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A Quantitative Reasoning Framework and the Importance of Quantitative Modeling in Biology

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Abstract: Biology is becoming more quantitative. If we are to support the future of quantitative biology, then the next generation of biologists must be prepared to consistently integrate quantitative reasoning into subject matter that has traditionally been considered through a qualitative lens. We introduce a quantitative reasoning framework and discuss the importance of quantitative modeling in biology. The framework includes the Quantitative Act as a support for Quantitative Modeling and Quantitative Interpretation. The QM BUGS diagnostic instrument was developed to assesses undergraduate biology students' abilities to create and apply models employing pre-calculus mathematics. A brief discussion of our research findings based on implementation of the instrument include the lack of student ability to develop quantitative models. We present items from the instrument as examples of the Quantitative Act elements: variable quantification through identifying variable and attributes, measurement, variation, quantitative literacy, and context. We also provide items representing quantitative modeling and quantitative interpretation. We then view quantitative biology from K-12 and collegiate perspectives, including instructional practices for teaching quantitative biology, motivating problem contexts that afford quantification, instructional strategies of repetition, scaffolding, peer teaching and learning, direct instruction and teacher moves on the K-12 level, as well as identifying five competencies for the next generation of biologists which require QA abilities.

Keywords: Quantitative, Biology, Modeling, Interpretation

1. Introduction

Biology is evolving. From its foundation in the Natural Philosophy of ancient civilizations such as Mesopotamia and Egypt, the origins of modern biology sprung from the work of Aristotle (384-322 BC) in his History of Animals [1, 2]. The method of observation was the primary means of study in biology. Physical sciences were then brought to bear in biology, primarily as a means of building tools to assist in biological investigations, such as Friedrich Wohler's and Justus Liebig's application of physical and chemical methods in organic chemistry to better understand biological processes. But the quantitative study of biology has moved far beyond that now. D'Arcy Thompson's On Growth and Form [3] was influential in that it ushered in the application of physical and geometric principles in biology [4] and resulted in a scientific explanation for one of the fundamental aspects of developmental biology: morphogenesis – the biological process that causes an organism to develop its shape. The outcome of Thompson's efforts was an interdisciplinary field at the intersection of biology, physical sciences, and mathematics having at it is center the mathematical formulation of the physical mechanisms underlying morphogenesis [4]. Pioneers in this transition include the work by Galvani and Volta on animal electricity which led to the first battery [5] and research by Hodgkin and Huxley on the action potentials in the squid giant axon [6]. Over the 78 years since Thompson's influential work pushed biology in a quantitative direction, the field of biology has transitioned from a primarily qualitative view of phenomena to include quantitative analysis that provides congruent explanations that go beyond description.

In the 21st century, the use of mathematics in biology has become widespread and is generally accepted as a powerful and important tool [4]. With the availability of extensive quantitative data, researchers can now develop and test mathematical models of biological phenomena. The increased use of mathematics, statistics, and computational techniques to study life and living organisms has resulted in a number of new areas of study including: cellular identities within regularity networks [7], computation applied in study of nervous system [8], social interactions of flocking birds [9], and structures of protein molecules [10]. The future of quantitative biology looks to go beyond mathematical models of complex biological phenomena and toward the formulation of theories that can be tested quantitatively [11], thus expanding the bounds of biology to include quantitative science and providing for an understanding of the living world where the distinction between biology and physical sciences will vanish [4].

If we are to support the future of quantitative biology, then the next generation of biologists must be prepared to consistently integrate quantitative reasoning into subject matter that has traditionally been considered through a qualitative lens. We define the concept below:

Quantitative reasoning is mathematics and statistics applied in real-life, authentic situations that impact an individual's life as a constructive, concerned, and reflective citizen [12].

Quantitative Reasoning (QR) includes quantifying a problem by conceptualizing the focal object and assigning units of measure to its attributes (i.e., the Quantitative Act [QA]), developing and revising models to explain phenomena related to the object and its attributes (i.e., Quantitative Modeling [QM], and using models to make predictions (i.e., Quantitative Interpretation [QI]). QA is fundamental to meaningful student engagement in QM and QI. QA is the mathematical process of conceptualizing an object and an attribute of it so that the attribute has a unit measure. Included in QA is quantitative literacy, the use of fundamental mathematical concepts in sophisticated ways, which allows the student to describe, compare, manipulate, and draw conclusions from the quantified variables. Models require building blocks and an understanding of how those blocks go together to form mathematical expressions. This is the realm of QA, mathematizing a scientific context by identifying variables, assigning measures to the variables, and determining attributes that allow for comparing, contrasting and combining variables to from quantitative expressions. Mathematical expressions are the meta-building blocks used to create quantitative models. Quantitative reasoning is underpinned by QA.

In this chapter we introduce a quantitative reasoning framework and discuss the importance of quantitative modeling in quantitative biology. We then shift the focus to the Quantitative Act as a support for QM and QI. A summary of our research on Quantitative Biology through the lens of the quantitative reasoning framework is provided before offering examples of QA supporting QM and QI from K-12 and collegiate perspectives, and exploring both K-12 teacher professional development and faculty professional development in teaching Quantitative Biology. The chapter addresses the following questions:

1) What is quantitative reasoning?
2) What is the role of modeling in Quantitative Biology?
3) How do you measure quantitative reasoning in Biology?
4) How does Quantitative Biology look like in the classroom?
5) How do K-12 teachers or university faculty improve student's Quantitative Act ability?
6) What are the barriers to implementing Quantitative Biology in K-12 courses and undergraduate biology courses?

2. Quantitative Reasoning Framework

Across all areas of STEM, national organizations support improving students' quantitative reasoning abilities, which include quantitative act, quantitative modeling, and quantitative interpretation. [13-17]. One of the mechanisms recommended for improving quantitative reasoning is engagement in the construction, assessment, and revision of models for real-world natural phenomena [18, 19]. Students develop a hypothesis, create a model, make predictions using the model and compare them with data they collected, then revise the model as needed. Reference [19] state that the desired outcome is to generate a "defensible explanation for the way the natural world works". This requires interpretation of the model (QI) within in context. Both QM and QI are dependent on the student's ability in QA.

QA, QM and QI are inexorably intertwined, but a clear understanding of the relationship between the three elements of quantitative reasoning was lacking. We created a QR Framework to address this need. The following discussion of our quantitative reasoning framework is based on the NSF funded Culturally Relevant Ecology, Learning Progressions, and Environmental Literacy (or simply the Pathways project). The Quantitative Reasoning (QR) framework was based on Pathways research exploring the trajectory of QR development in science across sixth to twelfth grades. This framework introduced and defined three components of quantitative reasoning: Quantitative Act (QA), Quantitative Interpretation (QI), and Quantitative Modeling (QM).

We conceptualized the QR framework as a learning progression having four levels: the lower anchor, upper anchor and two intermediate levels of understanding. The lower anchor was based on the students' current understanding of QR, which was established through distribution and analysis of a QR assessment and interviews with 6th to 12th grade students. The upper anchor was based on what experts identified as desired QR understanding at the 12th grade level. The framework provides three progress variables, which are essential categories for the overall QR construct across which the levels are established. The QR progress variables are:

Quantification Act (QA): Mathematical conceptual-ization of an object and an attribute of it so that the attribute has a unit measure. This includes the use of fundamental mathematical concepts in sophisticated ways, which allows the student to describe, compare, manipulate, and draw conclusions from the quantified variables (i.e., quantitative literacy).

Quantitative Interpretation (QI): Use of models to discover trends and make predictions.

Quantitative Modeling (QM): Creation of quantitative representations to explain phenomena and to revise them based on fit to reality.

Finally, each of the progress variables was elucidated by identifying a collection of elements determined through student interviews that indicated essential abilities within the categories:

a. Quantification Act Elements: Variable Quantification, Variation, Quantitative Literacy, Context.
b. Quantitative Interpretation Elements: Trends, Predictions, Translation, Revision.
c. Quantitative Modeling Elements: Create model, Refine model, Reason with model, Statistical analysis.

For a detailed presentation of the QR framework see Reference [12].

Table 1. *Quantitative modeling framework of students' abilities.*

Model Formulation	Model Deployment	Meta-Modeling
Analyze – decompose phenomena into quantifiable variables	*Empirical Testing* – investigate quantitative interactions within phenomena	*Nature and Purpose of Models* - describe role of modeling in science
Inductive Reasoning – hypothesize how elements interact conceptually and quantitatively	*Evaluate* – assess degree of fit and ways to change model	*Steps of Modeling* - describe the iterative nature of modeling practice
Quantify – formulate a quantitative model	*Analogical Testing* – compare models to identify the best	
	Apply – use the model to predict or explain other phenomena	

2.1. Quantitative Modeling

Quantitative modeling is a means for developing and enhancing a conceptual understanding of natural phenomena. Successful model building is centered in model-based reasoning and meta-modeling abilities [20, 21]. Model-based reasoning is the ability for students to constuct scientific models in order to explain observed phenomena. Meta-modeling is understanding the nature of models and their utility and purpose [22]. Here we focus on the first of these abilities, model-based reasoning. The MoDeLs Project [23] has students construct and revise models based on empirical evidence. An anchoring phenomena is observered, then a model of the phenomena is constructed, empirically tested, evaluated, compared to other models, and revised before finally being used to predict or explain. Model-based reasoning makes it possible to understand and predict aspects or characteristics of the phenomena [24-26]. In the model-based framework, science is a complex, dynamic network of models. Scientific modeling is defined as the process of developing concrete representations of abstract concepts in science and the underlying mechanisms that cause physical phenomena and are driven by observations of physical phenomena [27-29, 19]. Modeling thus plays a central role in the formation and justification of new knowledge [30, 25]. Learners use models to link new information to prior knowledge, adjusting the models as needed to accommodate new experiences to enhance their understanding of the phenomenon under study.

Modeling-based learning (MbL) is a theoretical framework whereby learning takes place via student construction of models as representations of physical phenomena [31, 19]. In science education, the MbL approach is grounded in inquiry, constructivism, and constructionism traditions [23] and is used to promote scientific literacy and authentic scientific inquiry [32-33]. The construction and refinement of models has been shown to improve conceptual understanding, operational understanding of the nature of science, and ability to employ procedural and reasoning skills [34-35]. Modeling also engages students in communication about science [33], collaboration with peers [36-37], and encourages metacognition [38]. Moreover, modeling is an interdisciplinary skill, where modeling improvement in one discipline can transfer to other disciplines [39]. Research on the elements or steps in modeling is extensive [40, 41, 42, 23, 29, 19] and generally suggests students' abilities fall under model formulation and model deployment categories (Table 1). For the quantitative reasoning framework by [43] presented in the next section, model formulation includes QA in the analyze component, QL in the inductive reasoning component, QM in the quantify component, and QI in model deployment encompasses.

Modeling takes on many forms, including experiential (physical manipulatives), visual, verbal (qualitative discourse), numerical (quantitative data), or symbolic quantitative models [44]. Multiple model representations can provide different perspective of a problem and thus, have the potential to improve students' comprehensive understanding [45-46] as they encode and retrieve knowledge in different modalities [47]. Computational modeling and the simulation of models are two examples of modern quantitative modeling approaches that permit a deeper understanding of underlying mechanisms and the ability to investigate a complex problem or process holistically [48]. For example, quantitative modeling requires learners to develop a quantitative account of the phenomena and understand mathematical and conceptual interactions among the model components [12]. Translation across multiple representations is strengthened through quantitative interpretation of models when determining trends and making predictions [12]. Importantly, students who are given the opportunity to develop the quantitative models themselves become owners of the modeling process since they are responsible for learning about the phenomena [22-23], and selecting the model form that provides the most appropriate account of the particular facet of the phenomenon in which they are interested.

It is important for students to move from a qualitative account to a quantitative account in their models, providing the capacity for data-based argumentation to support their conclusions [49-50, 16, 51]. For example, students in middle and high school often provide qualitative accounts of phenomena in environmental science and chemistry and avoid employing quantitative reasoning about them [52, 53, 12]. The students in the Mayes et al. study [12] relied on a force-dynamic discourse based on personal experiences rather than scientific discourse [54], whereas the students who provided a quantitative discourse were better able to provide a convincing data-based argument [12].

2.2. Quantitative Biology

Quantitative biology is the use of mathematics, statistics or computational techniques to study life and living organisms. As problems researched in the field of biology are becoming more complex and interdisciplinary, modern experimental and computational tools are transforming biology from an observational to a data-intensive science. Grand challenges for complex biological systems over the next 20 years, as identified by the Biological and Environmental Research Advisory Committee [55], include enabling predictive biology through development of simulation models, modeling the evolution and dynamics of a complex biological system; measuring and analyzing biological systems through advanced computational and analytic methods; and exploring ecosystem function and elemental cycling. Extensive data and advanced computing are supporting interdisciplinary teams in conducting this research, where biologists are joined by chemists, physicists and mathematicians to tackle these grand challenges. Quantitative models have taken on a major role given the study of complex global problems and the development of software and inexpensive hardware that permit data analysis and simulation [56]. If quantitative biology is essential to future biology research, then we need to know more about how to integrate it into biology courses to better prepare students today for engaging in quantitative biology tomorrow. In this section, we discuss the Quantitative Modeling in Biology for Undergraduate Students (QM BUGS) project which informed our understanding of the current status of QR in undergraduate biology.

Research on student learning in quantitative biology courses has focused on improving students' numeracy skills, graphical interpretation of data, and inferences from mathematical models [57-58]. At the undergraduate level, quantitative accounts of biological system dynamics are often left to biology courses with a lack of consensus on whether to first introduce biology, mathematics, modeling, or technology [59, 60, 51]. Such courses started with the assumption that a quantitative account yields greater biological knowledge, with no metric for measuring the current or changing quantitative modeling abilities of the students. However, no instrument existed that examined the entirety of the modeling practice, from building to evaluating to revising the model and understanding the model's purpose. This left a gap in the research concerning students' cognitive and metacognitive modeling abilities and the practices students utilized while they built and revised quantitative models in undergraduate biology courses [61].

3. Measuring Quantitative Reasoning

3.1. QM BUGS Diagnostic Instrument

To address this need, we developed the QM BUGS diagnostic instrument which assesses undergraduate biology students' abilities to create and apply models employing pre-calculus mathematics. The Pathways QR framework was applied in the development of the diagnostic instrument, which measures QA, QM, and QI.

The foundation for student engagement in modeling biological phenomena is the quantification act (QA). The student should begin by mathematizing the biological context. Quantification begins by drawing the variables out of the context. Once identified, the variables are assigned appropriate measures for the context. Finally, attributes are determined for the variables, allowing for comparing, contrasting and combining variables to form quantitative expressions. So how do instructors engage students in QA?

A common practice we observed in the Pathways Project with 6th to 12th grade science classes and in QM BUGS with undergraduate biology courses was instructors providing quantitative models to students and asking them to interpret the models (QI). The students would jump right in, plugging numbers into the models without consideration of the context the model represented. For example, we observed students interpreting graphs without a clear conception of the variables assigned to the axes of the graph. The instructors failed to begin the QI exercise with appropriate attention to quantification of the variables. The result is meaningless calculation and a lack of connection of results with the context of the problem. This is evident when students provide an answer with no measure and no connection to other variables in the problem. This is reminiscent of a question asked in The Hitchhiker's Guide to the Galaxy, a comic science fiction series and movie created by Douglas Adams. Hyper-intelligent beings task the supercomputer Deep Thought with finding the answer to the ultimate question of life, the universe, and everything. After 7.5 million years of computing Deep Thought proclaims the answer to be 42. The computer (student) failed to provide any measure or meaningful attributes for the variable. The hyper-intelligent beings (teacher/faculty) never actually knew what the question was. This is an example of poor QA.

Modeling is challenging, so students may need scaffolding to develop modeling abilities. While a good scaffolding technique is to engage the students in quantitative interpretation (QI) of a model provided for them, the instructor must ensure that students start their interpretation with QA. Quantitative interpretation of models provided by the instructor was common, but the instructor often over directed the interpretation, not allowing students to fully engage. Slow down the QI process by providing time for students to fully engage in QA, including having students work in small groups to identify potential variables, label the variables, determine attributes of the variables, and determine a measure for the attributes that are appropriate for the context of the problem. Have students apply the attributes and measures to explore how the variables are related, a first step in building a model.

The ultimate goal of incorporating quantitative reasoning into biology is for students to develop their own models. In our observations of 6-12 and undergraduate biology this goal was seldom achieved and often not even attempted. QA and QI were employed, but students were not asked to create a model (QM). Quantitative modeling requires learners to develop a quantitative account of the phenomena and understand mathematical and conceptual interactions among the model components [43]. Translation across multiple representations is strengthened through quantitative interpretation of models when determining trends and making predictions [12]. If developing students' quantitative biology understanding is a goal, then providing at least one or two open-ended modeling tasks is essential. Remember that developing QM starts with QA. So let's look in more detail at the quantitative act.

3.2. Quantitative Act

In our framework, the Quantitative Act (QA) has four elements: variable quantification, variation, quantitative literacy, and context. Here we provide definitions and examples of each of the four elements. Examples will be drawn from the Quantitative Modeling in Biology for Undergraduate Students (QM BUGS III) diagnostic assessment. The assessment was designed by our research team to measure students' proficiency in quantitative modeling. QM BUGS III is a third version of the assessment which underwent rigorous review, reliability and validity testing, The diagnostic assessment consisted of 5 subsections - quantitative act (QA, 6 items), quantitative interpretation (QI, 6 items), quantitative modelling (QM, 8 items), metamodeling knowledge (MMK, 6 items), and quantitative biology capability confidence (QBCC, 12 items). Across the five subsections of QM BUGS III there are 38 questions: 26 multiple choice questions addressing four subcategories of QA, QI, QM, and MMK, and 12 Likert questions using a 4-level scale (from 1 Strongly Disagree to 4 Strongly Agree) addressing QBCC. The self-ratings items (QBCC) include 1 on Quantitative Act capability, 7 on Quantitative Modelling capability, and 4 on Quantitative Interpretation capability.

Variable quantification is the development of a mental construct for an object within a context that includes both attributes and measure [62]. Variable quantification builds the capacity to communicate quantitative accounts of a solution, facilitate decisions concerning solving the problem, and set a course of action within the context of the problem. QM BUGS III item 3 and item 4 (Figures 1 and 2) assessed variable quantification. The biological context for the assessment was transpiration in plants. This provided a context which undergraduate students have been exposed to either in high school or college. The following information on the context was provided as an opening to the assessment.

Transpiration Context: Transpiration is the process by which water is carried through plants from roots to small pores on the underside of leaves called stomata, where it is released as vapor into the atmosphere. The process is critical for plant cooling, accessing nutrients from the soil, providing water to plant cells to maintain rigidity, and supplying cells with hydrogen and oxygen atoms for cellular respiration. There are a number of atmospheric factors affecting transpiration. Answer the following questions on transpiration.

3.2.1. Variable Quantification: Identifying Variable and Attributes

QM BUGS III Item 3 addressed variable quantification characteristics of identifying variables and attributes of those variables from context. The students were provided five possible variables to use to answer a research question. They had to identify the appropriate variables to use given attributes of the variables. The desired response was 3e. Item 3 had the second highest correct response rate across the 20 questions on QA, QI, and QM.

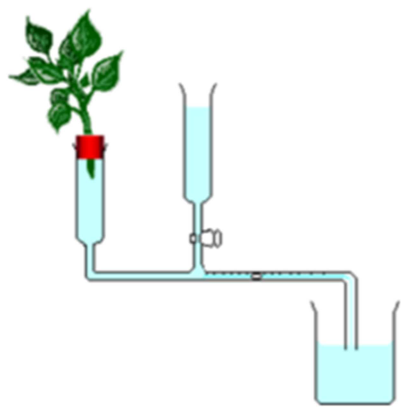

Figure 1. *Transpiration study apparatus.*

QM BUGS ITEM 3: Students were interested in the question: Will plants with larger leaves transpire more than those with smaller leaves? They conducted an experiment where they placed 3 different plants with small, medium and large leaves in sealed tubes (see apparatus in Figure 1). The plants were placed where they received the same amount of partial sunlight. The students measured temperature change (Co), relative humidity (%), and wind speed (mph) each day. They also measured the square area (cm^2) of each plant's leaf area. Over the course of two weeks they tracked the amount of water lost from the apparatus. Identify the variables that the students should use to answer their research question. (QA Variable Quantification – identify possible variables and attributes from context).

a. Temperature increase triggers plant to open stomata which increases transpiration rate, so they should use temperature and water lost data.
b. Relative humidity of air surrounding plant rises making it harder for water from plant to evaporate, so they should use relative humidity and leaf area data.
c. Leaf surface area and light to determine increased photosynthesis.
d. Light makes photosynthesis possible, plant opens stomata to take in CO_2 which increases transpiration, so they should use light and water lost data.
e. Leaf surface area increases number of stomata which would increase transpiration, so they should use leaf surface area and water lost data.

3.2.2. Variable Quantification: Measurement

QM BUGS III Item 4 addressed the variable quantification characteristic of measurement. The context provided information that indicated the appropriate measure to select, which was response 4c. Item 4 had the highest correct response rate of the 20 questions on QA, QI and QM.

QM BUGS ITEM 4: Transpiration is related to the rate of passage of water vapor exiting through the stomata of a leaf, which is called stomatal conductance. Stomatal conductance is a diffusion flux representing the movement of air from a region of high moisture concentration to a region of low moisture concentration across an area of leaf over a given time (Figure 2). Which of the following is an appropriate unit of measure for stomatal conductance? (QA Variable Quantification measure).

NOTE: A mole (mol) is the amount of substance, in this case water vapor, in a unit of air.

a. mol air/meter x second
b. mol air/meter^2
c. mol air/meter^2/second
d. mol air/meter^3/second
e. second/meter^2 x mol air

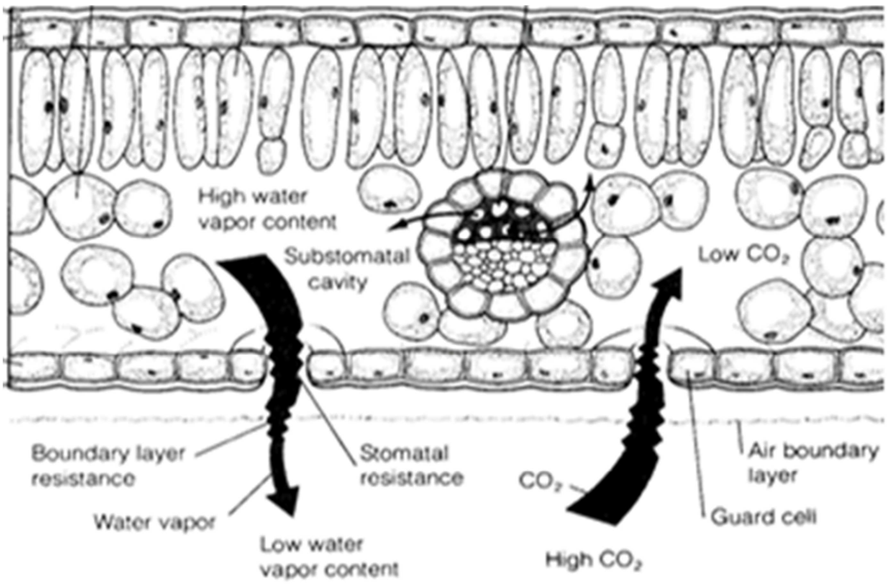

Figure 2. *Stomatal conductance in a leaf.*

3.2.3. Variable Quantification: Variation

Establishing relationships between variables within a context is reliant on the ability to reason about covariation of variables. Covariation entails comparing, contrasting, and relating variables in the context of the problem. Statistically covariance is a measure of joint variability between two variables. QM BUGS Item 2 addressed variation. A context is provided, then five possible variations are given. The student was to use the context of the study to determine the appropriate covariation between two variables. The desired response was 2e. This item did not rank in the highest five correct response rates of the 20 questions on QA, QI and QM.

QM BUGS ITEM 2: A biologist is studying transpiration of apple trees. She collected the data in Table 2 on two trees. The biologist compares variation in two variables to determine relationships. Which of the following is an important relationship between variables for her study? (Inductive Reasoning – hypothesize from data, Variation).

a. Variation in transpiration by variation of length of day.
b. Variation in stomatal resistance by variation in leaf area.
c. Variation in leaf area by variation in humidity.
d. Variation in transpiration rate by variation in temperature.
e. Variation in transpiration rate by variation in stomatal resistance.

Twelve selected sampling dates throughout year (DJ number of days since January 1, 1996), Temperature (T in Kelvin), Length of Day (N photoperiod duration in hours), Relative Humidity (RH mean percent), Leaf Area (LA in square meters), Transpiration Rate (TR in leaf water transpired per tree per day), and Stomatal Resistance is resistence to evaporation of water from stomata (s in meters per second).

3.2.4. Quantitative Literacy

Quantitative literacy is reasoning with quantities to explain relationships between variables. It is fundamental to moving from variable quantification to building quantitative expressions. Quantitative literacy includes ability to engage in proportional reasoning, numerical reasoning, and algebraic reasoning or higher mathematical reasoning. QM BUGS III.

Item 5 addressed quantitative literacy. A context was provided which included question on the quantitative relationship between four variables. Students were to translate the text into a quantitative expression modeling the relationship. The desired response was 5a. This item did not rank in the highest five correct response rates of the 20 questions on QA, QI and QM.

Table 2. *Data on transpiration of apple trees.*

Day DJ (1-1-96)	Temperature T (Kelvin)	Length of Day N (hours)	Relative Humidity RH (%)	Leaf Area LA (m^2)	Transpiration Rate (TR) (L H_2O tree^{-1} day^{-1})	Stomatal Resistance (s) (m s^{-1})
Tree 1						
184	289.8	15	92	9.3	7.3	302
188	292.3	15	75	9.3	15.1	509
191	292.5	15	73	9.3	15.3	550
198	292.1	15	68.3	9.3	12.8	572
200	293.5	15	68.9	9.3	16.6	602
205	296.4	15	82.6	9.3	10.9	475
Tree 2						
185	295.5	15	48	20.6	57.6	761
187	294.8	15	78	20.6	31.7	542
191	293.6	15	79	20.6	29.4	476
192	293.5	15	69.2	20.6	38.9	562
193	294.6	15	59.3	20.6	47.5	656
194	294.9	15	56.2	20.6	51.8	664

QM BUGS ITEM 5: Stomatal conductance Rvs is the rate of passage of water vapor exiting through the stomata (small pores) of a leaf. A steady state porometer (Figure 3) is an instrument that measures stomatal conductance by clamping it to the leaf surface, then computing the vapor flux between two locations on the diffusion path. The ratio of the change between vapor concentration at the leaf CvL and the concentration at the first sensor Cv1 to the combined stomatal resistance Rvs and resistance at the first sensor R1 is used in the vapor flux computation. Which of the following expressions represents this ratio? (QA Quantitative Literacy).

Figure 3. *Study state porometer measuring stomatal conductance.*

a. $\frac{C_{vL}-C_{v1}}{R_{vs}+R_1}$
b. $(C_{vL}-C_{v1})(R_{vs}+R_1)$
c. $\frac{C_{vL}-C_{v2}}{R_{vs}+R_2}$
d. $\frac{R_{vs}+R_1}{C_{vL}-C_{v1}}$
e. $\frac{C_{vL}-C_{v1}}{R_{vs}-R_1}$

3.2.5. Quantitative Context

Context involves taking a situated view of quantitative reasoning within a community of practice [63]. Ill-defined problems existing within socio-political contexts are solved by ad-hoc methods. Solutions require informal reasoning within the context [64]. QM BUGS III Item 1 (Figure 5) and Item 6 addressed elements of context. An anchoring phenomenon is provided in Item 1 within a real-world context and students are asked to reflect on it. The desired response was 1c. Item 1 had the fifth highest correct response rate of the 20 questions on QA, QI and QM.

QM BUGS ITEM 1: A biologist places a bag over a branch of a plant, leaves the bag on for a day, then comes back to observe the results (Figure 4). She sees water has collected in the bag. What does she hypothesize about her observation? (Analyze - Anchoring Phenomena, hypothesize from observation).

a. Bag is permeable and dew passed through bag.
b. High humidity air was trapped when she placed bag over limb.
c. There is water in the bag released from the leaves.
d. Photosynthetic process releases water into bag.
e. Water condensed in the bag due to temperature difference inside and outside the bag.

Figure 4. *Plant branch bag apparatus.*

QM BUGS III Item 6 addressed context through cultural appreciation for biology. A context related to applying biology to everyday life was provided. The students had to interpret a table to analyze the condition of their plant. The desired response was 6b. This item did not rank in the highest five correct response rates of the 20 questions on QA, QI and QM.

QM BUGS ITEM 6: Relative humidity has a big impact on growing your indoor plants. Growers pay attention to the impact of temperature on plants, but often ignore humidity. A rule of thumb for growers is to have 70% relative humidity for vegetative growth, but this does not take into account temperature. Vapor Pressure Deficit (VPD) is the difference between the vapor pressure inside a leaf compared to the vapor pressure of the air. VDP can be calculated from the air temperature and relative humidity. VPD provides the grower with a better indicator of how plants really "feel" and react to the combination of humidity and temperature. For best growing conditions you should keep the VPD between 7.5 and 10.5 kilopascal units of pressure (Table 3). Which of the following is the correct analysis of your plant condition with respect to VPD? (QA Context cultural appreciation).

Table 3. *Relative humidity by temperature chart.*

TEMP		Relative Humidity													
°C	°F	100%	95%	90%	85%	80%	75%	70%	65%	60%	55%	50%	45%	40%	35%
15	59	0.0	0.8	1.7	2.5	3.4	4.2	5.1	5.9	6.8	7.6	8.5	9.4	10.2	11.1
16	61	0.0	0.9	1.8	2.8	3.7	4.6	5.5	6.4	7.3	8.2	9.1	10.0	10.9	11.8
17	63	0.0	1.0	2.0	2.9	3.9	4.9	5.8	6.8	7.8	8.8	9.7	10.6	11.6	12.6
18	64	0.0	1.0	2.0	3.1	4.1	5.1	6.2	7.2	8.2	9.3	10.3	11.3	12.4	13.4
19	66	0.0	1.1	2.2	3.3	4.4	5.5	6.6	7.7	8.8	9.9	11.0	12.1	13.2	14.3
20	68	0.0	1.2	2.4	3.5	4.7	5.9	7.0	8.2	9.4	10.6	11.7	12.8	14.0	15.2
21	70	0.0	1.2	2.4	3.7	4.9	6.2	7.4	8.6	9.9	11.1	12.4	13.7	14.9	16.1
22	72	0.0	1.3	2.6	3.9	5.3	6.6	7.9	9.2	10.5	11.9	13.2	14.5	15.8	17.2
23	73	0.0	1.4	2.8	4.2	5.6	7.0	8.5	9.9	11.3	12.7	14.1	15.4	16.8	18.2
24	75	0.0	1.5	3.0	4.5	5.9	7.4	8.9	10.4	11.9	13.4	14.9	16.4	17.9	19.4
25	77	0.0	1.6	3.2	4.8	6.4	8.0	9.5	11.1	12.7	14.3	15.9	17.4	19.0	20.5
26	79	0.0	1.7	3.4	5.1	6.7	8.4	10.1	11.8	13.4	15.1	16.8	18.4	20.1	21.8
27	81	0.0	1.8	3.5	5.3	7.1	8.9	10.7	12.4	14.2	16.0	17.8	19.6	21.3	23.1
28	82	0.0	1.9	3.8	5.7	7.6	9.5	11.4	13.3	15.1	17.0	18.9	20.7	22.6	24.5
29	84	0.0	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.1	24.1	26.1
30	86	0.0	2.1	4.2	6.4	8.5	10.6	12.7	14.8	17.0	19.1	21.2	23.3	25.4	27.5
31	88	0.0	2.2	4.5	6.7	9.0	11.2	13.4	15.7	17.9	20.2	22.4	24.6	26.9	29.1
32	90	0.0	2.4	4.7	7.1	9.5	11.9	14.2	16.6	19.0	21.3	23.7	26.1	28.4	30.8
33	91	0.0	2.5	5.0	7.5	10.0	12.5	15.0	17.6	20.1	22.6	25.1	27.6	30.1	32.6
34	93	0.0	2.7	5.3	8.0	10.6	13.3	15.9	18.6	21.2	23.9	26.5	29.2	31.8	34.5

a. My plant has a leaf virus at 70°F with VPD 3.7 kilopascal so increase relative humidity.
b. My plant is exhibiting slow growth at 70°F with VPD 14.9 kilopascal so increase relative humidity.
c. My plant leaves wilted at 70°F with VPD 13.7 kilopascal so decrease relative humidity.
d. My plant is exhibiting slow growth at 70°F with VPD 8.6 kilopascal so decrease relative humidity.
e. My plant is forcing water out of the leaves at their edges at 70°F with VPD 2.4 kilopascal so increase relative humidity.

3.3. Quantitative Modeling and Interpretation

We provide an example of quantitative modeling and quantitative interpretation from the QM BUGS III diagnostic assessment to indicate the type of items addressing the other two components of quantitative reasoning.

Table 4. *Transpiration rate by temperature for trees.*

T (K)	TR (L/tree x d)
289.8	7.3
292.1	12.8
292.3	15.1
292.5	15.3
293.5	16.6
293.5	38.9
293.6	29.4
294.6	47.5
294.8	31.7
294.9	51.8
295.5	57.6
296.4	10.9

3.3.1. Quantitative Modeling

Developing closed form questions that address quantitative modeling is a challenge, since QM is centered on students developing a model for themselves. The eight items addressing QM included items on conceptual model development, phenomenological modeling, analytic models, graphic models, mechanistic models, statistical analysis, and refining a model. Here we share the phenomenological modeling Item 9. A phenomenological model describes the empirical relationship of phenomena to each other, in a way which is consistent with fundamental theory, but is not directly derived from theory. The desired response was 9b. This item ranked as the fifth most difficult in response rates of the 20 questions on QA, QI and QM.

QM BUGS ITEM 9: The biologist is working on building a model of Transpiration Rate (y-axis) by Temperature (x-axis) from the data in Table 4. She created a scatterplot of the data (Figure 5) and fit different models to the data. Which of the following is an appropriate strategy for modeling the data? (QM Create Model – phenomenological).

a. Fit a line that passes through the origin and another data point.
b. Fit multiple models and determine which has best fit.
c. Fit a linear model that passes through the most data points.
d. Fit a linear model that divides the data set in half.
e. Fit a non-linear model that passes through the first and last data points.

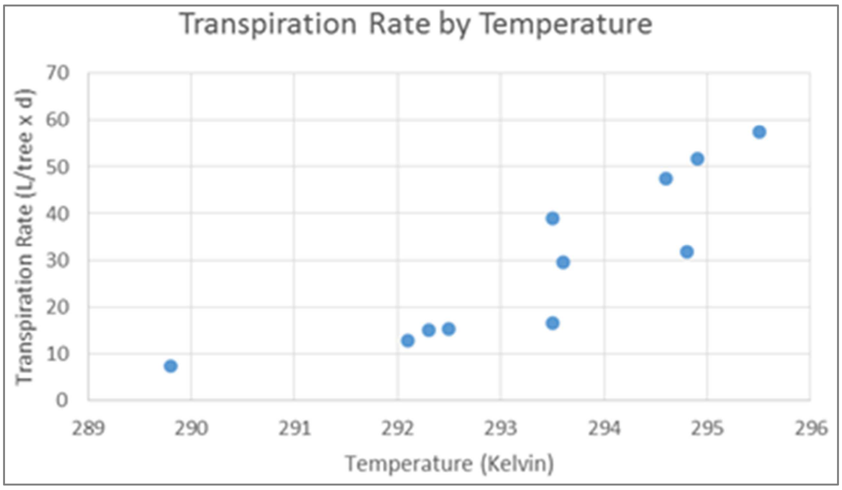

Figure 5. *Transpiration rate by temperature scatterplot.*

3.3.2. Quantitative Interpretation

The six questions on quantitative interpretation included items on empirical testing of models, model comparisons, applying trends in models, translation between models, making predictions from models, and revision of models. Here we share the prediction Item 19. Students were provided a graphic model and an analytic formula model and were asked to confirm a prediction. The desired response was 19c. This item had the third highest correct response rate of the 20 questions on QA, QI and QM.

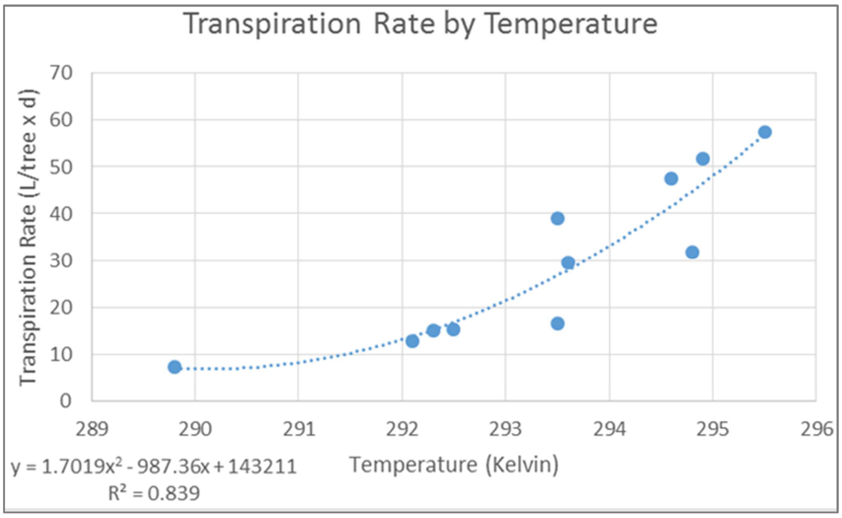

Figure 6. *Transpiration rate by temperature model.*

QM BUGS ITEM 19: Use the graph or equation model in Figure 6 to predict what happens to transpiration rate if the temperature is 300 K? Select the best answer. (Model Application QI Prediction).

a. Data was only collected between 289 and 296 degrees Kelvin, so you can't make a prediction for transpiration rate at 300 degree Kelvin.
b. You cannot extend the graph of the best fit curve to make an estimate above 296 degree Kelvin.
c. Transpiration rate is increasing at a nonlinear rate as temperature increases, at 300 degree Kelvin the transpiration rate is approximately 70 L/(tree x d).
d. Above 296 degrees Kelvin the transpiration rate remains at a constant value, so with 4 degrees increase from 296 to 300 Kelvin the transpiration rate grows to approximately 58.5 L/(tree x d).
e. Transpiration rate increases at a constant rate of about 1.7, so with 5 degrees increase from 295 to 300 degrees Kelvin the transpiration rate grows to approximately 56.5 L/(tree x d).

4. Quantitative Reasoning Results

Here we provide a short summary of the assessment findings on undergraduate biology students quantitative reasoning performance.

1) On the QM BUGS II assessment undergraduate biology majors tended to perform at a higher mean level on quantitative act ($M = 42.5$, $SD = 26$) than on quantitative interpretation ($M = 29.3$, $SD = 19$) and quantitative modelling ($M = 30.8$, $SD = 18$). However, the relatively low percentage correct (42%) provides reason for concern about their overall ability level.
2) Bonferroni post-hoc comparisons found QA performance differed significantly from QM (delta = 11.8, SE = 1.1), effect size of $d = .52$, $t(1) = 11.0$, $p < 0.001$ and from QI (delta = 13.2, SE = 1.1), effect size of $d = .58$, $t(1) = 12.0$, $p < 0.001$, while QI and QM were not significantly different (delta = 1.5, SE = 0.9), effect size of $d = .08$, $t(1) = 1.6$, $p = 0.339$. Overall, QI and QM performance levels were both over one half of one standard deviation below that of QA.
3) Evidence of relatively consistent associations among QA, QI, and QM was shown by Pearson correlations which indicated there was a significant positive correlation among all three subsections: QA and QM ($r = .33$, $p < .001$), QA and QI ($r = .32$, $p < .001$), and QM and QI ($r = .28$, $p < .001$).

Student performance on the QA subsection may have been higher than that of QI or QM due to the call for training, explicitly or implicitly, in quantitative literacy throughout secondary school [65]. Despite this type of exposure students have shown to be largely underprepared for QA [66-67]. The percentage correct for QA within our investigation is reflective of the 54% correct found by Johnson and Kaplan [68] in a study of quantitative literacy among undergraduate statistics students. Performance on the QA predicted performance on the QI and QM subsections, suggesting that students who reasoned about quantitative relationships were better prepared to conduct interpretation and modelling with the plant transpiration phenomena. Speth et al [57] found that incorporating quantitative literacy in undergraduate introductory biology courses through active-learning pedagogy improved quantitative skills, but construction of data-based scientific arguments was more of a challenge. Students in the present investigation performed particularly well on QA questions that included anchoring their understanding of transpiration by identifying a hypothesis, identifying relevant variables to study, and quantifying the variable of interest. The students were more comfortable with QA, given that four of the five easiest items for students were QA items. However, even the two easiest items, on quantifying a variable by determining an appropriate measure (item 4) or identifying variables with attributes in context (item 3), were correctly answered by only 57% (item 4) and 54% (item 3) of students. In addition, it is worth considering that the superior QA performance in this study may possibly be influenced by an order effect of QA being assessed first on the QM BUGS III assessment.

We hypothesized that students would perform at an intermediate level on the QI subsection given the push in biology to consider quantitative reasoning in the form of graphical analysis [13, 16] including how to interpret functional relationships [68]. Stanhope et al. [69] found that items related to visualizing data generally had low difficulty although a few questions about translating between a research question to a visual model (e.g., what is the best way to represent the data for temperature versus transpiration rate) had high difficulty levels. However, this intermediate level performance was not found in the present study and that likely contributed to the poor performance on QM. QM is not considered entirely dependent on QI performance in a hierarchical manner, although there are elements of QI that are important for QM elements. For example, the ability to refine a model will be predicated partially on one's ability to interpret the trends of the current model and make a mental prediction about a new model or new data. Goldstein and Flynn [70] found that even students who learned quantitative analysis skills had trouble applying the skills to interpret biological datasets. At the same time, refining a model also involves additional cognitive tasks like knowing the nature of models (quantitative relationships, biological significance) and the purpose of determining coherence with scientific evidence.

5. Quantitative Biology K-12 Level

The development of students' quantitative reasoning has become increasingly emphasized in K-12 education. In fact, one look at the framing of National Science Standards (i.e., Next Generation Science Standards [NGSS]) documents erases all doubt that the QR revolution has arrived [71]. For example, the bulk of science and engineering practices from the NGSS directly mirror aspects of QR (e.g., developing and using models, planning and carrying out investigations, analyzing and interpreting data, and engaging in argument from evidence), while the remaining practices are informed by QR at the very least (e.g., argumentation). Similarly, crosscutting concepts such as patterns, cause and effect, scale, proportion, and quantity, and stability and change, all reflect the importance of QR for contemporary science education in general and for biology education, specifically. Finally, given that a majority of the performance expectations include aspects of QR, it is clear that mastery of science standards requires fluency with QR and its constituent parts. In summary, the framing of core ideas, practices, and crosscutting concepts highlights the importance of quantitative literacy toward the preparation of K-12 students as informed citizens and underscores the significance of QR to a functional understanding of biology.

The inclusion of quantitative reasoning practices in the national science standards is certainly warranted – the real-world problems that schools are preparing students to solve require quantitative reasoning and as a result, is requisite to a functional understanding of biology. Current evidence supporting this assertion is the consideration of change in populations over time. For example, much of the news cycle in 2020 has focused on rates of COVID-19/coronavirus transfer among populations of countries around the world. COVID-19 is a real-world context with authentic opportunities to apply quantitative reasoning with the purpose of better understanding the coronavirus and slowing its spread. In response to the coronavirus outbreak, Sadler et al. [72] developed a module using the COVID-19 pandemic as an anchor and focusing on students' engagement in modeling practices. One of the modeling simulations enabled students to better understand how reproduction number (RO) related to the number of individuals infected with the disease. Another simulation used a computation model developed within Netlogo to simulate the impact that social distancing had on the spread of the virus. The modeling activities made it possible for students to analyze variables related to the growth and spread of the virus, adjust the variables to determine how they related to transmission rate, and situate quantitative reasoning in a real world context in which decisions had to be made to slow virus transmission (e.g., whether or not to socially distance). Decision-making regarding policy to quell the spread of COVID-19 would not be informed without a heavy dose of quantitative reasoning.

A favorite biology investigation of ours that requires students to engage in quantitative reasoning is called Life in Groups: Why Do Wolves Live in Groups from Sampson and Murphy's [73] Argument-Driven Inquiry in Third-Grade Science. The investigation explores why animals live in groups and focuses on wolves living in packs and the benefits to hunting that doing so affords. The goal of the investigation is for students to determine whether living in groups (packs) makes it easier for wolves to get food and survive. Students are tasked with watching a number of videos of wolves hunting prey of different sizes, including caribou (200-400 lbs.), elk (500 – 700 lbs.), and bison (1300 – 1500 lbs.), while looking for cause (group living) and effect (getting the food they need to survive) relationships. After devising methods for their study, students collect and analyze data with the purpose of supporting an argument for why wolves live in packs. The different elements of the QA are readily apparent in the problem (Table 5).

Table 5. *Elements of Quantitative Act necessary for 'Why Do Wolves Live in Groups?' investigation.*

Quantitative Act Element	Example
Variable quantification	Students must determine the aspects of hunting episodes that are important to the investigations so that they can devise a method for measuring each attribute as they engage in the investigation. Important attributes to account for in this investigation include the number of wolves, the type of prey (size), and the outcome of the hunt.
Variation	Students must consider how the important attributes in the investigation relate to one another. In this investigation, the number of wolves involved in the hunt would be expected to directly relate to the likelihood of a successful kill. Additionally, the size of the prey would also directly relate to the number of wolves necessary to succeed in hunting it.
Quantitative literacy	Students exhibit quantitative literacy by accounting for the relationships between variables. In this investigation, students should be able to reason that larger pretty are more difficult to kill and thus, likely require additional manpower to increase the likelihood of success. However, no matter the size, having a few wolves around to help separate weaker prey from the group should increase hunting success rate.
Context	Whereas this investigation as written by Sampson and Murphy did not include a socio-political context that in which the quantitative reasoning would need to be situated in order to resolve an issue, I often situate this investigation in the context of wolf reintroduction to Yellowstone, where quantitative reasoning about wolves' group living and hunting success would need to be integrated with perspective taking regarding a number of stakeholders (e.g., indigenous, farmers, ecologists, visitors, etc.) before a solution could be settled on.

Another example highlighting the importance of quantitative reasoning to biology education relates to "The Tragedy of the Commons" [74], where resources that are available to all will ultimately be overharvested and disappear. One context that is often used to illuminate the Tragedy of the Commons is fisheries biology, where overharvesting of fish has catastrophic effects on fish populations and can lead to their extinction. A popular activity that can be used effectively across grade levels is to have a bowl (lake) full of fish (goldfish snacks) and allow individuals who access the lake to fish at different rates to determine the effect of catch rate and reproduction rate on population size. We usually frame the activity with four fishers, each whom represents the head of a hungry family, whose only food source is the lake. The lake has a carrying capacity of 20 fish, and the fish remaining after each season (student fishing episode) double to account for reproduction (up to 20 fish carrying capacity, of course). The parameters are that the families of fisherman who catch < 2 fish starve but those that catch 2 fish survive. Each fish caught after the first 2 nets $1 from Captain D's, and the greatest provider of fish to Captain D's from the lake wins a $20 bonus and all you can eat hushpuppies. Before fishing ensues, each student writes what their daily catch will be and lists all factors that contributed to their decision, on their own and in silence.

Then, each individual tells their group how many fish they will take each season, an amount they cannot change (students are often aghast by the number of fish their peers chose to harvest). Once fishing begins, all fish usually disappear within the first few rounds (seasons) of fishing. The activity provides students with an opportunity to engage in quantitative reasoning by considering attributes and measure (e.g., number of fishers, number caught per day, reproduction rate), how each of those variables relate and why, functions that represent the relationships present, and the moral context provided by students selecting different daily catch and how they justify it through analysis of the sum of factors contributing to their determined harvest amount.

5.1. Instructional Practices for Teaching QR in Biology

All players in academia need additional support to usher in the era of quantitative reasoning. Seriously, whether we are talking about students, pre-service or in-service teachers, or even university faculty, most have generally not had substantive QR experiences, and those expected to teach QR could use significant support [75-76]. In the following paragraphs, we highlight instructional practices that can contribute to teachers' success in facilitating quantitative reasoning instruction.

5.2. Motivating Problem Contexts That Afford Quantification

Perhaps the first step to enhancing students' quantitative reasoning skills in biology is to identify contexts that afford quantification [77] that would be meaningful for students and require QR. Real world problems, such as the COVID-19 example, are often relatable to students and serve as meaningful contexts that can motivate learning, but even problem contests that are loosely based in reality (e.g., goldfish tragedy of the commons scenario) can garner significant interest and engagement. We find it helpful to make an effort to get them hooked before breaking out the quantitative reasoning provides motivation to consider the mathematics behind that which they got hooked on. For example, regarding COVID-19, a video clip of someone suffering on a respirator might engender a compassionate response and provide motivation to use quantitative reasoning in order to suggest solutions that would have the greatest likelihood of reducing that individuals' suffering. Alternatively, watching videos of wolves hunting and breaking down how they do it provides an interesting context for many to willingly engage in data collection with the purpose of generating meaningful explanations about the wolves hunting practices. The point is, quantitative reasoning for the sake of it may not interest the masses, but real world problems can be easily leveraged to hook students on using whatever means necessary to better understand the problem, including quantitative reasoning.

It is important to note teachers need not find new biological contexts for each grade level to engage their students in age appropriate quantitative reasoning. Rather, problem contexts that are ubiquitous in the biological sciences can be tailored for different age groups to ensure that the quantitative reasoning tasks you are expecting students to accomplish meet grade-level objectives and are within their 'zone of proximal development' [78, p. 86], or within their reach with the help of a peer or the teacher. For example, an educator can describe a Tragedy of the Commons activity around fisheries and carrying capacity that is appropriate for late elementary or middle school students. However, White, Timmons, and Medders [79] used the same context to meet learning objectives for their high school students.

5.3. Repetition

If students are to become adept at using quantitative reasoning skills to make sense of real-world problems, they need practice and repetition. Concepts like competition for resources and tragedy of commons are ubiquitous across sub disciplines of biology and can be used as a means for providing repeated bouts of practice with quantitative reasoning in different contexts using similar mathematics. For example, whereas we highlighted competition for resources in our Fisheries-Tragedy of the Commons activity, Bozzone [80] developed a different context for her students to engage in similar QR about phases of the amoebae lifecycle that are triggered by the depletion of food supplies. Similarly, Jessup, Ode, and Balgopal [81] described their implementation of a lesson about parasitic wasps, who inject hosts (e.g., moths) with venom and lay eggs on them. The larval offspring then compete for the food resources as they feed on the host. Providing students with multiple opportunities to practice QR skills in different contexts is a sure-fire way to aid their competence.

5.4. Scaffolding

Given that many students have not had ample opportunities to engage in quantitative reasoning, teachers cannot expect that students are comfortable with diving head first into quantitative reasoning and may wish to scaffold learning by providing supports at the beginning that are gradually removed as QR skills are mastered. For example, we have found that pre-service elementary teacher often struggle with variable quantification in the 'Wolves living in packs' investigation. We often find these students in the weeds, so to speak, considering variables that did not directly contribute to answering the research question at hand (such as whether it was winter or who the alpha male was). Teachers may choose to scaffold instruction such that skills can be developed that better prepare for problematic elements of QR, such as variable quantification in the case of my pre-service teachers. For example, in the parasitic wasp larvae activity [81], the investigation was framed to the students as a research project that had only been partially completed – the data had been collected but needed analysis. By providing the data set, the instructors could focus their students attention to whether and how the variables included in the data set were important to understanding parasitic wasp competition for resources (e.g., variation and quantitative literacy elements of QA), without first getting bogged down in trying to identify and quantify any number of variables that might be relevant. Scaffolding in such a manner could aid students in develop proficiency considering a handful of variables and their relation to a problem before turning them lose to resolve a different related problem. Using multiple contexts with similar themes, such as competition for resources as indicated in the previous paragraph, makes for smooth scaffolding, as students can gain confidence using similar skills to answer similar questions in novel situations.

5.5. Peer Teaching and Learning

Some of the most effective strategies for helping students gain proficiency with QR is to use peer teaching and learning strategies, such as think-pair-share, where students first consider and attempt to resolve problem contexts requiring QR before getting together with a peer(s) to see how their conception of meaningful objects, attributes, and their measurements in the context of their driving research question differed. For example, in the 'Why do wolves live in groups?' activity, students often struggled with QR practices, such as which variables to account for or how to represent findings with models. By having students first struggle with QR, they become conscious of their incompetence, and upon joining in groups with their peers, they see multiple means for reasoning about these things, and can discuss with their peers which strategies are best and why. We often have groups share the experimental design and model that they felt was most appropriate before highlighting those that did not make the cut and providing reasoning as to why they were inferior. Peer teaching and learning is not only effective, it also takes some of the work off the teachers' hands to identify and rectify each students' mistakes.

5.6. Direct Instruction and Teacher Moves

A final instructional practice for developing QR that is worth highlighting is direct instruction. There are a number of areas of QR that are common practices with which students often struggle. For example, students often struggle to formulate meaningful research questions, and even when they do, struggle to identify variables that are important for answering those questions. It is important for teachers to aid students in considering objects and attributes present in a problem set and determining their relevance to the research question, whether they are dependent or independent variables, and reason about their potential relation to one another. Similarly, students struggle to create models using data that meaningful inform the problem they are attempting to solve. It can be helpful to provide students with a number of graph types and discuss when and why each of the graphs are most appropriate. Finally, a teacher should take an evaluative stance and model for their students' constructive criticism and critique and normalizing their use in the classroom. Students often approach investigation through different lenses, and the research products they produce are diverse, which provides ample opportunity for deeply considering the pros and cons of the different methods that were used to solve problems and answer questions using QR.

6. Quantitative Biology on Collegiate Level

What guidelines, skill sets and competences should be considered in undergraduate and graduate quantitative biology programs? First, an overall training on the use of massive data sets and modern technology is needed. Bialek and Botstein [49] laid out an integrated curriculum that addressed this need by proposing that prerequisite mathematics, physics, chemistry and computation courses be replaced with quantitative biology courses that use authentic biology problems as contexts for meaningfully integrating concepts from these courses. Quantitative biology should be truly interdisciplinary by incorporating quantitative approaches and technology to analyze biological systems and construct model engineered life systems [82]. These programs need to develop students and future scientists that are adept at transdisciplinary approaches to solving tomorrows problems in biology. Tan et al. [83] identified a list of competencies for the next generation of biologists:

1) Basic knowledge in the specific domains of computer science, statistics, and mathematics that support modern biology.
2) Expertise in communicating and representing biological knowledge and processes in mathematical, statistical, and computing terms and concepts.
3) Ability to use or develop efficient bioinformatics and biocomputational tools and techniques for the acquisition, interpretation, analysis, prediction, modeling, simulation, and visualization of experimental and other biological data.
4) Proficiency in the search, retrieval, processing, curation, organization, classification, management, and dissemination of biological data and information in databases for deriving biological insights and knowledge discovery.
5) Critical thinking and problem-solving skills in quantitative aspects of biology.

All five of these competencies require QA abilities.

The barriers to changing university biology programs to become quantitative biology-focused are significant. Bioinformatics education, which is the teaching and learning of the use of computer and information technology to gather, store, analyze, interpret, and integrate data to solve biological problems [84], is closely tied to QB, and effectively integrating bioinformatics into university programs has been challenging due to (1) its cross-disciplinary nature; (2) disparate methods, outlooks and cultures of its related disciplines; (3) the lack of an integrated training support structure [85]. The Course Source Bioinformatics Learning framework [86] supports the effort by providing course curricula that includes the competencies above. Other curricula supporting the competencies have been developed and piloted, including the course Computational Approaches for Life Scientists [87] which integrates abstract, algorithmic, and logical thinking in a computational culture. If creating a new program is not possible, college programs might consider creating quantitative biology interest groups, peer-to-peer learning events such as hackathons and workshops such as the Santa Barbara Advanced School of Quantitative Biology, or QB bootcamps [82]. Eaton et al [88] identify a number of additional reform efforts in undergraduate biology education to improve QB, including the Professional Society Alliance for Life Science Education, Quantitative Undergraduate Biology Education and Synthesis - QUBES [89], Intercollegiate Biomathematics Alliance, Mathematical Biosciences Institute, National Institute for Mathematical Biology and Synthesis – NIMBioS, Interdisciplinary Training for Undergraduates in Biological and Mathematical Sciences – UBM, and National Consortium for Synergistic Undergraduate Mathematics via Multi-institutional Interdisciplinary Teaching Partnerships – SUMMIT-P.

Efforts to implement QB courses are reported in the literature with many focused specifically on aspects of QR. For example, Speth et al [57] incorporated quantitative literacy skills into large-enrollment introductory biology courses for science majors. They focused on representing and interpreting data and articulating data-based arguments. The sample of 175 freshmen and sophomore students included life science majors and prehealth or preveterinary students. The students completed active, inquiry-based modules, homework, and assessments that incorporated QL. QL skills included graphing data, labeling axes appropriately, using an appropriate graph to represent data. Preassessment results indicated that students had difficulties with representing data on a graph, labeling axes, and formulating complete and correct arguments. Students made significant gains in the first two skills, but still struggle with formulating arguments. They concluded that student-centered inquiry-based learning environments were well-suited to support development of quantitative literacy skills. The three major challenges to incorporating QL in introductory biology courses they identified were: how to fit QL into a crowded curriculum, what quantitative skills are most important to include, and how to assess students' development of those skills. First, infusion of QL does not district from content, but greatly supports teaching for conceptual understanding. Second, the QL skills they began with were frequencies, substituting numbers into a formula, ability to perform simple calculations, distinguish expected versus observed results, and interpreting a statistical test of significance. Third, use immediate formative assessment to provide feedback on skill development and incorporate authentic assessment where students have to demonstrate their ability within a biological context.

Hoffman et al. [58] integrated quantitative reasoning modules into an introductory undergraduate ecology and evolution biology course. The modules were designed to improve quantitative numeracy, which is a component of QA, interpreting data (QI) and making inferences using models (QI). Four modules were integrated into the courses using active-learning approaches: Mendelian Genetics, Introduction to Mathematical Modeling, Population Genetics I – National Selection, and Population Geneitics II – Gene Flow and Genetic Drift. Students improved their quantitative numeracy abilities from pre-test to post-test and regardless of initial abilities, they obtained comparable levels of proficiency. Students also showed significant improvement in interpreting data but gains in making inferences were highly variable. Interviews with students in the classes indicated that few students understood covariation, a QA ability. They concluded that greater and sustained QA improvements requires consistent integration throughout the biology curriculum.

7. Conclusion

A number of promising efforts in and resources for quantitative biology are provided in this chapter. The QA outcomes of these efforts include improved quantitative numeracy, facility with the language of mathematics, extracting relevant information from large data sets, understanding covariation, and simply quantifying a biological context. But there are other important potential outcomes of integrating quantitative reasoning, including QA, QI, and QM, into biology. The active learning strategies that were used in some of the examples promote collaboration across disciplines, engender positive student attitudes about quantitative methods that reduce student anxiety and increase their willingness to engage in biology, and increase student persistence in taking more biology courses [90-91]. Aikens and Dolan [61] believe that development of quantitative skills in biology can have the following impacts:

1) More positive emotional responses to quantitative work – more enjoyment, less anxiety.
2) More positive beliefs about the ability to do quantitative work – increase confidence and self-efficacy.
3) Greater sense of centrality of mathematics, statistics, and computation to the practice of life sciences.
4) Improved ability to work in interdisciplinary teams.
5) Increased persistence in pursuing and careers in quantitative biology.

The integration of quantitative methods into biology has deep learning and future workforce implications. It all begins with the Quantitative Act (QA).

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