
Effects of viscous dissipation and heat generation on magneto hydrodynamics natural convection flow along a vertical wavy surface

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Abstract: In this paper we consider the combined effects of viscous dissipation and heat generation on MHD natural convection flow along a vertical wavy surface are studied. The governing Navier-Stokes equations with associated boundary conditions are transformed into non-dimensional boundary layer equations using appropriate variables. Implicit finite difference method based on Keller-box scheme is used to solve these governing equations. The numerical results in terms of the skin friction coefficient, the rate of heat transfer in terms of local Nusselt number, the streamlines as well as the isotherms are discussed and shown graphically for different values of viscous dissipation parameter N , heat generation parameter Q and magnetic parameter M .

Keywords: Viscous Dissipation, MHD, Heat Generation, Natural Convection, Keller Box Method, Wavy Surface

1. Introduction

The viscous dissipation effect plays an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotational speeds and also in strong gravitational field processes on large scales (on large planets), in geological process and in nuclear engineering in connection with the cooling of reactors. Natural convection flow is often encountered in cooling of nuclear reactors or in the study of the structure of stars and planets. The study of temperature and heat transfer is of great importance to the engineers because of its almost universal occurrence in many branches of science and engineering. It is also necessary to study the heat transfer from an irregular surface because irregular surfaces are often present in many applications, such as radiator, heat exchangers and heat transfer enhancement devices.

Yao [1] investigated natural convection along a vertical wavy surface. He has found that the frequency of the local heat transfer rate is twice that of the wavy surface. Hossain and Rees [2] analyzed combined heat and mass transfer in natural convection flow from a vertical wavy surface. In this investigation they focused on the boundary layer regime promoted by the combined events near the wavy surface when the surface is at a uniform temperature and a uniform mass

diffusion. Wang and Chen [3] studied transient force and free convection along a vertical wavy surface in micropolar fluid. The natural convection of fluid with variable viscosity from a heated vertical wavy surface was studied by Hossain et al. [4]. Natural convection heat and mass transfer along a vertical wavy surface were investigated by Jang et al [5]. Molla et al [6] studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Molla and Hossain [7] investigated the radiation effect on mixed convection laminar flow along a vertical wavy surface. Alam et al [8] studied viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation. Mamun et al [9] presented a paper on MHD-conjugate heat transfer analysis for a vertical flat plate in presence of viscous dissipation and heat generation. Jha and Ajibade [10] observed the effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions. Recently Joule heating and MHD free convection flow along a vertical wavy surface with viscosity and thermal conductivity dependent on temperature have been investigated by Parveen and Alim [11]. None of the above investigations considered the effects of viscous dissipation and magnetic field on natural convection flow in presence of heat generation/ absorption along a vertical wavy surface. An investigation of the effects

of viscous dissipation and magnetic field in presence of heat generation/ absorption on free convection flow along a vertical wavy surface is proposed in the present study. The results will be obtained for different values of relevant physical parameters and will be shown in graphs.

2. Formulation of the Problem

The boundary layer analysis outlined below allows $\bar{\sigma}(X)$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$Y_w = \bar{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right) \quad (1)$$

where L is the wave length associated with the wavy surface.

The geometry of the wavy surface and the two-dimensional cartesian coordinate system are shown in Figure-1.

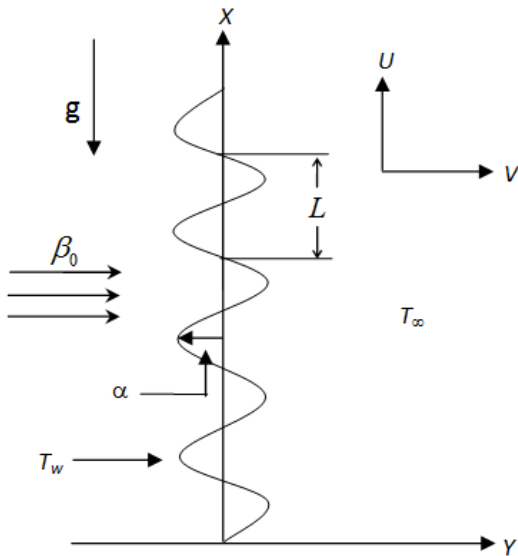


Figure 1. The coordinate system and the physical model.

The conservation equations for the flow characterized with steady, laminar and two-dimensional boundary layer; under the usual Boussinesq approximation, dimensionless form of the continuity, momentum and energy equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Gr^{1/4} \sigma_x \frac{\partial p}{\partial y} + (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} - Mu + \theta \quad (3)$$

$$\sigma_x \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -Gr^{1/4} \frac{\partial p}{\partial y} + \sigma_x (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} - \sigma_{xx} u^2 \quad (4)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} (1 + \sigma_x^2) \frac{\partial^2 \theta}{\partial y^2} + Q \theta + N \left(\frac{\partial u}{\partial y} \right)^2 \quad (5)$$

Where $Pr = \frac{C_p \mu}{k}$ is the Prandtl number, $Q = \frac{Q_0 L^2}{\mu C_p Gr^{1/2}}$ is the heat generation parameter, $N = \frac{\nu^2 Gr}{L^2 C_p (T_w - T_\infty)}$ is the viscous dissipation parameter, and $M = \frac{\sigma_0 \beta_0^2 L^2}{\mu Gr^{1/2}}$ is the magnetic parameter.

Using Prandtl's transposition theorem to transform the irregular wavy surface into a flat surface as extended by Yao [2] and boundary-layer approximation, the following dimensionless variables are introduced for non-dimensionalizing the governing equations,

$$x = \frac{X}{L}, \quad y = \frac{Y - \bar{\sigma}}{L} Gr^{1/4}, \quad p = \frac{L^2}{\rho \nu^2} Gr^{-1} P$$

$$u = \frac{L}{\nu} Gr^{-1/2} U, \quad v = \frac{L}{\nu} Gr^{-1/4} (V - \sigma_x U),$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \sigma_x = \frac{d\bar{\sigma}}{dX} = \frac{d\sigma}{dx}, \quad Gr = \frac{g \beta (T_w - T_\infty) L^3}{\nu^2}$$

where θ is the non-dimensional temperature function and (u, v) are the dimensionless velocity components.

It can easily be seen that the convection induced by the wavy surface is described by equations (2)–(5). We further notice that, equation (11) indicates that the pressure gradient along the y -direction is $O(Gr^{-1/4})$, which implies that lowest order pressure gradient along x -direction can be determined from the inviscid flow solution. For the present problem this pressure gradient ($\partial p / \partial x = 0$) is zero. Equation (4) further shows that $Gr^{1/4} \partial p / \partial y$ is $O(1)$ and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p / \partial y$ from equations (3) and (4) leads to

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_x \sigma_{xx}}{1 + \sigma_x^2} u^2 + \frac{1}{1 + \sigma_x^2} \theta \quad (6)$$

The corresponding boundary conditions for the present problem are:

$$\left. \begin{aligned} u = v = 0, \quad \theta = 1 \quad \text{at } y = 0 \\ u = \theta = 0, \quad p = 0 \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (7)$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi = x^{3/4} f(x, \eta), \quad \eta = yx^{-1/4}, \quad \theta = \theta(x, \eta) \quad (8)$$

where $f(\eta)$ is the dimensionless stream function, η is the pseudo similarity variable and ψ is the stream function that satisfies the Eq. (2) and is defined by (8) where $f(\eta)$ is the dimensionless stream function, η is the pseudo similarity variable and ψ is the stream function that satisfies the Eq. (2) and is defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (9)$$

Introducing the transformations given in Eq. (8) and into Eqs. (6) and (5) the following system of non linear equations are obtained,

$$(1+\sigma_x^2)f'' + \frac{3}{4}f'' - \left(\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2 + \frac{1}{1+\sigma_x^2}\theta \frac{Mk^{1/2}}{1+\sigma_x^2}f' = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \quad (10)$$

$$\frac{1}{Pr}(1+\sigma_x^2)\theta'' + \frac{3}{4}f\theta' + x^{1/2}Q\theta + Nx f'^2 = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \quad (11)$$

The boundary conditions (7) now take the following form:

$$\left. \begin{aligned} f(x,0) = f'(x,0) = 0, \quad \theta(x,0) = 1 \\ f'(x,\infty) = 0, \quad \theta(x,\infty) = 0 \end{aligned} \right\} \quad (12)$$

In the above equations prime denote the differentiation with respect to η .

The local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x takes the following form:

$$C_{fx}(Gr/x)^{1/4}/2 = \sqrt{1+\sigma_x^2}f''(x,0) \quad (13)$$

$$Nu_x(Gr/x)^{-1/4} = -\sqrt{1+\sigma_x^2}\theta'(x,0) \quad (14)$$

3. Numerical Procedure

The transformed boundary layer equations are solved numerically with the help of implicit finite difference method together with the Keller-box scheme [9]. To begin with the partial differential equations (10) and (11) are first converted into a system of first order differential equations. Then these equations are expressed in finite difference forms by approximating the functions and their derivatives in terms of the central difference approximations. The above central difference approximations reduces the system of first order differential equations to a set of non-linear difference equations for the unknown at x_i in terms of their values at x_{i-1} . The resulting set of non-linear difference equations are solved by using the Newton's quasi-linearization method. The Jacobian matrix has a block-tridiagonal structure and the difference equations are solved using a block-matrix version of the Thomas algorithm.

4. Results

Here we have shown the combined effects of viscous dissipation and heat generation on MHD natural convection flow of viscous incompressible fluid along a vertical wavy surface. The skin friction coefficient C_{fx} , the rate of heat transfer in terms of Nusselt number Nu_x , the streamlines as well as the isotherms are shown graphically in Figs. 2-10 for different values of the aforementioned physical parameters.

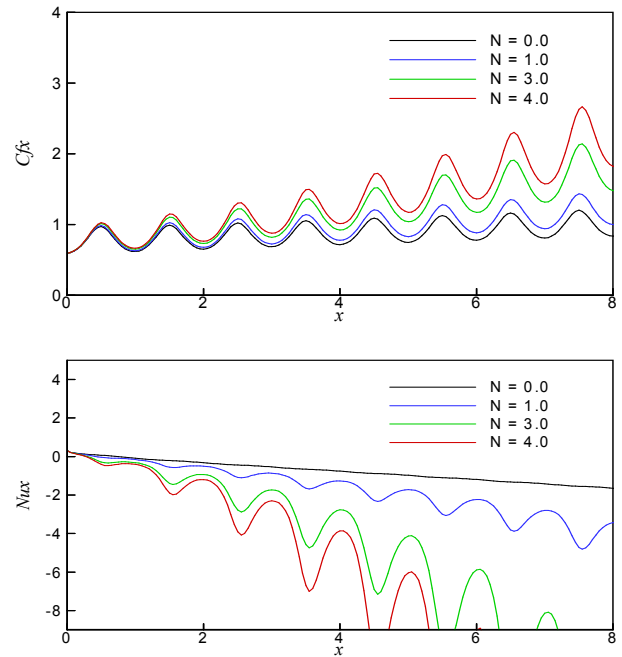


Figure 2. Effect of N on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x .

The variation of local skin friction C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x against x from the wavy surface while $\alpha = 0.3$, $Q = 0.4$, $M = 0.5$, and $Pr = 0.73$ are illustrated in Fig. 2(a) and 2(b) respectively. Since the higher value of N accelerates the fluid flow and increases the temperature so from the figure it is noted that for the viscous dissipation parameter $N = (0.0, 1.0, 3.0, 4.0)$, the skin friction coefficient increases along the upstream direction of the surface and to decrease of the heat transfer rates.

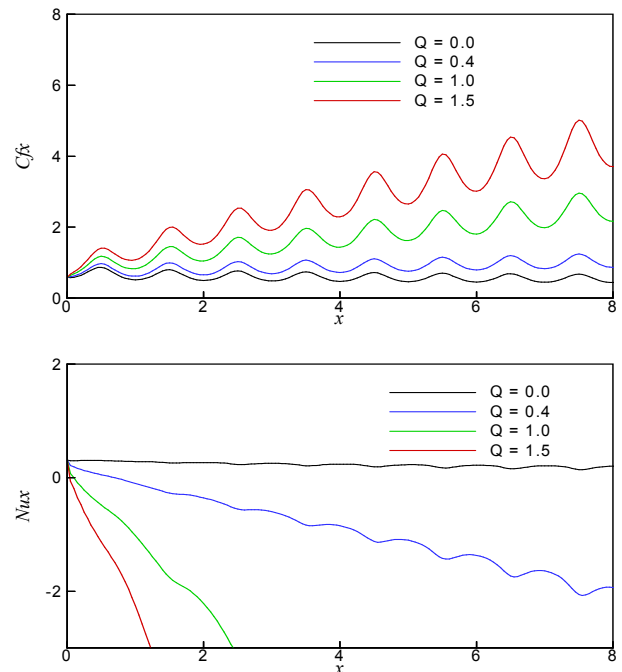


Figure 3. Effect of Q on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x .

The influence of the parameter Q , on the skin friction coefficient C_{fx} and local rate of heat transfer Nu_x are illustrated in Figs 3(a) and 3(b) respectively while $\alpha = 0.3$, $N = 0.2$, $M=0.5$ and $Pr = 0.73$. From those it is observed that an increase in the heat generation parameter $Q = (0.0, 0.4, 1.0, 1.5)$ leads to increase the local skin friction coefficient C_{fx} and decrease the local rate of heat transfer Nu_x at different position of x . These are happened, since the heat generation mechanism creates a layer of hot fluid near the surface and finally the resultant temperature of the fluid exceed the surface temperature and temperature gradient decreases. For this reason the rate of heat transfer decreases. Increasing temperature increases the viscosity of the fluid. Hence the corresponding shearing stress in terms of local skin friction coefficient increases.

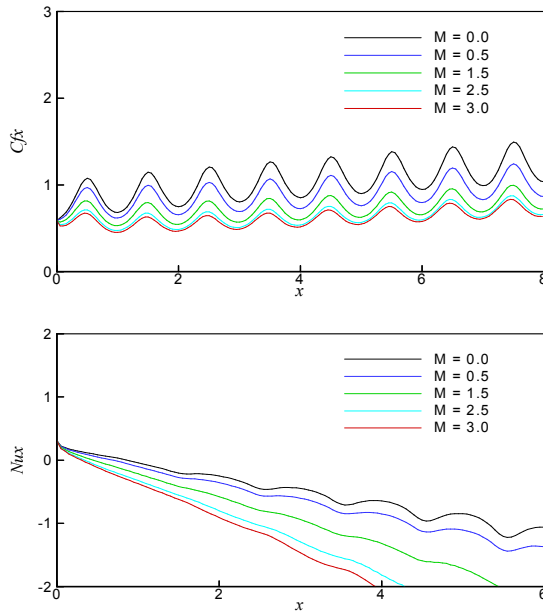


Figure 4. Effect of M on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x .

In Figures 4(a) and 4(b), the skin friction coefficient C_{fx} and local rate of heat transfer Nu_x are illustrated in for different values of M while $\alpha = 0.3$, $N = 0.2$, $M=0.5$ and $Pr = 0.73$. Here it is observed that an increase in the magnetic parameter $M=(0.0,0.5,1.5,3.0)$ leads to decrease the local skin friction coefficient and local rate of heat transfer at different position of x . The magnetic field acts against the flow and reduces the skin friction and the rate of heat transfer.

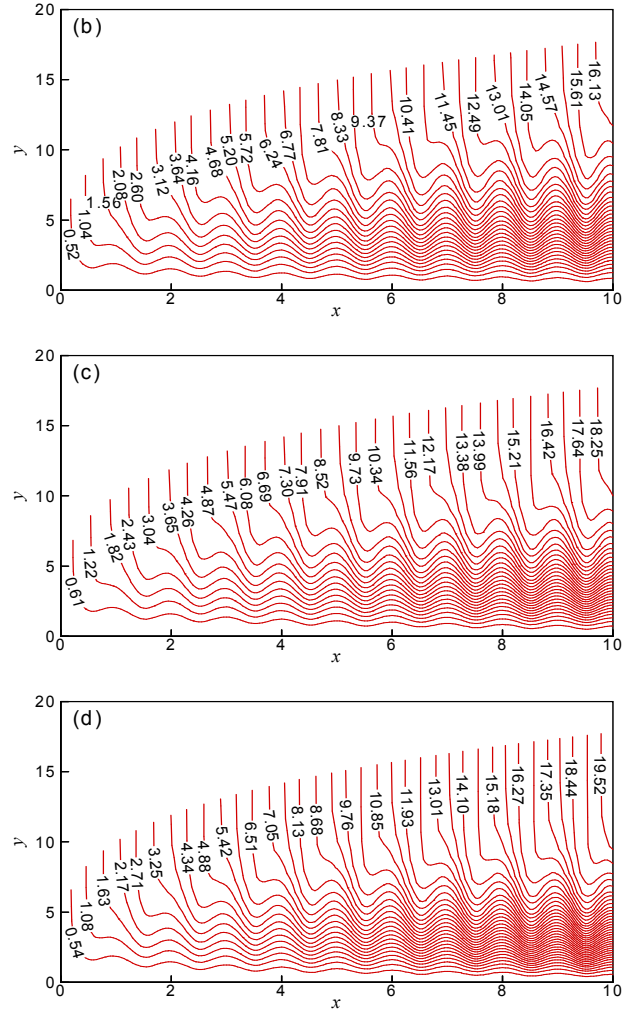
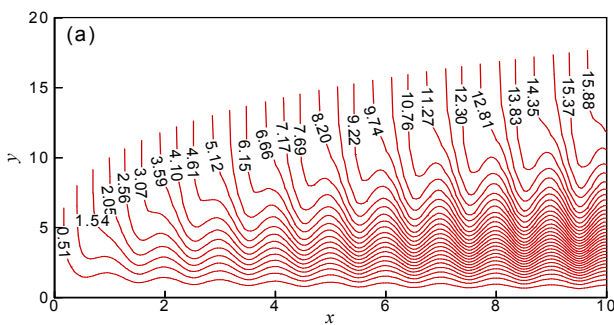
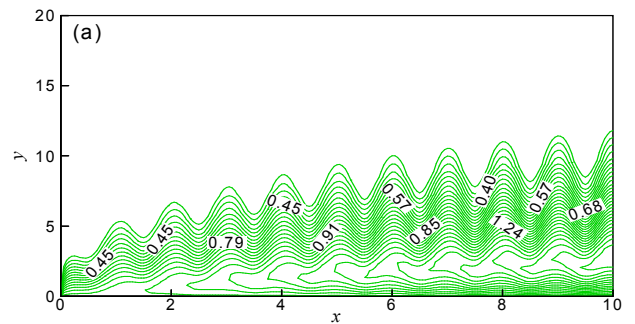


Figure 5. Streamlines for (a) $N = 0.0$ (b) $N = 1.0$ (c) $N = 3.0$ and (d) $N = 4.0$ while $Pr = 0.73, \alpha = 0.3, Q=0.4$ and $M=0.5$.

Fig. 5 and Fig. 6 show the effect of viscous dissipation parameter $N = (0.0, 0.5, 1.0, 2.0)$ on the formulation of streamlines and isotherms respectively while $Pr = 0.73$, $Q = 0.4$, $M=0.5$ and $\alpha = 0.3$. We find that for $N = 0.0$ the value of ψ_{max} is 15.88, for $N = 3.0$ ψ_{max} is 18.25 and for $N = 4.0$ ψ_{max} is 19.52. From Fig. 5, it is seen that the effect of viscous dissipation parameter N , the flow rate in the boundary layer increases. From Fig 6, it is also observed that due to the effect of N , the thermal state of the fluid increases. Finally, the thermal boundary layer becomes thicker.



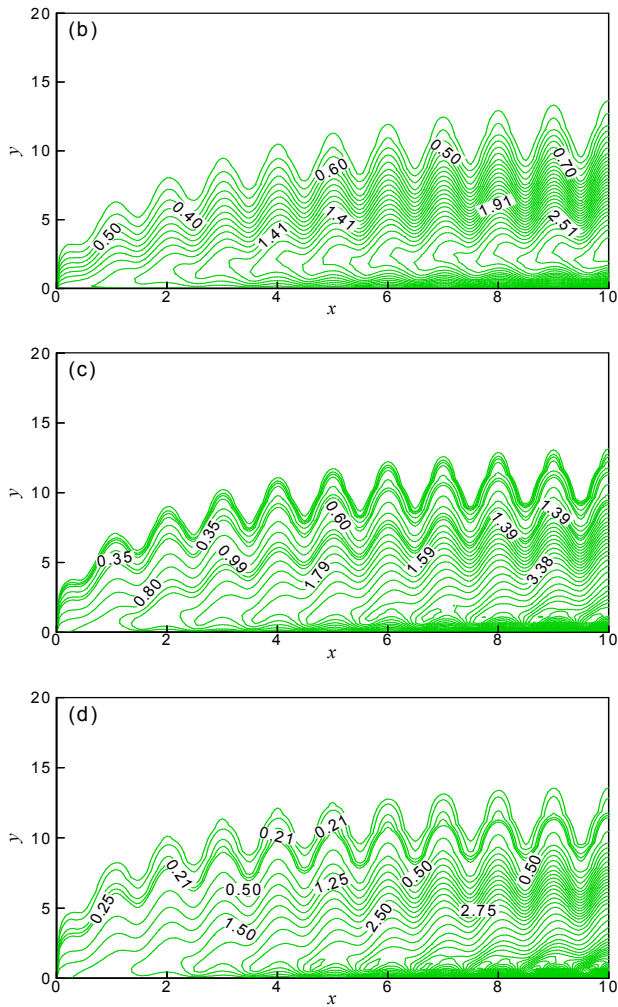


Figure 6. Isotherms for (a) $N = 0.0$ (b) $N = 1.0$ (c) $N = 3.0$ and (d) $N = 4.0$ while $Pr = 0.73$, $\alpha = 0.3$, $Q = 0.4$ and $M = 0.5$.

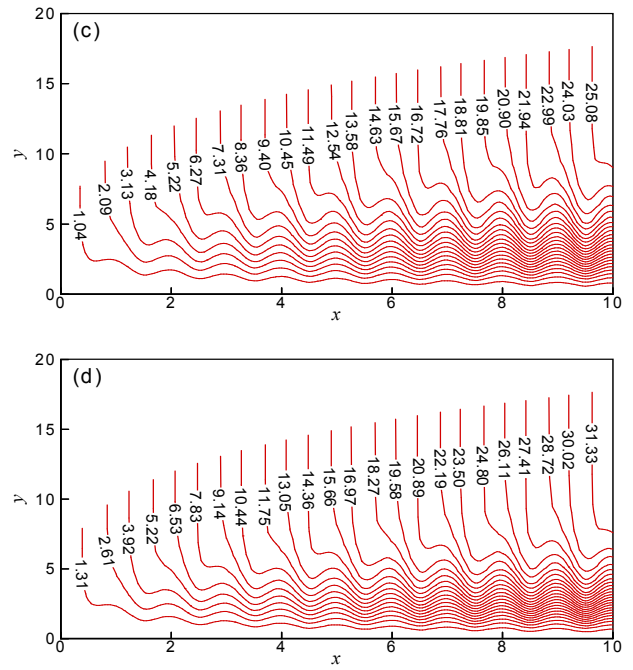


Figure 7. Streamlines for (a) $Q = 0.0$ (b) $Q = 0.4$ (c) $Q = 1.0$ and (d) $Q = 1.5$ while $Pr = 0.73$ $\alpha = 0.3$, $N=0.2$ and $M=0.5$.

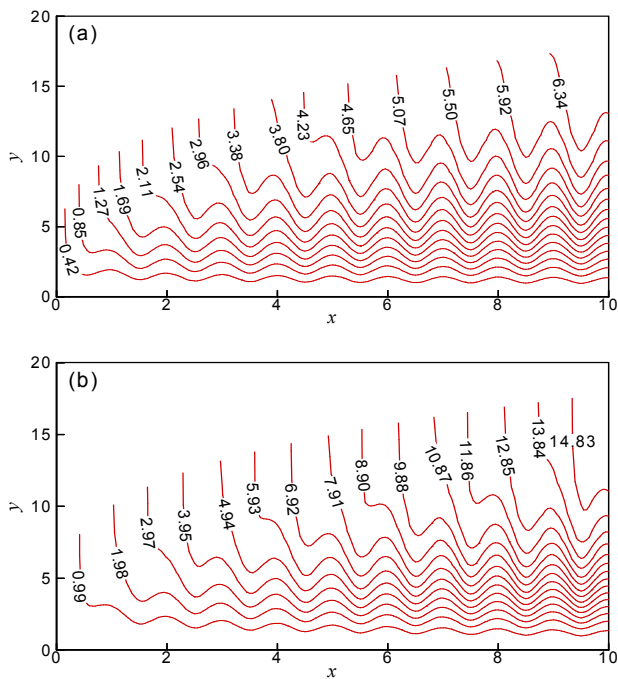


Figure 8. Isotherms for (a) $Q = 0.0$ (b) $Q = 0.4$ (c) $Q = 1.0$ and (d) $Q = 1.5$ while $Pr = 0.73$, $\alpha = 0.3$, $N=0.2$ and $M=0.5$.

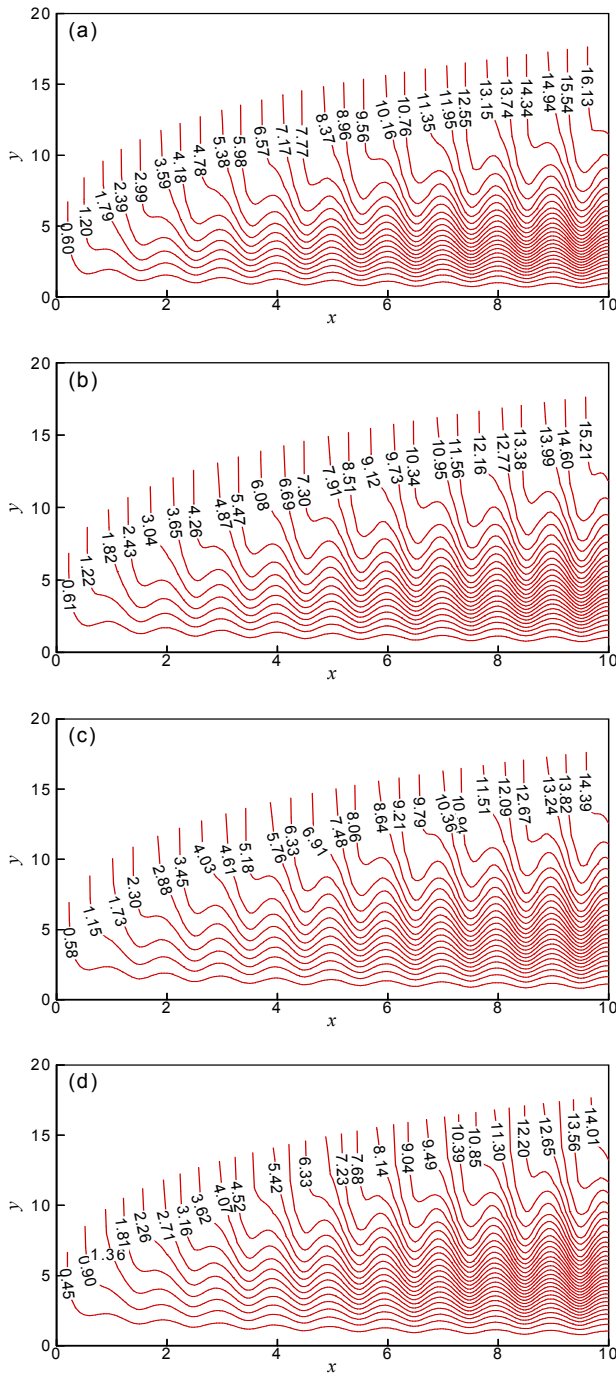


Figure 9. Streamlines for (a) $M = 0.0$ (b) $M = 0.5$ (c) $M = 1.5$ and (d) $M = 3.0$ while $Pr = 0.73$, $\alpha = 0.3$, $Q = 0.4$ and $N = 0.2$.

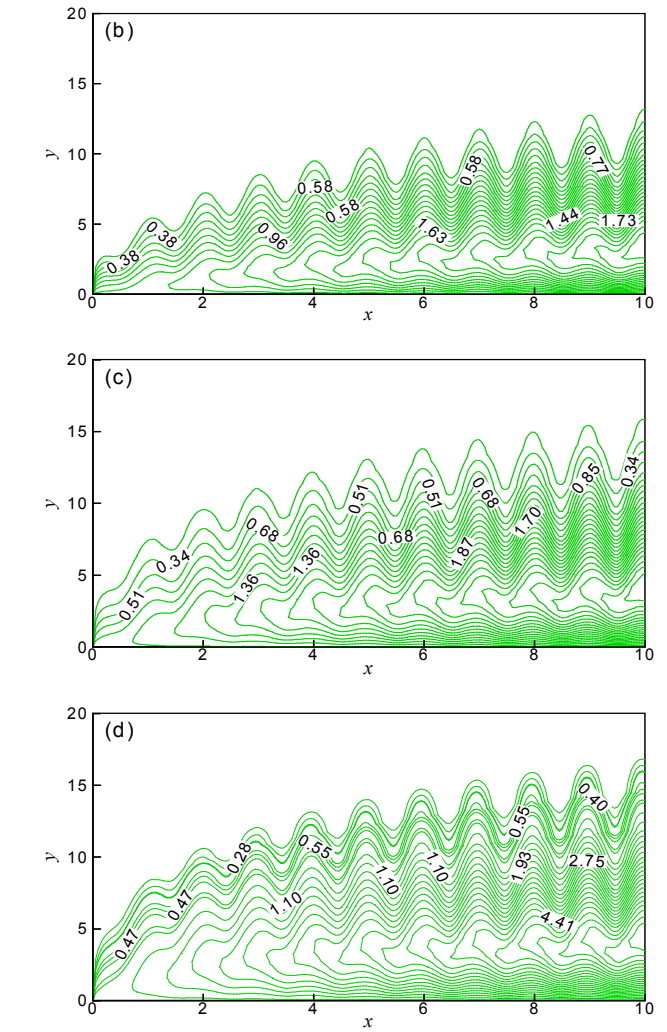
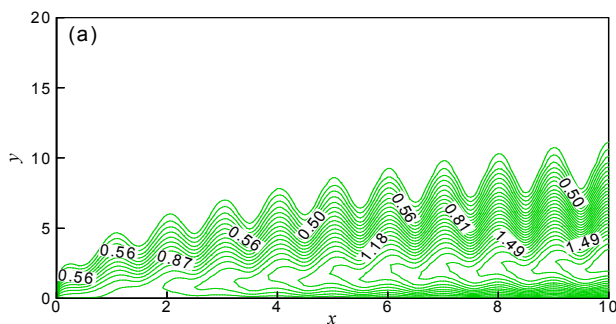


Figure 10. Isotherms for (a) $M = 0.0$ (b) $M = 0.5$ (c) $M = 1.5$ and (d) $M = 3.0$ while $Pr = 0.73$, $\alpha = 0.3$, $Q = 0.4$ and $N = 0.2$.

The effect of variation of the Q equal to 0.0, 0.4, 1.0 and 1.5 on the streamlines and isotherms are depicted by the Fig. 7 and 8 respectively while $\alpha = 0.3$, $N = 0.2$, $M = 0.5$ and $Pr = 0.73$. Figure 7 depicts that the maximum values of ψ increases while the values of Q increases that is ψ_{max} are 6.34, 14.83, 25.08 and 31.33 for $Q = 0.0, 0.4, 1.0$ and 1.5 respectively. It is noted from Fig. 8 that as the value of Q increases the thermal boundary layer becomes thicker gradually. So the isotherms increases while the values of Q increases.

The effect of variation of the surface roughness on the streamlines and isotherms for the values of M equal to 0.0, 0.5, 1.5, and 3.0 are depicted by Figure 9 and Figure 10 while $Pr = 0.73$, $\alpha = 0.3$, $Q = 0.4$ and $N = 0.2$. Figure 9 depicts that the maximum values of streamline decreases steadily while the values of M increases. The maximum values of streamline are 16.13, 15.21, 14.39 and 14.01 for $M = 0.0, 0.5, 1.5$, and 3.0 respectively. We observe in Figure 10 that as the values of M increases the thermal boundary layer thickness becomes higher gradually.

5. Conclusion

The combined effects of viscous dissipation and heat generation on natural convection flow along a vertical wavy surface have been studied. From the present investigation the following conclusions may be drawn:

- The skin friction coefficient C_{fx} has increased and the rate of heat transfer in terms of Nusselt number Nu_x has decreased for the effect of viscous dissipation parameter N and heat generation parameter Q .
- Streamlines have changed slightly too upper and the same results are observed for thermal boundary layer thickness with the increasing values of viscous dissipation parameter N and heat generation parameter Q .
- The skin friction coefficient C_{fx} , the rate of heat transfer in terms of Nusselt number Nu_x , the stream line as well as isotherms have decreased for increasing values of M .

Nomenclature

| | |
|---------------|--|
| C_{fx} | local skin friction coefficient |
| C_p | specific heat at constant pressure.....J.kg ⁻¹ .K ⁻¹ |
| f | dimensionless stream function |
| g | acceleration due to gravity..... m.s ⁻¹ |
| Gr | Grashof number |
| k | thermal conductivity..... W.m ⁻¹ .K ⁻¹ |
| L | wave length associated with the wavy surface..... m |
| N | viscous dissipation parameter |
| Nu_x | local Nusselt number |
| P | pressure of the fluid..... N.m ⁻² |
| Pr | Prandtl number |
| Q | heat generation parameter |
| Q_0 | heat generation constant |
| T | temperature of the fluid in the boundary layer..... K |
| T_w | temperature at the surface..... K |
| T_∞ | temperature of the ambient fluid..... K |
| u, v | dimensionless velocity components along the (x, y) axes..... m.s ⁻¹ |
| x, y | axis in the direction along and normal to the tangent of the surface |
| Greek symbols | |
| α | amplitude-to-length ratio of the wavy surface |
| η | dimensionless similarity variable |
| θ | dimensionless temperature function |
| ψ | stream function..... m ² .s ⁻¹ |
| μ | viscosity of the fluid..... kg.m ⁻¹ .s ⁻¹ |
| μ_∞ | viscosity of the ambient fluid |
| ν | kinematic viscosity..... m ² .s ⁻¹ |
| ρ | density of the fluid..... kg.m ⁻³ |
| σ_0 | electrical conductivity |

τ_w shearing stress

$\sigma(x)$ surface profile function defined in equation (1)

Subscripts

w wall conditions

∞ ambient conditions

Superscripts

' differentiation with respect to η

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