Applied and Computational Mathematics
2014; 3(6): 303-306
Published online December 23, 2014 (http://www.sciencepublishinggroup.com/j/acm)
doi: 10.11648/j.acm.20140306.13
ISSN: 2328-5605 (Print); ISSN: 2328-5613 (Online)

(α, β)- infimum and supremum of Q- Fuzzy subgroups over implication operator of M* ([0,1])

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To cite this article:
R. Nagarajan, K. Balamurugan. (α, β)- Infimum and Supremum of Q- Fuzzy Subgroups over Implication Operator of M* ([0,1]). *Applied and Computational Mathematics.* Vol. 3, No. 6, 2014, pp. 303-306. doi: 10.11648/j.acm.20140306.13

Abstract: In this paper, the concept of (α,β)- inf-sup Q-fuzzy set is generalized and there after we defined (α,β)- inf-sup Q-fuzzy group and a few of its properties are discussed. On the other hand we give the definition of the upper normal Q- fuzzy subgroups, and study the main theorem for this. We also give new results on this subject. Characterization of inf-sup normal Q-fuzzy subgroups also investigated.

Keywords: Fuzzy Set, (α,β)- Inf-Sup Q-Fuzzy Group, (α,β)- Inf-Sup Q-Fuzzy Normal Subgroups, Q-Fuzzy Subset, Fuzzy Group

1. Introduction

The fundamental concept of fuzzy sets was initiated by L.Zadeh[13] in 1965 and opened a new path of thinking to mathematicians, engineers, physicists, chemists and many others due to its diverse application is various fields. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information science, coding theory, group theory, real analyses, hectare theory etc. In 1971, Rosenfeld[9] first introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic structures. Thereafter the notion of different fuzzy algebraic structures such as fuzzy ideals in rings and semi rings etc, have seriously studied by mathematicians. In 1965, Zadeh introduced the concepts of interval-valued fuzzy sets, where the values of member instead of the real points.

His definitions has been generalized by Anthony and Sherwood[1]. They introduced the concept of fuzzy normal subgroups also. Mukherjee and Bhattacharya [2,3] studied the normal fuzzy groups and fuzzy cosets, on the other hand the notion of a fuzzy subgroup abelian group was introduced by Murali and Makamba[5,6]. The concept of Q-fuzzy group is introduced in [8,10,11].The purpose of this paper is to generalize new definition of (α,β)- inf-sup Q-fuzzy groups and using this definition to study some properties for this subject.

2. Preliminaries

Definition 2.1:[13] Let X be a set. Then a mapping that is $\mu : X\to[0,1]$ is called a fuzzy subset of X.

*Definition 2.2:[10]*Let Q and G be a set and a group respectively. A mapping $A:GxQ\to [0,1]$ is called Q-fuzzy set in G. For any Q-fuzzy set A in G and t Є [0,1]. The set U (A:t) ={xЄG/ A(x,q)≥t, qЄQ} which is called an upper cut of A.

*Definition 2.3:[9]*Let G be any group. A mapping μ: $G\to[0,1]$ is called a fuzzy group of G if

(1) $\mu (x,y)\leq \max\{ \mu (x), \mu (y)\}$
(2) $\mu (x^{-1}) \leq \mu (x)$ for all x, y ЄG

*Definition 2.4:[10]*A Q-fuzzy set A is called Q-fuzzy group of G if

(QFG1) : $A(xy,q) \geq \text{Min}\{A(x,q),A(y,q)\}$
(QFG2): $A(x^{-1},q) = A(x,q)$
(QFG3) : A(x,q)=1 for all x,yЄG and qЄG.

*Proposition 2.1:*If μ is a Q-fuzzy group of a group G having identity e, then

(1) $\mu(x^{-1},q)=\mu(x,q)$(ii) $\mu(e,q) \leq \mu(x,q)$ $\forall X\epsilon g$.

Proof: it is obivious.

*Definition 2.5: :[10]*Let μ be a Q-fuzzy group of G. then "μ" is called a Q-fuzzy normal group if μ (xy , q) = μ (yx,q) ∀ x, y ЄG

Definition 2.6 :[8] Let X be a set then a mapping $\mu:X\times Q\to M^{*}([0,1])$ is called inf-sup subset of X, where

$M^*([0,1])$ denotes the set of all non empty subset of [0,1]

*Definition 2.7:[3]*Let X be a non empty set and μ and λ be two inf-sup Q-fuzzy subset of X. Then the intersection of μ and λ denoted by $\mu\cap\lambda$ and defined by

$\mu\cap\lambda=\{\min\{a,b\}/\ a\in\mu(x),b\in\lambda(x)\}$ for all $x\in X$.The union of μ and λ and denoted by $\mu\cup\lambda$ and defined by $\mu\cup\lambda=\{\max\{a,b\}/\ a\in\mu(x),b\in\lambda(x)\}$ for all $x\in X$.

Definition 2.8:Let X be a groupoid that is a set which is closed under a binary relation denoted multiplicatively. A mapping is called(α,β)- inf-sup Q-fuzzy groupoid if for all $x,y\in X$, following conditions hold:

(1) $\inf\mu(xy,q)\cap\alpha\leq S\{\inf\mu(x,q),\inf\mu(y,q)\}\,⌂\,\beta$

(2) $\sup\mu(xy,q)\cap\alpha\leq S\{\sup\mu(x,q),\sup\mu(y,q)\}\,⌂\,\beta$

Definition2.9:Let G be a group. A mapping $\mu: G\times Q\rightarrow M^*([0,1]$ is called (α,β)- inf-sup Q-fuzzy group of G if for all $x,y\in G$, following conditions hold:

(1) $\inf\mu(xy,q)\cap\alpha\leq S\{\inf\mu(x,q),\inf\mu(y,q)\}\,⌂\,\beta$

(2) $\sup\mu(xy,q)\cap\alpha\leq S\{\sup\mu(x,q),\sup\mu(y,q)\}\,⌂\,\beta$

(3) $\inf\mu(x^{-1},q)\cap\alpha\leq\inf\mu(x,q)\,⌂\,\beta$

(4) $\sup\mu(x^{-1},q)\cap\alpha\leq\sup\mu(x,q)\,⌂\,\beta$

*Note:*In definition if $\mu: G\times Q\rightarrow[0,1]$ then $\mu(x,q)\ \forall\ x\in G$ are real points in [0,1] and also $\inf(\mu(x,q))=\sup\mu(x,q)=\mu(x,q)$, $x\in G$ and $q\ \varepsilon\ Q$. Thus definition (2.9) reduces to definition of (9)..

So (α,β)- inf-sup Q-fuzzy group is a generalization of (9).

3. Main Results

*Proposition 3.1:*An (α,β)- inf -sup Q-fuzzy subset μ of a group G is (α,β)- inf-sup

Q-fuzzy group iff for all $x,y\in G$ followings are hold.

(1) $\inf\mu(xy^{-1},q)\cap\alpha<S\{\inf\mu(x,q),\inf\mu(y,q)\}\,⌂\,\beta$

(2) $\sup\mu(xy^{-1},q)\cap\alpha<S\{\sup\mu(x,q),\sup\mu(y,q)\}\,⌂\,\beta$

Proof: At first let μ be (α,β)- inf-sup Q-fuzzy group of G and $x,y\in G$. Then

$$\inf\mu(xy^{-1},q)\cap\alpha<S\{\inf\mu(x,q),\inf\mu(y^{-1},q)\}\,⌂\,\beta$$

$$=S\{\inf\mu(x,q),\inf\mu(y,q)\}\,⌂\,\beta \text{ and}$$

$$\sup\mu(xy^{-1},q)\cap\alpha<S\{\sup\mu(x,q),\sup\mu(y,q)\}\,⌂\,\beta$$

$$=S\{\sup\mu(x,q),\sup\mu(y,q)\}\,⌂\,\beta$$

Conversely, let μ be (α,β)- inf-sup Q-fuzzy subset of G and given conditions hold. Then for all $x\in G$, we have

$$\inf\mu(e,q)\cap\alpha=\inf\mu(xx^{-1},q)\cap\alpha\leq S\{\inf\mu(x,q),\inf\mu(x,q)\}\,⌂\,\beta=\inf\mu(x,q)\,⌂\,\beta \quad (1)$$

$$\text{Sup}\,\mu(e,q)\cap\alpha=\sup\mu(xx^{-1},q)\cap\alpha\leq S\{\sup\mu(x,q),\sup\mu(x,q)\}\,⌂\,\beta=\sup\mu(x,q)\,⌂\,\beta \quad (2)$$

So,

$$\inf\mu(x^{-1},q)\cap\alpha=\inf\mu(ex^{-1},q)\cap\alpha\leq S\{\inf\mu(e,q),\inf\mu(x,q)\}\,⌂\,\beta=\inf\mu(x,q)\,⌂\,\beta \text{ by (1)}$$

And

$$\sup\mu(x^{-1},q)\cap\alpha=\sup\mu(ex^{-1},q)\cap\alpha\leq S\{\sup\mu(e,q),\sup\mu(x,q)\}\,⌂\,\beta=\sup\mu(x,q)\,⌂\,\beta \text{ by (2)}$$

Again

$$\inf\mu(xy,q)\cap\alpha\leq S\{\inf\mu(x,q),\inf\mu(y^{-1},q)\}\,⌂\,\beta$$

$$\leq S\{\inf\mu(x,q),\inf\mu(y,q)\}\,⌂\,\beta$$

And

$$\sup\mu(xy,q)\cap\alpha\leq S\{\sup\mu(x,q),\sup\mu(y,q)\}\,⌂\,\beta$$

$$\leq S\{\sup\mu(x,q),\sup\mu(y,q)\}\,⌂\,\beta$$

Hence μ is (α,β)- inf-sup Q-fuzzy group of G.

Proposition 3.2: If μ is an(α,β)- inf-sup Q-fuzzy groupoid of a infinite group G, then μ is an(α,β)- inf-sup Q-fuzzy group of G.

Proof:

Let $x\in G$. Since G is finite, x has finite order, say p. then $x^p=e$, where e is the identity of G. Thus $x^{-1}=\mu^{\,p-1}$ using the definition of (α,β)- inf-sup Q-fuzzy groupoid, we have

$$\inf\mu(x^{-1},q)\cap\alpha=\inf\mu(x^{p-1},q)\cap\alpha=\inf\mu(x^{p-2},q)\cap\alpha\leq S\{\inf\mu(x^{p-2},q),\mu(x,q)\}\,⌂\,\beta$$

Again

$$\inf\mu(x^{p-2},q)\cap\alpha=\inf\mu(x^{p-3},x,q)\cap\alpha\leq S\{\inf\mu(x^{p-3},q),\mu(x,q)\}\,⌂\,\beta$$

Then we have

$$\inf\mu(x^{-1},q)\cap\alpha\leq S\{\inf\mu(x^{p-3},q),\inf\mu(x,q)\}\,⌂\,\beta$$

So applying the definition of (α,β)- inf-sup Q-fuzzy groupoid repeatedly, we have that $\inf\mu(x^{-1},q)\cap\alpha\leq\inf\mu(x,q)\,⌂\,\beta$

Similarly we have $\sup\mu(x^{-1},q)\cap\alpha\leq\sup\mu(x,q)\,⌂\,\beta$

Therefore μ is (α,β)- inf-sup Q-fuzzy group.

Proposition 3.4: The Intersection of any two (α,β)- inf-sup Q-fuzzy groups is also (α,β)- Inf-sup Q-fuzzy group of G.

Proof : Let A and B be any two (α,β)- inf-sup Q-fuzzy groups of G and $x,y\in G$ then

$\inf(A\cap B)(xy^{-1},q)\cap\alpha=S\{\inf A(xy^{-1},q),$

$\inf B(xy^{-1},q)\}\,⌂\,\beta\leq S\{S\{\{\inf A(x,q),$

$\inf A(x,q)\}\,⌂\,\beta,\{\inf B(x,q),\inf B(y,q)\}\}\}\,⌂\,\beta\ S\{S\{\inf A(x,q),\inf B(x,q)\}\,⌂\,\beta,S\{\inf A(x,q),\inf B(y,q)\}\}\}\,⌂\,\beta$

$$=S\{\inf A\cap B(x,q),\inf A\cap B(y,q)\}\,⌂\,\beta \quad (3)$$

Again

$$\sup(A\cap B)(xy^{-1},q)\cap\alpha=S\{\sup A(xy^{-1},q),\sup B(xy^{-1},q)\}\,⌂\,\beta \text{ by definition}$$

$$\leq S\{S\{\{\sup A(x,q),\sup A(x,q)\}\,⌂\,\beta,\{\sup B(x,q),\sup B(y,q)\}\}\}\,⌂\,\beta\ S\{S\{\sup A(x,q),\sup B(x,q)\}\,⌂\,\beta,S\{\sup A(x,q),\sup B(y,q)\}\}\}\,⌂\,\beta$$

$$=S\{\sup A\cap B(x,q),\sup A\cap B(y,q)\}\,⌂\,\beta \quad (4)$$

Hence by (3) and (4) and using proposition we say $A \cap B$ is (α,β)- inf-sup Q-fuzzy group of G.

*Proposition 3.5:*If A is a (α,β)- inf-sup Q-fuzzy group of a group G having identity e, then $\forall x \in X$

(1) $\inf A(x^{-1},q) \cap \alpha = \inf A(x,q) \triangle \beta$ and $\sup A(x^{-1},q) \cap \alpha = \sup A(x,q) \triangle \beta$

(2) $\inf A(e,q) \cap \alpha \leq \inf A(x,q) \triangle \beta$ and $\sup A(e,q) \cap \alpha = \sup A(x,q) \triangle \beta$

Proof :

(1) As A is a (α,β)- inf-sup Q-fuzzy group of a group G, Then $\inf A(x^{-1},q) \leq \inf A(x,q)$

Again

$$\inf A(x,q) \cap \alpha = \inf A((x^{-1})^{-1},q) \cap \alpha \leq \inf A(x^{-1},q) \cap \alpha$$

So

$$\inf A(x^{-1},q) \cap \alpha = \inf A(x,q) \triangle \beta$$

Similarly we can prove that

$$\sup A(x^{-1},q) \cap \alpha = \sup A(x,q) \triangle \beta$$

(2) $\inf A(e,q) \cap \alpha = \inf A(xx^{-1},q) \cap \alpha \leq S\{\inf A(x,q), \inf A(x^{-1},q)\} \triangle \beta$

And $\sup A(e,q) \cap \alpha = \sup A(xx^{-1},q) \cap \alpha \leq S\{\sup A(x,q), \sup A(x^{-1},q)\} \triangle \beta$

Proposition 3.6: Let μ and λ be two (α,β)- inf-sup Q-fuzzy group of G_1 , G_2 respectively and let Q be a homomorphism from G_1 to G_2. Then

(1) $Q(\mu,q)$ is (α,β)- inf-sup Q-fuzzy group of G_2

(2) $Q(\lambda,q)$ is (α,β)- inf-sup Q-fuzzy group of G_1

Proof : It is trivial

Remark: If μ is (α,β)- inf-sup Q-fuzzy group of G and K is subgroup of G then the restriction of μ to $K(\mu/K)$ is (α,β)- inf-sup Q-fuzzy group of K.

4. INF-SUP Normal Q-Fuzzy Group

Definition 4.1 : If μ is an (α,β)- inf-sup Q-fuzzy group of a group G then μ is called a (α,β)- inf-sup normal Q-fuzzy group of G if for all $x,y \in G$

$$\inf \mu(xy,q) \cap \alpha = \inf \mu(yx,q) \triangle \beta \text{ and}$$

$$\sup \mu(xy,q) \cap \alpha = \sup \mu(yx,q)\} \triangle \beta$$

Proposition 4.1: The Intersection of any two (α,β)- inf-sup normal Q-fuzzy groups of G is also (α,β)- inf-sup normal Q-fuzzy group of G.

Proof : Let A and B be any two (α,β)- inf-sup normal Q-fuzzy groups of G. By proposition 3.4 $A \cap B$ is an(α,β)- inf-sup Q-fuzzy group of G.

Let $x,y \in G$ then by definition

$$\inf (A \cap B)(xy,q) \cap \alpha = S\{\inf A(xy,q), \inf B(xy,q)\} \triangle \beta \text{ by definition}$$

$$= S\{\inf A(yx,q), \inf B(yx,q)\} \triangle \beta$$

$$= \inf A \cap B(yx,q) \triangle \beta$$

Similarly $\sup (A \cap B)(xy,q) \cap \alpha = \sup (A \cap B)(yx,q) \triangle \beta$

This shows that $A \cap B$ is(α,β)- inf-sup normal Q-fuzzy group of G.

Proposition 4.2: The Intersection of any arbitrary collection of (α,β)- inf-sup normal Q-fuzzy groups of a group G is also (α,β)- inf-sup normal Q-fuzzy group of G.

Proof:

Let $x,y \in G$ and $\alpha \in G$

$$\inf A(xy^{-1},q) \cap \alpha = \inf A(\alpha^{-1}xy^{-1}\alpha,q) \cap \alpha \text{ by definition}$$

$$= \inf A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha,q) \cap \alpha$$

$$= \inf (A(\alpha^{-1}x\alpha,q),A((\alpha^{-1}y\alpha)^{-1},q)) \cap \alpha$$

$$\leq S\{\inf (A(\alpha^{-1}x\alpha,q), \inf A((\alpha^{-1}y\alpha),q))\} \triangle \beta$$

$$= S\{\inf (A(x,q),A((y,q))\} \triangle \beta$$

Again

$$\sup A(xy^{-1},q) \cap \alpha = \sup A(\alpha^{-1}xy^{-1}\alpha,q) \cap \alpha \text{ by definition}$$

$$= \sup A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha,q) \cap \alpha$$

$$= \sup (A(\alpha^{-1}x\alpha,q),A((\alpha^{-1}y\alpha)^{-1},q)) \cap \alpha$$

$$\leq S\{\sup (A(\alpha^{-1}x\alpha,q), \sup A((\alpha^{-1}y\alpha),q))\} \triangle \beta$$

$$= S\{\sup (A(x,q),A((y,q))\} \triangle \beta$$

A is (α,β)- inf normal Q-fuzzy group of G.

5. Conclusion

In this paper, the concept of (α,β)- inf-sup Q-fuzzy set is generalized and there after we defined (α,β)- inf-sup Q-fuzzy group and a few of its properties are discussed.

Acknowledgement

The authors are highly grateful to the referees for their valuable comments & suggestions for improving the paper. Also we thankful to the editors & members of Science publishing group for providing proper guidance.

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Biography

R. Nagarajan has been working as Associate Professor and Head of Mathematics in J.J College of engineering &Technology, Trichy. He received his Ph.D degree in the field of *Fuzzy Techniques in Algebra* from Bharathidasan University, Trichy. He has Published more than 60 research articles in various International and National Journals. His area of interests are Fuzzy Algebraic Structures, Fuzzy soft Structures, Fuzzy Decision making and Fuzzy Optimizations.