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Modification in Two-Connected Graph with Gallai's Property in 2-Dimensional and 3-Dimensional Graph Containing 19 Vertices

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Abstract: The graph theory plays an important role in the network analysis, social networking as well as in many engineering fields such as electrical circuits, artificial intelligence, architecture, making the design or pattern of roads, buildings, shopping mall and etc. Due to this wide range application human enjoying her life with peacefully, Graph theory creates a way for human being to connect among themselves by social network. All above applications based on graph or molecule which may be the planer, non-planer and Peterson graph or etc. Peterson graph is the most important and reasonable example of Hypo-Hamiltonian graph. In the earlier, it was found as a hypo-traceable graph (graph which has not Hamiltonian graph. Naeem et al has worked on "A Two-Connected Graph with Gallai's Property" In his research paper he has applied the property and has found the longest path and cycle in the graph. In this research paper we will develop the 3-dimensional graph of computational molecule contains 19 vertices and will split it into three different planes (xy, xz and yz-plane), and will find the longest path, longest cycle the molecule. The designed graphs can be useful in various fields of science and technology including computational geometry, networking, theoretical computer science and circuit designing.

Keywords: Gallai Property, Hamiltonian Path, Hamiltonian Cycle, Hypo Hamiltonian Graph, Graph Theory, Traceable Graph

1. Introduction

Graph theory is essential part of mathematics and his usages are in computer sciences especially in switching theory and logical design, artificial intelligence, formal languages, computer graphics, operating systems, compiler writing, information organization and retrieval electric wires, mobile networks and relationship among human and living organism.

Actually, graph is the combination of vertex V(G) and lines segments called edges E(G). Each graph has its own identity the strategy of figure is

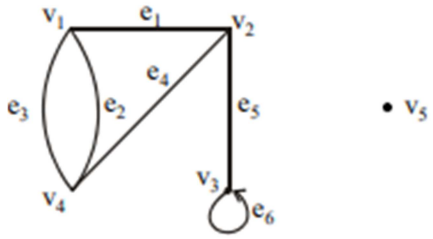

Figure 1. *Five vertices graph containing loops.*

Here it can be defined as the edge which is connected single or double vertices it is said to be endpoints here in figure e_1 is connected with v_1 qnd v_2 or other these two points are the end points of e_1. furthermore if an edge has a only one

endpoint is called loop it can also defined as the edge which is connected to a vertex to itself it is loop, It can be visualized in graph on vertex v_3 and e_6 and adjacent is beingdefines as the two vertices connected by an edge and a vertex that is an endpoint of a loop is said to be adjacent to itself. An edge is said to be incident on each of its endpoints (i.e. e_1 is incident on v_1 qnd v_2). Graph theory has several types and so many mathematicians have worked onto find out the cycle and path of the graph. A cycle is the process which passes through each vertexwithout repeating the vertex and end on the starting or end point of the graph it is generally called as Hamiltonian circuit or cycle. Similarly, A Hamiltonian path is the path which starts from a vertex with did not end at the initial vertex or starting point of vertex.

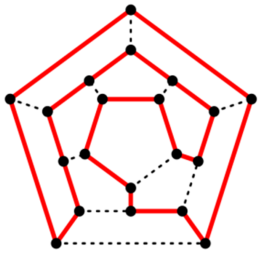

Figure 2. *Finding highest cycle in (Hamiltonian Cycle & Path).*

Here in figure 2, it is represented a Hamiltonian graph containing 20 points nodes and the highest order of cycle as well as path is 20. Furthermore, the graph which has both Hamiltonian cycle as well as Hamiltonian path is said to be traceable graph. Some examples are given below

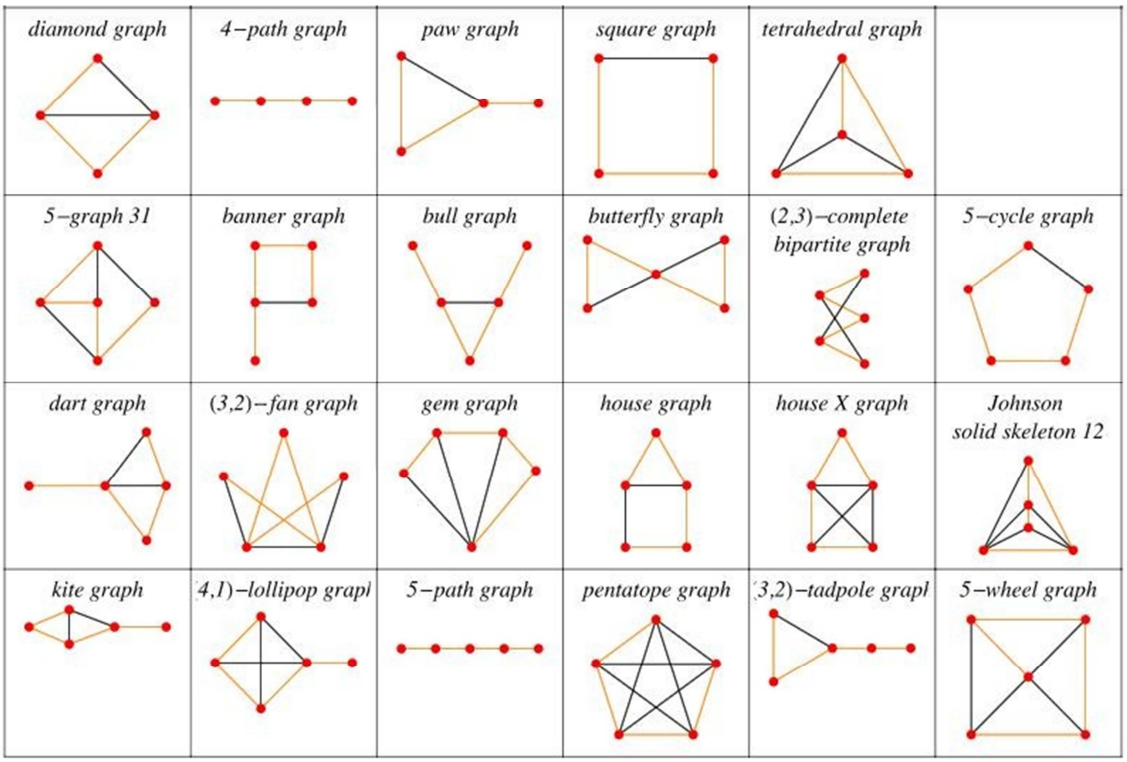

Figure 3. *Examples of Traceable Hamiltonian Graph.*

In the investigation on graph theory we able to find the strategy regarding hypo Hamiltonian graph it says if G is the graph which have not Hamiltonian cycle but every graph developed by eliminating a single vertex from G is Hamiltonian. Hypo Hamiltonian graphs are virtually Hamiltonian and Peterson graph is the best example of it [1, 2].

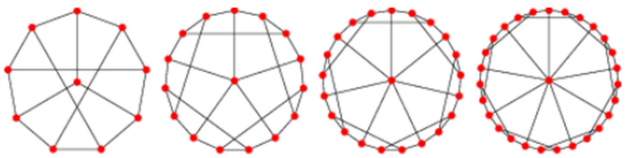

Figure 4. *Most useful examples of Hypo Hamiltonian graphs.*

In the 1966 a mathematician T. Gallai [3, 4] had queried about the exit of proposed graph with the property that missing the vertex by getting longest path in the occurrence of hypo-Hamiltonian graphs with former upgrading of hypo-traceable graphs. After this statement in 1969 firstly H. Walther [5] had replied the quarried of Gallai's asked question, he developed a planer graph on 25 vertices which were satisfying Gallai's property. After H. Walther many mathematicians just like H. Walther and H. Voss [6], & Tudor Zamfirescu [7], proposed such type of graph with 12 vertices. W. Schmitz has worked on the planer graph and have taken lowest number of 17-vertices [8] and Thomason give his idea on smallest non-planer graph of order 34 including 82 vertices [9]. Now a day's 26 vertices is the lowest example [10] but other side the lowest example is of 32 orders [11-13]. Tudor Zamfirescu had asked question regarding the Gallai's property. He said if $p_j^i = \infty$ ($p_i^{-j} = \infty$) and there is not planer graph or $i - connected$ graph such that independently set of

j points persist separate from the longest path condition$p_j^i \neq \infty\ (p_i^{-j} \neq \infty)$. Suppose $p_j^i p_i^{-j}$ shows shortest number of vertices of a $j-connected$ graph or panner graph such that independently the set of j selected vertices be the disjoint from the longest path. Equally the cases $C_i^j\ and\ C_i^{-j}$ are clearly used for longest circuits as the exchange of longest path [14]. Furthermore this types of graph have wide range application in new computational models which is often based on different types of relationships among human beings, peoples and network communications. Besides of this it is also useful in data structure as well as social structure which plays every important role in science and technology specially internet, mobile services and so many social connection or networks. In this predictable graph organization is being symbolize by nodes or vertices and the association among the nodes or vertices characterize edges or sides.

2. Results and Discussion

We have developed the molecule having nineteen vertices in three dimensional, which the combination of three planes $xy, xz\ and\ yz\ plane$ given below.

3. Two Dimensional Cycle in Molecule under 19-Points Graph

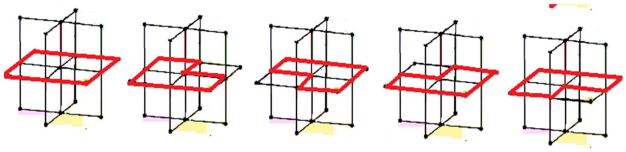

Figure 5. *Variation in XY-PLANE & Z=0.*

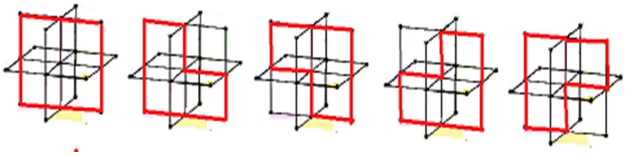

Figure 6. *Variation in XZ-PLANE& Y=0.*

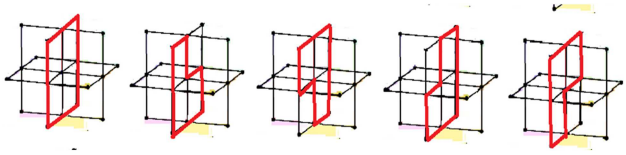

Figure 7. *Variation in YZ-PLANE& X=0.*

This is the 3-dimensional molecule containing 19 vertices. we have converted the molecule into the planes just like $xy-plane, yz-plane\ and\ xz\ plane$ and have find the cycle in the graph. In this investigation we found almost same 8 vertices order of cycle in 2D molecule shown in Figures 5, 6, 7 and missing one vertex in each cycle in the graph.

4. Two Dimensional Path in Molecule Containing 19 Points Nodes

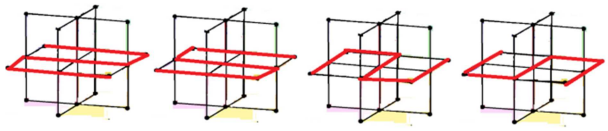

Figure 8. *Variation in XY-PLANE& Z=0.*

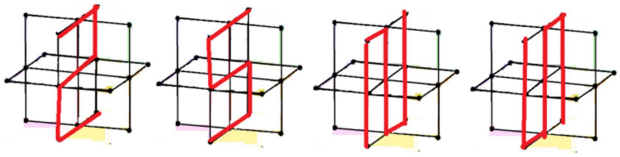

Figure 9. *Variation XZ-PLANE& Y=0.*

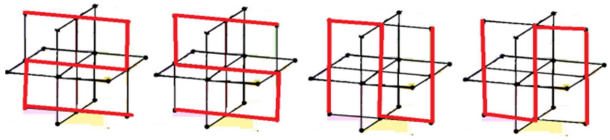

Figure 10. *Variation YZ-PLANE& X=0.*

The investigation has been carried out from the molecule that from $xy-plane$ we have found the maximum path of order 9 shown in figure 4, in the $xz-plane$ again we have found same kind of order and similarly in $yz-plane$ nine orders of path has been found in the molecule. Furthermore, two kinds of numeral just like $2\ and\ 5$ appear in the shape of path.

5. Three Dimensional Cycle in Molecule Under Nineteen Points or Vertices

This is the molecule obtained by octane number picture and some time it can also used in cylindrical mesh system, The molecule contains three planes and detail of the plane are given table 1. In this investigation we have found out the longest cycle in the 3D graph. We have found maximum 12^{th} order cycle in the proposed molecule of 19 vertices which can be visualized in mention below in pictures $(a\ to\ h)$. During the process of cycle some vertices has been missed which we can also be seen in table 2.

Table 1. *Planes, centre Angle, Axis vertices and Missed vertices.*

Planes	Centre Vertex	Axis Vertex	Mixed Vertex
xy	1	4, 8	15, 10, 11, 14, 13, 12
yz	1	11, 14	16, 17, 11, 14, 18, 19
xz	1	2, 6	1, 3, 4, 5, 7, 8

Finding the longest cycle shown in red color, we have focused from vertex 1 and tried to make a cycle in maximum time here we have put some examples and missed vertex in Figure.

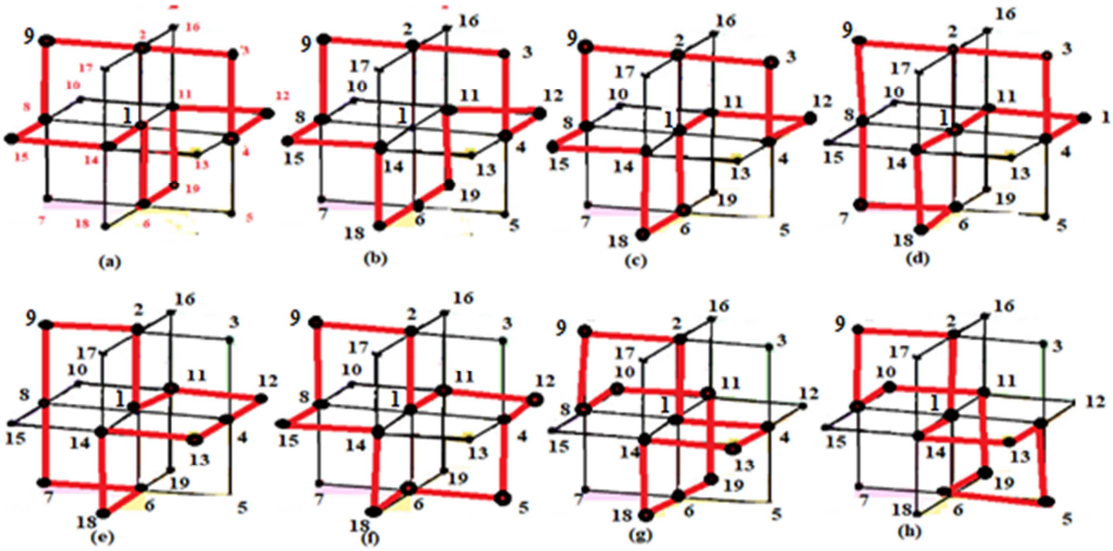

Figure 11. *Longest cycle of the graph containing 19-points.*

Table 2. *Longest cycle with missed vertices of 19-points graph.*

Cycle	Longest Cycle	Missed Vertex
A	9, 2, 3, 4, 12, 11, 19, 6, 1, 14, 15, 8	5, 7, 10, 13, 17, 18, 19
B	9, 2, 3, 4, 12, 11, 19, 6, 18, 14, 15, 8	5, 7, 1, 10, 13, 16, 17
C	9, 2, 3, 4, 6, 8, 1, 11, 12, 14, 15, 18	5, 7, 10, 13, 16, 17, 19
D	9, 2, 3, 4, 12, 11, 1, 14, 18, 6, 7, 8	13, 15, 10, 16, 17, 19, 5
E	9, 2, 1, 11, 12, 4, 13, 14, 18, 6, 7, 8	10, 15, 3, 16, 17, 5, 19
F	9, 2, 1, 11, 12, 4, 5, 6, 18, 14, 15, 8	3, 16, 17, 10, 13, 7, 19
G	9, 2, 1, 4, 13, 14, 6, 18, 19, 11, 10, 8	3, 16, 17, 12, 15, 7, 5
H	9, 2, 1, 14, 13, 4, 5, 6, 19, 11, 10, 8	3, 16, 17, 12, 15, 7, 18

6. Three Dimensional Path in Molecule

In this research paper we have found the path not only in 2 dimensions but as well as in 3-dimensionalmolecule or graph. During the investigation on 3D graph containing 19 vertices, we found 13 order longest path (indicates in red color lines) with missing vertices which can be shown in the figure 12.

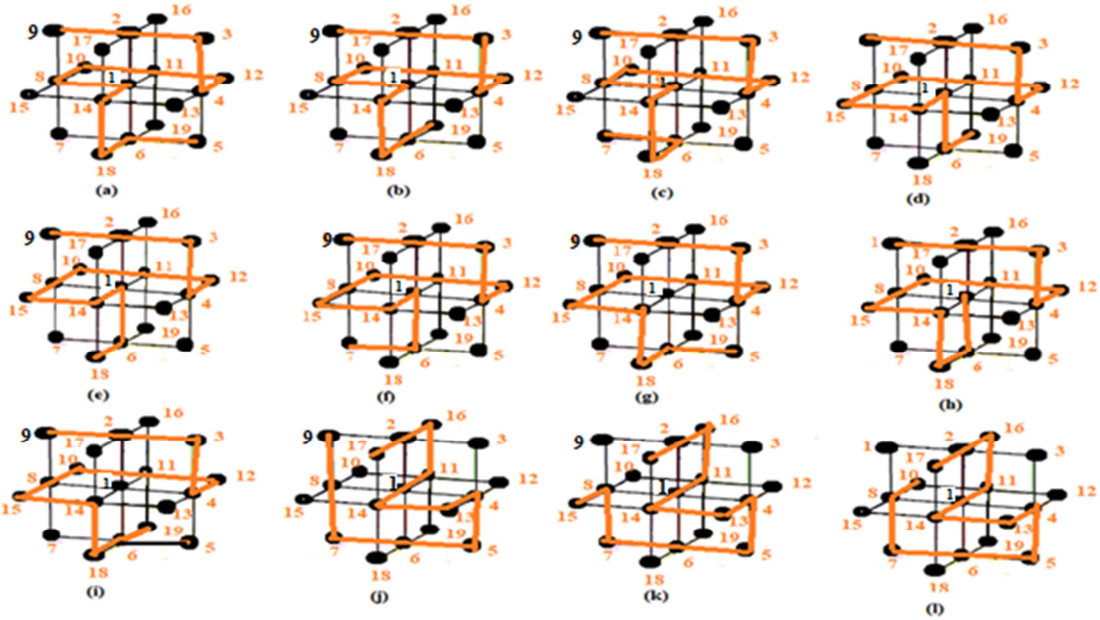

Figure 12. *Obtained longest Path and different types.*

Table 3. *Longest Path with missed vertices in 19 points graph.*

Path	Longest Path	Missed Vertex
A	9, 2, 3, 4, 12, 11, 10, 1, 8, 14, 18, 6, 5	7, 13, 15, 16, 17, 19
B	9, 2, 3, 4, 12, 11, 10, 1, 8, 14, 18, 6, 19	5, 7, 13, 15, 16, 17
C	9, 2, 3, 4, 12, 11, 10, 1, 8, 14, 18, 6, 7	5, 13, 15, 16, 17, 19
D	9, 2, 3, 4, 12, 11, 10, 8, 15, 14, 1, 6, 19	5, 7, 13, 15, 16, 17, 18
E	9, 2, 3, 4, 12, 11, 10, 8, 15, 14, 1, 6, 18	5, 7, 13, 15, 16, 17
F	9, 2, 3, 4, 12, 11, 10, 8, 15, 14, 1, 6, 7	5, 13, 16, 17, 18, 19
G	9, 2, 3, 4, 12, 11, 10, 8, 15, 14, 18, 6, 5	7, 9, 13, 16, 17, 19
H	9, 2, 3, 4, 12, 11, 10, 8, 15, 14, 18, 6, 9	5, 7, 13, 16, 17, 19
I	9, 2, 3, 4, 12, 11, 10, 8, 15, 14, 18, 6, 19	5, 7, 1, 13, 15, 16, 17
J	17, 2, 16, 11, 1, 14, 13, 4, 5, 6, 7, 8, 1	3, 10, 12, 15, 18, 19
K	17, 2, 16, 11, 1, 14, 13, 4, 5, 6, 7, 8, 15	1, 10, 3, 12, 18, 19
L	17, 2, 16, 11, 1, 14, 13, 4, 5, 6, 7, 8, 10	1, 3, 12, 15, 18, 19

In table 3, the obtained results of the proposed graph containing nineteen vertices shows the longest path with different position and similar number of order. This indicates the reliability and accuracy of the graph. In this table 3, (A-L) paths have been taken for finding longest path, the table 2 shows the longest path of same dimension with missing vertices, it can be seen in Figure 12.

In this investigation we have established two connected graph in which some vertices have been missed by some longest cycle and path which be seen in above tables, Claiming the consequences the exits graph is satisfying the Gallai's property.

7. Conclusions

In this research article 3 dimensional graphs has been taken which contains 19-vertices and providing huge number of applications in graph theory as well as artificial intelligence. In this research article, longest path and cycle has been found with missed vertices. After all, the graph can be used in social structures, engineering fields and circuit analysis. The basic aim of this research article is to find out the best way for social network as well as in electrical circuit, data analysis, and so many designed or undersign able structure in 3-D and due to longest cycle and path it will help the engineer in making roads, circuits, architectures, building, shopping mall infrastructure and etc.

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