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Comments on the Adomian Decomposition Methods applied to the KdV equation

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Abstract: Based on previous works, especially [1] and [2], we try in the present contribution to study some new aspects of the numerical solution of the KdV equation through the standard Adomian Decomposition Method. The use of the multistage Adomian Decomposition Method, applied to this equation, will be presented and discussed.

Keywords: KdV Equation - Standard and Multistage Adomian Decomposition Methods

1. The Adomian Decomposition Method (ADM)

Consider the KdV equation:

$$\frac{\partial u}{\partial t}+6u\frac{\partial u}{\partial x}+\frac{\partial^3 u}{\partial x^3}=0, \qquad u(x,0)=f(x)$$

which can be rewritten as follows:

$$\frac{\partial u}{\partial t}=-Ru-6N(u), \qquad u(x,0)=f(x)$$

where $R=\partial^3/\partial x^3$ represents the linear operator of the KdV equation and $N(u)=u\partial u/\partial x$ is the non-linear function. According to the ADM, the solution is expressed by:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t),$$

and the non-linear part:

$$N(u)=\sum_{n=0}^{\infty}A_n,$$

with:

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0},$$

By integrating with respect to time and using the initial conditions we have:

$$u(x,t)=f(x)-\int_0^t\left[Lu+6.N(u)\right]ds.$$

So,

$$u(x,t)=f(x)-\int_0^t\left[L\left(\sum_{n=0}^{\infty}u_n(x,s)\right)+6.\sum_{n=0}^{\infty}A_n\right]ds.$$

For the KdV equation, the Adomian polynomials can be expressed as follows:

$$A_n=\sum_{i=0}^{n}u_i\frac{\partial u_{n-i}}{\partial x}$$

This allows us to deduce $u_n(x,t)$, namely

$$\begin{cases}u_0(x,t)=f(x)\\ u_{n+1}(x,t)=-\int_0^t\left[Ru_n+A_n\right]ds, \quad n\geq 0\end{cases},$$

with the following initials conditions:

$$u(x,0)=\frac{1}{2}\operatorname{sech}^2\left(\frac{x}{2}\right)$$

We proceed in the following to compute $A_n(x,t)$ and $u_n(x,t)$ for $n\in[0,2]$ and try to determine the approximate solution $\tilde{U}_n(x,t)$ using the following formula:

$$\tilde{U}_n(x,t)=\sum_{i=0}^{n}U_i(x,t)$$
$$=f(x)-\int_0^t\left[L\left(\sum_{i=0}^{n}u_i(x,s)\right)+6.\sum_{i=0}^{n}A_i\right]ds.$$

2. Study of the Discrepancy of the ADM Applied to the KdV Equation

2.1. Calculation of Discrepancy

We present in this section some calculations by making comparison between the values of approximated function and the exact solution through space and time evolution. We also evaluate the percentage of the discrepancy over the exact solution.

Table 1: *Discrepancy calculation at X=0*

T	U_0	U_1	U_2	$\tilde{U}_2$	U_{Exact}	Δ	Δ/U_{Exact}
0	0,5	0	0	0,5	0,5	0	0,00%
0,1	0,5	0	-0,00125	0,49875	0,49875208	2,08039E-06	0,00%
0,2	0,5	0	-0,005	0,495	0,495033145	3,31454E-05	0,01%
0,3	0,5	0	-0,01125	0,48875	0,488916623	0,000166623	0,03%
0,4	0,5	0	-0,02	0,48	0,480521491	0,000521491	0,11%
0,5	0,5	0	-0,03125	0,46875	0,470007424	0,001257424	0,27%
0,6	0,5	0	-0,045	0,455	0,457568481	0,002568481	0,56%
0,7	0,5	0	-0,06125	0,43875	0,443425747	0,004675747	1,05%
0,8	0,5	0	-0,08	0,42	0,427819393	0,007819393	1,83%
0,9	0,5	0	-0,10125	0,39875	0,411000615	0,012250615	2,98%
1	0,5	0	-0,125	0,375	0,393223866	0,018223866	4,63%
1,1	0,5	0	-0,15125	0,34875	0,374739759	0,025989759	6,94%
1,2	0,5	0	-0,18	0,32	0,355788881	0,035788881	10,06%
1,3	0,5	0	-0,21125	0,28875	0,336596725	0,047846725	14,21%
1,4	0,5	0	-0,245	0,255	0,317369795	0,062369795	19,65%
1,5	0,5	0	-0,28125	0,21875	0,298292904	0,079542904	26,67%
1,6	0,5	0	-0,32	0,18	0,279527584	0,099527584	35,61%
1,7	0,5	0	-0,36125	0,13875	0,261211494	0,122461494	46,88%
1,8	0,5	0	-0,405	0,095	0,243458681	0,148458681	60,98%
1,9	0,5	0	-0,45125	0,04875	0,226360519	0,177610519	78,46%
2	0,5	0	-0,5	0	0,209987171	0,209987171	100,00%

Table 2: *Discrepancy calculation at X=0,5*

T	U_0	U_1	U_2	$\tilde{U}_2$	U_{Exact}	Δ	Δ/U_{Exact}
0	0,470007424	0	0	0,470007424	0,470007424	0	0,00%
0,1	0,470007424	0,011511359	-0,000963568	0,480555216	0,480521491	-3,37243E-05	-0,01%
0,2	0,470007424	0,023022718	-0,00385427	0,489175872	0,488916623	-0,000259249	-0,05%
0,3	0,470007424	0,034534077	-0,008672108	0,495869393	0,495033145	-0,000836248	-0,17%
0,4	0,470007424	0,046045436	-0,015417081	0,500635779	0,49875208	-0,001883699	-0,38%
0,5	0,470007424	0,057556795	-0,024089189	0,50347503	0,5	-0,00347503	-0,70%
0,6	0,470007424	0,069068154	-0,034688432	0,504387146	0,49875208	-0,005635066	-1,13%

T	U_0	U_1	U_2	$\tilde{U}_2$	U_{Exact}	Δ	$Δ/U_{Exact}$
0,7	0,470007424	0,080579513	-0,047214811	0,503372127	0,495033145	-0,008338981	-1,68%
0,8	0,470007424	0,092090872	-0,061668324	0,500429972	0,488916623	-0,011513349	-2,35%
0,9	0,470007424	0,103602231	-0,078048973	0,495560683	0,480521491	-0,015039191	-3,13%
1	0,470007424	0,11511359	-0,096356756	0,488764258	0,470007424	-0,018756833	-3,99%
1,1	0,470007424	0,126624949	-0,116591675	0,480040698	0,457568481	-0,022472217	-4,91%
1,2	0,470007424	0,138136308	-0,138753729	0,469390003	0,443425747	-0,025964256	-5,86%
1,3	0,470007424	0,149647667	-0,162842918	0,456812173	0,427819393	-0,02899278	-6,78%
1,4	0,470007424	0,161159026	-0,188859242	0,442307208	0,411000615	-0,031306593	-7,62%
1,5	0,470007424	0,172670385	-0,216802702	0,425875107	0,393223866	-0,032651241	-8,30%
1,6	0,470007424	0,184181744	-0,246673296	0,407515872	0,374739759	-0,032776113	-8,75%
1,7	0,470007424	0,195693102	-0,278471026	0,387229501	0,355788881	-0,03144062	-8,84%
1,8	0,470007424	0,207204461	-0,31219589	0,365015995	0,336596725	-0,028419271	-8,44%
1,9	0,470007424	0,21871582	-0,34784789	0,340875355	0,317369795	-0,02350556	-7,41%
2	0,470007424	0,230227179	-0,385427025	0,314807579	0,298292904	-0,016514675	-5,54%

Table 3: *Discrepancy calculation at X=1*

T	U_0	U_1	U_2	$\tilde{U}_2$	U_{Exact}	Δ	$Δ/U_{Exact}$
0	0,393223866	0	0	0,393223866	0,393223866	0	0,00%
0,1	0,393223866	0,01817155	-0,000353256	0,41104216	0,411000615	-4,15455E-05	-0,01%
0,2	0,393223866	0,036343099	-0,001413023	0,428153942	0,427819393	-0,000334549	-0,08%
0,3	0,393223866	0,054514649	-0,003179302	0,444559213	0,443425747	-0,001133466	-0,26%
0,4	0,393223866	0,072686198	-0,005652093	0,460257972	0,457568481	-0,002689491	-0,59%
0,5	0,393223866	0,090857748	-0,008831395	0,475250219	0,470007424	-0,005242795	-1,12%
0,6	0,393223866	0,109029297	-0,012717209	0,489535955	0,480521491	-0,009014463	-1,88%
0,7	0,393223866	0,127200847	-0,017309534	0,503115179	0,488916623	-0,014198555	-2,90%
0,8	0,393223866	0,145372396	-0,022608372	0,515987891	0,495033145	-0,020954746	-4,23%
0,9	0,393223866	0,163543946	-0,02861372	0,528154092	0,49875208	-0,029402012	-5,90%
1	0,393223866	0,181715495	-0,035325581	0,539613781	0,5	-0,039613781	-7,92%
1,1	0,393223866	0,199887045	-0,042743952	0,550366959	0,49875208	-0,051614879	-10,35%
1,2	0,393223866	0,218058594	-0,050868836	0,560413625	0,495033145	-0,06538048	-13,21%
1,3	0,393223866	0,236230144	-0,059700231	0,569753779	0,488916623	-0,080837156	-16,53%
1,4	0,393223866	0,254401693	-0,069238138	0,578387422	0,480521491	-0,097865931	-20,37%
1,5	0,393223866	0,272573243	-0,079482556	0,586314553	0,470007424	-0,116307129	-24,75%
1,6	0,393223866	0,290744793	-0,090433486	0,593535173	0,457568481	-0,135966692	-29,72%
1,7	0,393223866	0,308916342	-0,102090928	0,600049281	0,443425747	-0,156623534	-35,32%
1,8	0,393223866	0,327087892	-0,114454881	0,605856877	0,427819393	-0,178037484	-41,62%
1,9	0,393223866	0,345259441	-0,127525346	0,610957962	0,411000615	-0,199957347	-48,65%
2	0,393223866	0,363430991	-0,141302322	0,615352535	0,393223866	-0,222128669	-56,49%

As we can easily observe, the convergence is limited relatively to the time parameter, and it's shown that it diverge consistently beyond the value of τ_0 [1] defining the time limit of convergence.

2.2. Determination of the Contribution of Each Element of the Adomian Polynomial

In order to obtain a behavior of the evolving manner of the elements of the Adomian polynomial, we elaborate the calculation for $x = 1$, to precise the percentage of each one separately, as exposed in the following table:

Table 4: *The evolving of Ui*

T	U_0	U_1	U_2	U_3	U_4	U_5
0	1	0,00%	0,00%	0,00%	0,00%	0,00%
0,1	95,66%	4,42%	-0,09%	0,01%	0,00%	0,00%
0,2	91,80%	8,48%	-0,36%	0,05%	0,00%	0,00%
0,3	88,33%	12,25%	-0,77%	0,16%	-0,01%	0,00%
0,4	85,16%	15,74%	-1,22%	0,34%	-0,02%	0,01%
0,5	82,23%	19,00%	-1,85%	0,64%	-0,06%	0,03%
0,6	79,50%	22,04%	-2,57%	1,06%	-0,11%	0,07%
0,7	76,92%	24,88%	-3,39%	1,63%	-0,20%	0,16%
0,8	74,44%	27,52%	-4,28%	2,36%	-0,33%	0,29%
0,9	72,03%	29,96%	-5,24%	3,25%	-0,51%	0,51%
1	69,67%	32,20%	-6,26%	4,31%	-0,75%	0,84%
1,1	67,32%	34,22%	-7,32%	5,54%	-1,07%	1,30%
1,2	64,95%	36,02%	-8,40%	6,95%	-1,46%	1,94%
1,3	62,56%	37,58%	-9,50%	8,50%	-1,93%	2,79%
1,4	60,11%	38,89%	-10,58%	10,21%	-2,50%	3,88%
1,5	57,59%	39,92%	-11,64%	12,03%	-3,15%	5,25%
1,6	55,01%	40,67%	-12,65%	13,94%	-3,90%	6,93%
1,7	52,35%	41,13%	-13,59%	15,91%	-4,73%	8,93%
1,8	49,63%	41,28%	-14,44%	17,91%	-5,64%	11,27%
1,9	46,85%	41,13%	-15,19%	19,88%	-6,60%	13,94%
2	44,03%	40,69%	-15,82%	21,79%	-7,62%	16,93%

2.3. Interpretation of the Results

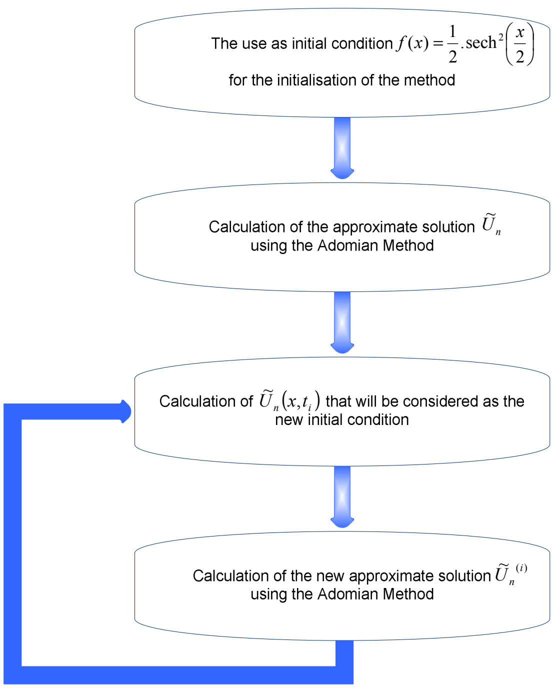
As described in the previous table, the contribution of the first elements of U_i, is more important at the beginning, while we progress in the time calculation, and it gradually decreases in favor of the other U_i elements. The U_i are evolving in positive and negative values and progressively their sum $\tilde{U}$ diverges from the exact solution.

So, the divergence cannot be attributed to special categories of U_i, which we can avoid by implementing a filter, but it is a mass fact. From where the interest to develop other technique, that allow us to overcome the limitations of the standard ADM.

3. Multistage Adomian Decomposition Method (MS-ADM) for KdV Equation

3.1. Principe of the Method

To simplify, we present the MS-ADM by the following diagram:



Based on this principle, we can write

$$f(x) = \tilde{U}_2(x,t)\Big|_{t_0=0,1}$$

Hence, after calculation we found:

$$f(x) = 2{,}21.e^{x}.\frac{\left(e^{2x} + 1{,}79186.e^{x} + 0{,}819005\right)}{\left(e^{x}+1\right)^{4}}$$

Using the previous backgrounds, we have to recalculate the approximated function $\tilde{U}_2^*(x,t)$ based on the new expression of the initial condition $f(x)$, for: $t = t_2$.

As made before for the standard Adomian decomposition method and also taking into account the new formulation, we have to precise the following expressions:

$$A_0^* = U_0^* \frac{\partial U_0^*}{\partial x} \quad ;$$

$$U_1^* = -\int_0^t \left[R U_0^* + A_0^* \right] ds \quad ;$$

$$A_1^* = U_0^* \frac{\partial U_1^*}{\partial x} + U_1^* \frac{\partial U_0^*}{\partial x} \quad ;$$

$$U_2^* = -\int_0^t \left[R U_1^* + A_1^* \right] ds \ .$$

3.2. Discrepancy Study

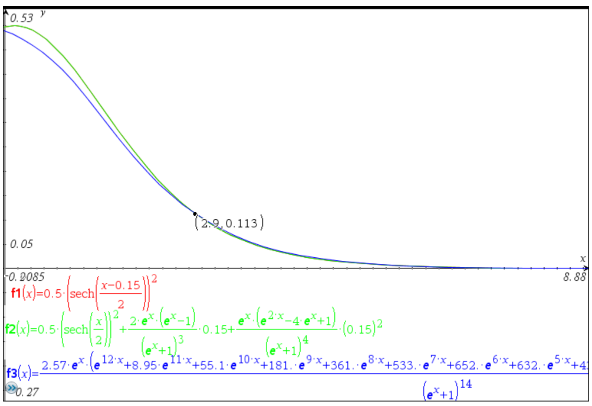
After calculation for $t_2 = 0{,}15$, and in comparison with the standard Adomian decomposition method, we present the following results in the table below:

Table5: *Discrepancies calculation*

	Values of X				
	0	**0,25**	**0,5**	**0,75**	**1**
Exact Solution	0,4971980	0,4987521	0,4849948	0,4575685	0,4195456
ADM	0,4971875	0,4988099	0,4851064	0,4577083	0,4196864
Δ/ADM	0,0000105	-0,0000578	-0,0001116	-0,0001399	-0,0001407
MS-ADM	0,4881485	0,4734446	0,4525302	0,4256124	0,3918466
Δ/MS-ADM	0,0090495	0,0253075	0,0324646	0,0319561	0,0276990

For establishing a better evaluation of these results, we will elaborate a graphical representation of these functions:

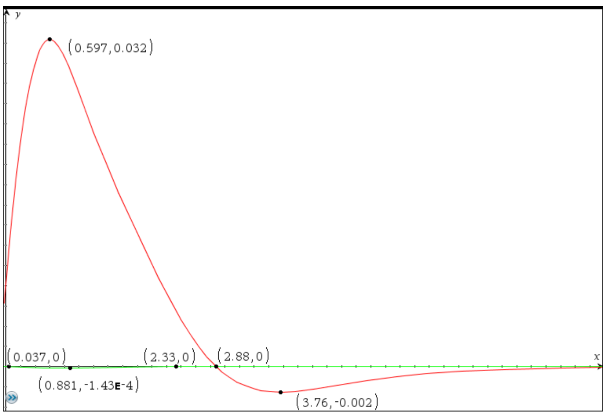
- f_1 : the exact solution;
- f_2 : approximate solution using the standard Adomian decomposition method;
- f_3 : approximate solution using the Multistage Adomian decomposition method.



Graph 1: *Comparative graphical representations*

And for further richest visualization, we plot also these function:

$$\begin{cases} F_{\Delta/ADM} = U_{Exact} - \tilde{U}_{ADM} \\ F_{\Delta/MS-ADM} = U_{Exact} - \tilde{U}_{MS-ADM} \end{cases}$$



Graph 2: *Discrepancies representation*

Where:

- $F_{\Delta/ADM}$: is represented by the green color ;
- $F_{\Delta/MS-ADM}$: is represented by the red color.

3.3. Discussion

Based on the previous numerical results and plotting graphical representations, we can deduce the following results:

1 Asymptotically, the both approximated functions converge to the exact solution for this time value;

2 The discrepancy graphic for $F_{\Delta/MS-ADM}$, show that this function evolve between +0,032 and -0,002 and have single null value at $x = 2,88$ and after that conserve a negative value;

3 The discrepancy graphic for $F_{\Delta/ADM}$, show that this function have two null values for $x = 0,037$ and $x = 2,33$ and minimum for $x = 0,881$, which it means that this function conserve after his second zero, a positive value.

Therefore, the both approximated functions are reaching the exact solution, but not by the same manner. The positive evolution for the function $F_{\Delta/ADM}$, lead for the next estimation processes of calculation of the Adomian decomposition method, to the amplification of the approximation error, contributing to it divergence.

Contrariwise, the Multistage Adomian decomposition method have a more important positive discrepancy at the beginning (in comparison with the Adomian decomposition method), but it becomes negative for the rest of the spatial parameter, while remaining near of 0.

4. Conclusion

Through this work, we were able to evolve with precision the nature and the progression of the discrepancy between the exact solution of KdV equation and the approximated one using the standard Adomian decomposition method.

We have also, applied the Multistage Adomian decomposition method to the resolution of the KdV equation, which it allows us to elaborate a non-divergent method with an acceptable accuracy level. Hence, the amplification of the approximation error, as it occurs for the standard Adomian decomposition method, is avoided.

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