American Journal of Applied Mathematics
2013; 1(1): 17-23
Published online May 30, 2013 (http://www.sciencepublishinggroup.com/j/ajam)
doi: 10.11648/j.ajam.20130101.14

Measuring the forecast performance of GARCH and Bilinear-GARCH models in time series data

Akintunde Mutairu Oyewale, D. K. Shangodoyin, P. M Kgosi

Department of Statistics, University of Botswana, Botswana

Email address:
waleakintunde2004@yahoo.com(A. M. Oyewale)

To cite this article:
Akintunde Mutairu Oyewale, D. K. Shangodoyin, P. M Kgosi. Measuring the Forecast Performance of GARCH and Bilinear-GARCH Models in Time Series Data. *American Journal of Applied Mathematics.* Vol. 1, No. 1, 2013; pp. 17-23.
doi: 10.11648/j.ajam.20130101.14

Abstract: In most of the literature in time series modeling, generalized autoregressive conditional heterosceasticity (GARCH) models has been used as a traditional model to forecast both the economic and financial time series data. Though literature has shown that it is not suitable for non-linear time series. For this reason, this model was augmented with bilinear model in order to make it more relevant in forecasting both economic and financial time series data. After the augmentation, the new model called Bilinear-GARCH (BL-GARCH) shows a better performance based on performance measures indices, models variances and out-of–samples forecast performances. In term of these three criteria the new models outperformed the traditional or classical GARCH model. To drive home this point, these two models were illustrated with Botswana inflation rates data. We observed that the new model (BL-GARCH) outperformed the classical GARCH model.

Keywords: GARCH Models, BL-GARCH Models, Forecasting, Inflation Rates and Non-Linear

1. Introduction

Recent developments in financial econometrics require the use of model that will bring to the investor(s) high returns on their investments. In this context inflation rates play a very important role as these are major determinants in financial markets. The Generalization of ARCH model known as GARCH was introduced by Bollerslev (1986), which various researchers has extended. Deficiency of GARCH model was noted by many researchers like Hinich (1998), Liew, et.al.(2003), Lim et.al.(2005),Claudio and Jean (2011) and so many other researchers, they all concluded that GARCH model cannot capture non-linear aspect of the series adequately hence the need to find a more suitable model. An augmented GARCH model is an hybrid of the GARCH model formed by combining bilinear model with GARCH model (BL-GARCH). It allows us to capture asymmetries in the conditional mean and variance of financial and economic time series by means of interactions between past shocks and volatilities. The bilinear-GARCH models take into account variations between the independent variables as well as co-variations between the variables. This is very important in the study of financial market data where the covariance between independent variables may play a significant role in determining market volatility.

The remaining part of this paper is organized as follows: Section 2 covers the specification of GARCH models, estimation of the parameters of Bilinear-GARCH model (BL-GARCH), Section performance adequacy measurement 3, empirical illustration, identification of non-linearity status of the series, estimation of classical GARCH and BL-GARCH models Section 4 Forecast performance and section 5: conclusion

2. Specification of Generalized Autoregressive Conditional Heteroscedasticity Model

The Generalized Autoregressive Conditional Heteroscedasticity Model (GARCH) was proposed by Bollerslev (1986). The specification of GARCH (p,q) is as follows:

let (y_t) be the time series of an inflation rate return, then
$y_t = \sigma_t \varepsilon_t$ and

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + ... + \alpha_q y_{t-q}^2 + \beta_2 \sigma_{t-1}^2 + ... + \beta_p \sigma_{t-p}^2$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (1)$$

Where $\alpha_0 > 0, \alpha_i \geq 0$ and innovation sequence $\{\varepsilon\}_{i=-\infty}^{\infty}$ is independent and identically distributed (iid) with $E(\varepsilon_0)$=0 and $E(\varepsilon_0^2)$=1. The model described in equation (1) is used to parameterize financial time series and in particular foreign exchange.

To derive the variance of y_t from the conventional expression given as:

$$Var(y_t) = E(y_t^2) - (E(y_t))^2 \quad (2)$$

$$E(y_t^2) = \alpha_0 + \sum_{i=1}^{p} (\alpha_i + \beta_j) E(y_{t-1}^2) - \sum_{j=1}^{q} \beta_j E(Z_{t-j}) + E(Z_t)$$

$$\alpha_0 + E(y_t^2) \sum_{i=1}^{p} (\alpha_i + \beta_j)$$

reduces to

$$E(y_t^2) = \frac{\alpha_0}{1 - \sum_{i=1}^{p} (\alpha_i + \beta_j)} \quad (3)$$

Using (2) and (3) gives (4)

$$Thus\ Var(y_t) = \frac{\alpha_0}{1 - \sum_{l=1}^{p} (\alpha_i + \beta_j)} \quad \forall\ i \neq j \quad (4)$$

2.1. Bilinear- GARCH Model (BL-GARCH)

The mean and variance of augmented GARCH model are given as follows:

$$y_t = \sigma_t \varepsilon_t + \sum_{i=1}^{p} \sum_{j=i}^{q} \tau_{ij} y_{t-i} \varepsilon_{t-j} \quad (5)$$

$$y_t^2 = \sigma_t^2 \varepsilon_t^2 + \tau_I^2 y_{t-1}^2 \varepsilon_{t-1}^2$$

$$E(y_t^2) = E(\sigma_t^2 \varepsilon_t^2) + \tau_i^2 E(y_{t-1}^2)$$

That is, $$E(y_t^2) = \frac{E(\sigma_t^2)}{(1 - \tau_i^2)}$$

Since $E(\sigma_t^2) = E\left[\alpha_0 + \sum \alpha_i y_{t-1}^2 + \sum \beta_j \sigma_{t-1}^2\right]$

$$E(y_t^2) = \frac{E(\sigma_t^2)}{(1 - \tau_i^2)} = \frac{\alpha_0 + \sum \alpha_i \sigma_{t-1}^2 + \sum \beta_j \sigma_{t-1}^2}{1 - \tau_i^2} \quad (6)$$

$$Var\ (BL - GARCH)$$

$$= \frac{\alpha_0 + \sum \alpha_i \sigma_{t-1}^2 + \sum \beta_j \sigma_{t-1}^2}{1 - \tau_i^2} - \sigma_\varepsilon^4 \left(\sum \tau_i\right)^2 \quad \forall i=j \quad (7)$$

2.2. Model Evaluation Statistics Indices

$$\tilde{y}_{t+h}^2 = \alpha_0(1) + \sum_{i=1}^{p} \alpha_i y_{t-i}^2(h) + \sum_{j=i}^{q} \beta_j y_{t-j}^2(h) \quad (8)$$

The square root of equation (8) shall be used as a forecast function of GARCH models while the equation (9) below shall be used as a forecast function of BL-GARCH

$$\tilde{y}_t^2(h) = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2(h) + \sum_{j=i}^{q} \beta_j y_{t-j}^2(h) + \sum_{i=1}^{p} \sum_{j=1}^{q} \mathrm{T}_{ij}^2 y_{t+j}^2(h) \quad (9)$$

We shall compare the forecast performance of equations (8) and (9) using the following

2.3. Performance Adequacy Measures

Several error indices are commonly used in model evaluation. These include mean absolute error (MAE), root mean square error (RMSE), mean absolute deviation (MAD), mean absolute precision error (MAPE) and THEIL U. These indices are valuable because they indicate error in the units (or squared units) of the constituent of interest, which aids in analysis of the results. RMSE, MAE, MAPE, MAD and Theil U values of 0 indicate a perfect fit. Singh et al. (2004) state that RMSE and MAE values less than half the standard deviation of the measured data may be considered low and that either is appropriate for model evaluation.

1. $$RMSE = \sqrt{\left\{T^{-1} \sum_{t=1}^{T} (Y_t - \hat{Y}_t)\right\}} \quad (10)$$

2. $$MAE = T^{-1} \sum_{t=1}^{T} \left| Y_t - \hat{Y}_t \right| \quad (11)$$

3. $$MAD = T^{-1} \sum_{t=1}^{T} (Y_t - \overline{Y_t}) \quad (12)$$

4. $$MAPE = T^{-1} \sum_{t=1}^{T} \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| X\ 100 \quad (13)$$

5. $$\text{THEIL } U = \frac{\sqrt{T^{-1}\sum_{i=1}^{T}\left(X_t - \hat{X}_t\right)^2}}{\sqrt{T^{-1}X_t^2} + \sqrt{T^{-1}\hat{X}_t^2}} \qquad (15)$$

If the forecast error values are smaller, we say the forecast performance is good otherwise it is bad. If the results are not consistent among the first three we choose the MAPE to be the benchmark.

3. Results and Discussion

3.1. Checking the Stationarity of the Series

Before using any series, there is need to determine the stationarity of such series otherwise the whole exercise will be a nullity and as a result of this, three important methods are used in this paper to determine the stationarity of the series. These are graph, correlogram and unit root test.

Figure1 shows evidence of non-stationarity of the series, since volatile values are evident and these do not fluctuate around a constant mean.

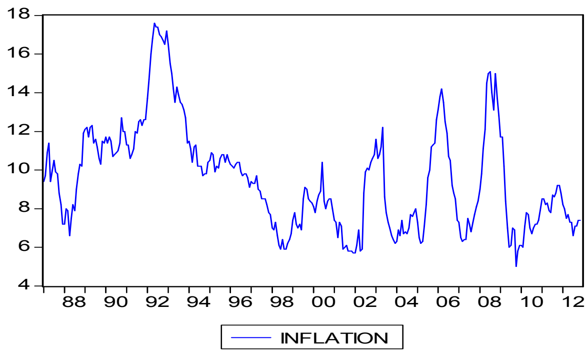

Figure 1. *Line graph of the leveled Botswana inflation rates*

We therefore transform the non-stationary series using the following formula:

$$R_t = \ln\left\{\frac{y_t}{y_{t-1}}\right\} * 100$$

After the transformation of the data, the new time series plot is shown below. The plot indicated that the mean of the series is now constant and as such we assume it is stationary.

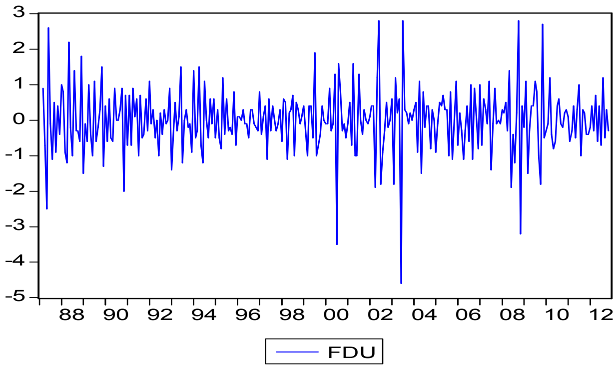

Figure 2. *Line graph of the first difference of Botswana inflation rates*

The results on the correlogram of the leveled for the series (as shown in table1 below) shows stronger evidence of non-stationarity since its autocorrelation coefficient function (ACF) of the residuals does not quickly decay to zero. it seems this series is not stationary.

So also the examination of correlogram of the first difference shows that there is no persistent trend and its values fluctuate around a constant mean of zero. hence this is suggesting that the exchange rate series is stationary

The table below shows that the result of Augmented Dickey-Fuller test on inflation rates series. since the statistic value for ADF test is greater than their corresponding critical values, so we do not reject the null hypothesis of the presence of unit root in the series and therefore conclude that the exchange rate is not stationary.

Table 1. *Leveled Correlogram for Botswana inflation rates*

Sample: 1987M01 2012M12						
Included observations: 311						
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|*******|	.|*******|	1	0.966	0.966	292.72	0.000
.|*******|	**|. |	2	0.917	-0.220	557.78	0.000
.|*******|	*|. |	3	0.864	-0.069	793.50	0.000
.|****** |	*|. |	4	0.802	-0.133	997.32	0.000
.|****** |	*|. |	5	0.732	-0.117	1167.7	0.000
.|***** |	.|. |	6	0.661	-0.012	1307.2	0.000
.|***** |	.|. |	7	0.592	-0.005	1419.3	0.000
.|**** |	*|. |	8	0.517	-0.113	1505.3	0.000
.|*** |	.|. |	9	0.446	0.023	1569.4	0.000
.|*** |	.|. |	10	0.377	-0.028	1615.5	0.000
.|** |	.|. |	11	0.311	-0.020	1646.9	0.000

.|** |	.|. |	12	0.246	-0.035	1666.6	0.000
.|** |	.|*** |	13	0.213	0.435	1681.4	0.000
.|* |	*|. |	14	0.190	-0.061	1693.2	0.000
.|* |	.|. |	15	0.174	0.048	1703.1	0.000
.|* |	.|. |	16	0.165	-0.017	1712.1	0.000
.|* |	*|. |	17	0.161	-0.087	1720.7	0.000
.|* |	.|. |	18	0.159	-0.031	1729.1	0.000
.|* |	.|. |	19	0.157	0.026	1737.3	0.000
.|* |	.|. |	20	0.164	0.027	1746.3	0.000
.|* |	.|* |	21	0.176	0.102	1756.7	0.000
.|* |	*|. |	22	0.189	-0.067	1768.7	0.000
.|** |	.|* |	23	0.207	0.120	1783.2	0.000
.|** |	.|* |	24	0.236	0.069	1802.0	0.000
.|** |	.|** |	25	0.268	0.286	1826.5	0.000
.|** |	.|. |	26	0.300	-0.018	1857.3	0.000
.|*** |	.|. |	27	0.329	-0.019	1894.5	0.000
.|*** |	.|. |	28	0.357	-0.017	1938.2	0.000
.|*** |	**|. |	29	0.374	-0.234	1986.5	0.000
.|*** |	*|. |	30	0.386	-0.060	2038.0	0.000
.|*** |	.|. |	31	0.393	-0.029	2091.8	0.000
.|*** |	.|. |	32	0.397	0.026	2146.7	0.000
.|*** |	.|* |	33	0.396	0.118	2201.5	0.000
.|*** |	*|. |	34	0.387	-0.147	2254.2	0.000
.|*** |	.|. |	35	0.369	-0.010	2302.3	0.000
.|*** |	.|. |	36	0.341	-0.034	2343.5	0.000

Table 2. *First difference Botswana inflation rates*

Sample: 1987M01 2012M12						
Included observations: 310						
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|* |	.|* |	1	0.171	0.171	9.1256	0.003
.|* |	.|. |	2	0.070	0.043	10.686	0.005
.|* |	.|* |	3	0.133	0.118	16.235	0.001
.|* |	.|* |	4	0.127	0.088	21.355	0.000
.|. |	.|. |	5	0.028	-0.017	21.607	0.001
.|. |	.|. |	6	0.013	-0.012	21.658	0.001
.|* |	.|. |	7	0.071	0.049	23.270	0.002
.|. |	*|. |	8	-0.048	-0.082	24.011	0.002
*|. |	.|. |	9	-0.065	-0.054	25.383	0.003
.|. |	.|. |	10	-0.023	-0.013	25.551	0.004
.|. |	.|. |	11	-0.018	-0.007	25.657	0.007
****|. |	****|. |	12	-0.473	-0.472	98.168	0.000
*|. |	.|. |	13	-0.150	0.006	105.46	0.000
*|. |	*|. |	14	-0.104	-0.068	109.01	0.000
*|. |	.|. |	15	-0.123	-0.003	113.96	0.000
*|. |	.|* |	16	-0.059	0.084	115.12	0.000
.|. |	.|. |	17	-0.043	0.005	115.72	0.000
.|. |	.|. |	18	-0.038	-0.019	116.21	0.000
*|. |	.|. |	19	-0.116	-0.044	120.66	0.000

*|. |	*|. |	20	-0.068	-0.095	122.19	0.000
.|. |	.|. |	21	0.003	0.002	122.19	0.000
.|. |	.|. |	22	-0.050	-0.056	123.04	0.000
*|. |	*|. |	23	-0.111	-0.084	127.21	0.000
.|. |	**|. |	24	-0.015	-0.263	127.29	0.000
.|. |	.|. |	25	0.030	0.010	127.59	0.000
.|* |	.|. |	26	0.073	0.037	129.41	0.000
.|. |	.|. |	27	0.034	-0.014	129.81	0.000
.|. |	.|* |	28	0.051	0.104	130.70	0.000
.|. |	.|. |	29	0.005	-0.037	130.71	0.000
.|. |	.|. |	30	0.020	0.001	130.85	0.000
.|. |	.|. |	31	0.048	-0.033	131.66	0.000
.|. |	*|. |	32	0.060	-0.060	132.93	0.000
.|* |	.|* |	33	0.090	0.095	135.76	0.000
.|* |	.|. |	34	0.103	0.041	139.52	0.000
.|* |	.|* |	35	0.177	0.098	150.58	0.000
.|. |	*|. |	36	0.063	-0.166	151.97	0.000

Table 3.

Null Hypothesis: INFLATION has a unit root			
Exogenous: Constant			
Lag Length: 12 (Automatic based on SIC, MAXLAG=15)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.186148	0.2119
Test critical values:	1% level	-3.452141	
	5% level	-2.871029	
	10% level	-2.571897	
*MacKinnon (1996) one-sided p-values.			

Table 4. *first difference Augmented Dickey-Fuller test statistic*

Null Hypothesis: D(INFLATION) has a unit root			
Exogenous: Constant			
Lag Length: 11 (Automatic based on SIC, MAXLAG=15)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.467909	0.0000
Test critical values:	1% level	-3.452141	
	5% level	-2.871029	
	10% level	-2.571897	
*MacKinnon (1996) one-sided p-values.			

3.2. Estimation of Classical GARCH Model

To generate parameter estimates for the GARCH model, we used E-view to analyzed differenced data for the study as follows:

Based on tables 5 the estimated GARCH(1,1) model are obtained for the series as follows:

$y_{Botswana\ Inflation} = \sigma_t \varepsilon_t$ where σ_t and ε_t are obtainable from the fitted model:

$$y\ y_{Botswana\ Inflation} = 0.995353 y_{t-1} + \varepsilon_t \text{ and } \sigma_t^2 = 0.386450 + 1.0103\ 97\ \varepsilon_{t-1}^2 - 0.040497^*\ \sigma_{t-1}^2 \tag{15}$$

Table 5. *GARCH model estimates for Botswana inflation rates*

Dependent Variable: INFLATION				
Included observations: 311 after adjustments				
Convergence achieved after 291 iterations				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
	Coefficient	Std. Error	z-Statistic	Prob.
DATE	0.003989	3.52E-05	113.4602	0.0000
	Variance Equation			
C	0.386450	0.110917	3.484122	0.0005
RESID(-1)^2	1.010397	0.281102	3.594408	0.0003
GARCH(-1)	-0.040497	0.052512	-0.771194	0.4406
T-DIST. DOF	414.8035	18607.56	0.022292	0.9822
R-squared	-0.345878	Mean dependent var		9.540836
Adjusted R-squared	-0.363471	S.D. dependent var		2.699959
S.E. of regression	3.152682	Akaike info criterion		4.131431
Sum squared resid	3041.457	Schwarz criterion		4.191556
Log likelihood	-637.4375	Durbin-Watson stat		0.049754

3.3. Estimation of Augmented GARCH Model

Estimation of parameters here was done here in two stages as the standard deviation obtained from classical GARCH was used to obtain the parameters of augmented GARCH models. The reduced form in equation (10) was estimated by making use of Bilinear (1,1) the reason for the choice of bilinear (1,1) was due to the fact that few parameters make the models to be parsimonious; from where sets of data were generated and OLS applied and the following results were obtained for the series By using the values generated in table 6 the BL-GARCH fitted for the series is as follows:

$$\text{Naira: } y_t = \sigma_t \varepsilon_t + \underset{(0.000044)}{0.087210}\, y_{t-1}\varepsilon_{t-1}$$

with variance of the model 0.855926982 (16)

Table 6. *Bilinear-GARCH model for Botswana inflation rates*

Dependent Variable: $y_t - \sigma_t \varepsilon_t$ **=ACMINFIT (Botswana inflation rates)**				
ACMINFIT=C(1)*ZT				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.030362	0.000722	42.02450	0.0000
	Variance Equation			
C	0.198439	0.069874	2.839955	0.0045
RESID(-1)^2	0.934091	0.265796	3.514319	0.0004
GARCH(-1)	0.059534	0.077003	0.773148	0.4394
T-DIST. DOF	2067.880	402682.1	0.005135	0.9959
R-squared	0.346199	Mean dependent var		1.564199
Adjusted R-squared	0.337624	S.D. dependent var		2.716843
S.E. of regression	2.211143	Akaike info criterion		3.756116
Sum squared resid	1491.192	Schwarz criterion		3.816383
Log likelihood	-577.1980	Durbin-Watson stat		0.058222

4. Forecast Performance

4.1. Forecast Evaluation Indices

Botswana inflation rates were subjected to all performance evaluation indices cross tabulated with GARCH and
Bilinear-GARCH models, it is clear from here that Bilinear-GARCH is far better than GARCH model as it produced the minimum of all this indices as seen in table (7) above.

Next we looked at the variance of the two models. A model that gives the minimum variance is adjudged as the better model. For instance from the table below (8). Bilinear-GARCH gave the minimum variance of 4.8892 compared to GARCH model which gave the variance of 9.9394 as shown below:

Table 7. *Forecast indices for the models*

model/indices	GARCH	BL- GARCH
MSE	3.1272	2.1932
MAE	2.3533	1.8188
MAPE	22.2050	18.6544
THEIL-U	0.1748	0.3966
BIAS PROPORTION	0.2501	0.1156
U-VAR PROPORTION	0.9266	0.8842
U-COV PROPORTION	0.0232	0.0002

Table 8. *Variances of the models*

Model	GARCH	BL-GARCH
Variance	9.9394	4.8892

Forecast performance of fitted GARCH (1,1) and BL-GARCH (1,1) models for the inflation rates has investigated shows that out-of-sample forecast performance for classical GARCH model for the inflation rate series under consideration failed to produced good forecast, whereas the BL-GARCH performed very well as showing in the following tables (9) and (10) below:

Table 9. *out-of-sample forecast performance for GARCH (2012)*

Date	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug	Sept.	Oct.	Nov.	Dec.
Actual	8.8	8.2	8.0	7.5	7.7	7.3	7.3	6.6	7.1	7.1	7.4	7.4
Forecast	6.03	6.03	6.03	6.03	6.03	6.03	6.03	6.03	6.03	6.03	6.03	6.03

Table 10. *Out-of-samples forecast performance for Bilinear-GARCH (2012)*

Date	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug	Sept.	Oct.	Nov.	Dec.
Actual	8.8	8.2	8.0	7.5	7.7	7.3	7.3	6.6	7.1	7.1	7.4	7.4
Forecast	8.83	8.35	8.02	7.68	7.89	7.78	7.48	6.71	7.13	7.13	7.42	7.42

4.2. Forecast Evaluation

Looking at table (9) and (10) which are the tables of out-of-samples forecast performance for the two models under study, it is evident that the forecast performance of classical GARCH model is not good as it gave constant values which under estimated the actual values, however, the forecast performance of Bilinear-GARCH model gave result that is comparable to the actual data. This shows that BL-GARCH outperformed classical GARCH model and as such recommended for would-be researcher(s).

5. Conclusion

This paper compared the two models used in measuring the forecast performance of Botswana inflation rates. The models are GARCH and Bilinear-GARCH, Monthly inflation rates of Botswana from 1978 to 2012 were used for empirical illustration. We examined the stationarity of the model using three tests (graph, correlogram and unit root), all these tests indicated that the series were not stationary. To ensure that the series are stationary we transformed the data and after the transformation the series was stationary. GARCH (1,1) and Bilinear-GARCH (1,1) was fitted to the model and there after we examined the forecast performance for the two models, in term of forecast performance indices and measurement of models variances Bilinear-GARCH model outperformed classical GARCH model. We also compared the out-of-sample forecast performance for the two models; Bilinear-GARCH also gave better forecast performance as seen from tables (9) and (10) above. So for would be forecasters, investors and other policy analysts the use of Bilinear-GARCH is recommended as it gave an excellent forecast compared to classical GARCH model

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