 \rightarrow d_\pi \omega^p = 0, d_\pi^* \omega^p = 0$$

The relations obtained

$$d_\pi \omega^p = 0, d_\pi^* \omega^p = 0 \tag{8}$$

are closure conditions for exterior inexact form and for dual form. The dual form is a metric form of manifold. The closed dual form describes the pseudostructure  $\pi$ , on which closed inexact (only on pseudostructure) exterior form is defined. (Integral structures and manifolds, such as the characteristics, potential surfaces, eikonal surfaces, singular points, are examples of pseudostructures and relevant manifolds.)

[The degenerate transformation is realized as the transition from nonintegrable manifold, made up by the trajectories of the material system elements (on which the unclosed evolutionary form is defined), to the integral structures and surfaces.]

The realization of the closure conditions (8) points to the fact that the exterior form closed on pseudostructure is realized.

In this case, on the pseudostructure  $\pi$  evolutionary relation (7) converts into the relation

$$d_{\pi} \psi = \omega_{\pi}^p \quad (9)$$

which proves to be an identical relation. Since the form  $\omega_{\pi}^p$  is a closed one, on the pseudostructure this form turns out to be a differential. There are differentials in the left-hand and right-hand sides of this relation. This means that the relation obtained is an identical one.

[For example, the identical relation realized from the evolutionary relation obtained from the noncommutative conservation laws for energy and linear momentum, points out to the fact that the conservation laws become commutative. However, this is true only along a certain direction, i.e. energy and impulse (realized from linear momentum) commute rather than energy and linear momentum do. As examples of the identical relation it may be the second principle of thermodynamics, which follows from the first one [10]. The second principle of thermodynamics follows from the first principle under the fulfillment of the condition of integrability, i.e. a realization of the integrating factor (the inverse temperature).]

From identical relation one can obtain the differential of the state functional  $d_{\pi} \psi$  and find the state function. This points to that the material system state is an equilibrium state. But this state is realized only locally since the state differential is interior one defined exclusively on pseudostructure. (The total state of material system turns out to be nonequilibrium because the evolutionary relation itself remains to be nonidentical one.)

The transition of the material system from nonequilibrium state into a locally equilibrium one means that the unmeasurable quantity described by the nonzero commutator of the unclosed evolutionary differential form  $\omega^p$ , that acted as an internal force, transforms into the measurable quantity. In material system this reveals as an occurrence of certain observable formations, which develop spontaneously. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, and others. The

intensity of such formations is controlled by a quantity accumulated by the evolutionary form commutator. (In paper [9] the process of production of vorticity and turbulence is described.)

Thus, it has been shown that the evolutionary nonidentical relation, which possesses the unique physical sense, follows from the balance conservation law equations for material systems. This relation describes the mechanism of evolutionary processes that proceed in material systems and are accompanied by the origin of some observable formations.

Below it will be shown that the process of origin of observable formations relates to generation of physical structures, on which exact conservation laws are satisfied.

### 3. Exact Conservation Laws. Connection between Exact Conservation Laws and the Balance Conservation Laws

The exact conservation laws are those that state the existence of conserved physical quantities or objects.

The exact conservation laws are described by closed exterior differential forms.

From the closure conditions for exterior differential form (vanishing the differential of the form)

$$d\theta^k = 0 \quad (10)$$

one can see that the closed exterior differential form is a conservative quantity ( $\theta^k$  is the exterior differential form of degree  $k$  ( $k$ -form)). This means that it can correspond to conservation law, namely, to existence of a certain conservative physical quantity.

If the exterior form is a closed inexact form, i.e. is closed only on pseudostructure, the closure condition is written as

$$d_{\pi} \theta^k = 0 \quad (11)$$

And the pseudostructure  $\pi$  obeys the condition

$$d_{\pi} {}^* \theta^k = 0 \quad (12)$$

where  ${}^* \theta^k$  is a dual form. From conditions (11) and (12) one can see that the closed exterior form and the dual form constitute a conservative object, namely, the pseudostructure with conservative quantity. Hence, such an object can correspond to some conservation law.

The closure conditions for the exterior differential form ( $d_{\pi} \theta^k = 0$ ) and the dual form ( $d_{\pi} {}^* \theta^k = 0$ ) are mathematical expressions of the exact conservation law.

#### 3.1. Generation of Physical Structures by Material Systems

Closed inexact exterior form and relevant closed dual form describe the differential-geometrical structure: the

pseudostructure (dual form) and the conservative quantity (closed exterior form). It is evident that the pseudostructures with conservative quantity are structures, on which exact conservation laws are satisfied.

Below it will be shown that such structures have a physical meaning, and therefore, they can be named as physical structures. (It should be noted that the differential-geometrical structures have also a mathematical meaning. The integral structures and relevant integrable manifolds such as the characteristics, singular points, characteristic and potential surfaces, which are obtained while solving the mathematical physics equations, are such differential-geometrical structures. G-structures are mathematical examples of such structures.)

In section 1 it has been shown that the transition from nonidentical evolutionary relation to identical one (which describes the transition of material system from nonequilibrium state to equilibrium one) relates to realization of closed dual form and exterior inexact skew-symmetric form. This points out to the occurrence of pseudostructure (dual form) with conservative quantity (closed exterior form), i.e. an occurrence of physical structure on which the exact conservation law is fulfilled.

On the other hand, as it has been shown, the transition of material system from nonequilibrium state to locally-equilibrium one is accompanied with origin of any observable formations. From this it follows that the process of origination of observable formation relates to the occurrence of physical structure. This fact also fixes by identical relation (9), which possesses the duality.

The left-hand side of identical relation (9) includes the differential, which specifies material system and whose presence points to the locally-equilibrium state of material system. And the right-hand side includes the closed inexact form, which describes physical structure. An existence of the state differential (left-hand side of relation (9)) points to the transition of material system from nonequilibrium state to the locally-equilibrium state (and origination of observable formations). And the emergency of the closed (on pseudostructure) inexact exterior form (right-hand side of relation (9)) points to the origination of the physical structure. [Physical structures and the formations of material systems observed are a manifestation of the same phenomena. The light is an example of such a duality. The light manifests itself in the form of a massless particle (photon) and as a wave. On the other hand, the observed formation and the physical structure are not identical objects. If the wave be such a formation, the element of wave front made up the physical structure while its motion.]

Thus, the analysis of the balance conservation law equations for material systems show that material systems generate physical structures, which correspond to exact conservation laws. This means that it exists two types of conservation laws and this points to the connection between the balance and exact conservation laws.

Below it will be shown that the physical structures, from which physical fields are formatted, are generated by ma-

terial systems. This result follows from the analysis of the field-theory equations.

### **3.2. Closed Inexact Exterior Forms as the Basis of Field Theories**

One can see that the existing field theories are based on the properties of closed exterior forms. Closed inexact exterior or dual forms are solutions of the field-theory equations. And there is the following correspondence:

-Closed exterior forms of zero degree correspond to quantum mechanics.

-The Hamilton formalism bases on the properties of closed exterior and dual forms of first degree.

-The properties of closed exterior and dual forms of second degree are at the basis of the equations of electromagnetic field.

-The closure conditions of exterior and dual forms of third degree form the basis of equations for gravitational field.

[One can see that field theories equations are connected with closed exterior forms of a certain degree. This enables one to introduce a classification of physical fields and interactions in degrees of closed exterior forms. If to denote the degree of closed exterior forms by  $k$ , then  $k = 0$  corresponds to strong interaction,  $k = 1$  does to weak one,  $k = 2$  does to electromagnetic one, and  $k = 3$  corresponds to gravitational interaction. Such a classification shows that there exists an internal connection between field theories that describe physical fields of various types. It is evident that the degree of closed exterior forms is a parameter that integrates field theories into unified field theory. This can serve as a step to constructing the unified field theory.]

The connection of field theory and the theory of closed exterior forms, corresponding to exact conservation laws, underlines the fact that field theories are based on the properties of exact conservation laws.

### **3.3. Connection between the Field-Theory Equations and the Equations of Conservation Laws for Material Systems**

A connection between the field-theory equations and the equations for material systems is executed by the nonidentical evolutionary relation.

The analysis of functional properties of the field-theory equations showed that the field-theory equations, as well as the evolutionary relation obtained from the balance conservation laws for material systems, prove to be nonidentical relations.

The field-theory equations differ in their functional properties from the equations for material systems. The equations for material systems are differential equations, its solutions are functions (which describe physical quantities such as a velocity, pressure and density). And the solutions to the field-theory equations are closed inexact exterior forms, i.e. they are differentials. Only the equations that have the form of relations (nonidentical) may have the solutions which are differentials rather than functions.

One can verify that all equations of existing field theories have the form of nonidentical relations in differential forms or in the forms of their tensor or differential (i.e. expressed in terms of derivatives) analogs.

The Einstein equation is a relation in differential forms.

The Dirac equation relates Dirac's bra- and cket- vectors, which made up the differential forms of zero degree.

The Maxwell equations have the form of tensor relations.

The field equation and Schrodinger's one have the form of relations expressed in terms of derivatives and their analogs.

Another specific feature of the field-theory equations consists in the fact that all field-theory equations are non-identical relations for functionals such as a wave function, action functional, the Pointing vector, Einstein's tensor and so on [7]. (Entropy is such a functional for the fields generated by thermodynamical and gas-dynamical systems.)

The evolutionary relation obtained from the equations for material systems is a nonidentical relation for all these functionals. That is, all field-theory equations are an analog to the evolutionary relation.

The correspondence between the field-theory equations and the evolutionary relation points to a connection between field theories and the equations for material systems.

Connection of the field-theory equations with the equations for material system points to the fact that physical structures, from which physical fields are formatted, are generated by material systems. This means that there exists a connection of physical fields with material systems.

The results obtained show that when building the general field theory it is necessary to take into account the connection of existing field theories (that describe physical fields and are based on exact conservation laws) with the equations of noncommutative conservation laws for material media (the balance conservation laws for energy, linear momentum, angular momentum and mass).

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